DIRECTION FINDING AFTER BLIND IDENTIFICATION OF SOURCES STEERING VECTORS: THE BLIND-MAXCOR AND BLIND-MUSIC METHODS

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ABSTRACT

To find the direction of arrival (DOA) of P sources impinging on an array of N sensors, actual second and fourth order direction finding (DF) methods try to solve a P-dimensional problem from the statistics of the data. The purpose of this paper is to present a new approach of DF, based on a first step of blind identification of sources steering vectors, aiming, for some of these methods, at reducing the problem dimension before DF. Two new methods, the Blind-MAXCOR and the Blind-MUSIC methods, are proposed and their performance are compared to that of MUSIC method.

1. INTRODUCTION

Actual second order DF methods such as superresolution methods (Capon, AR..), subspace methods (MUSIC, MIN-NORM, ESPRIT..) or Maximum Likelihood (ML) methods and their higher order extensions such as, for example, 4-th MUSIC or 4-th ESPRIT, try to find the DOA of P sources impinging on an array of sensors by solving a P-dimensional problem from the statistics of the data. Indeed, for most of these techniques, the DOA estimation of the P sources is deduced from the estimation of either the P maxima of a one (or two) variable function called pseudo-spectrum (Capon, MUSIC..) or the maximum of a P variable function (ML methods). Besides, for ESPRIT techniques, the DOA information is deduced from the P generalized eigenvalues of a couple of data correlation matrix. The multidimensionality of these DF approaches generates interaction between the sources DOA estimates, especially for short observation interval, when the sources are not well angularly separated or in the presence of temporally correlated sources, which may degrade the estimation accuracy.

The purpose of this paper is to present a new approach of DF, based on a first step of blind identification of sources steering vectors, aiming, for some of these methods, at reducing the problem dimension before DF. Such a DF concept is now possible due to the development, these last years, of several blind sources separation methods which allow the blind identification of sources steering vectors [1-2]. Two new methods of DF, corresponding to the Blind-MAXCOR and the Blind-MUSIC methods respectively, are proposed in this paper and their performance are compared to that of the classical second order MUSIC method [3].

2. HYPOTHESES

We consider, in this paper, an array of N narrow-band (NB) omnidirectional sensors and we call $\boldsymbol{x}(t)$ the vector of the complex amplitudes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of P stationary sources corrupted by a noise. Under these assumptions, the observation vector can be written as follows

$$\boldsymbol{x}(t) = \sum_{i=0}^{P} m_i(t)\, \boldsymbol{a}_i + \boldsymbol{b}(t) \stackrel{\Delta}{=} A\, \boldsymbol{m}(t) + \boldsymbol{b}(t) \qquad (2.1)$$

where $\boldsymbol{b}(t)$ is the noise vector, $\boldsymbol{m}(t)$ is the vector which components $m_i(t)$ are the complex amplitudes of the sources, A is the $(N\xi P)$ matrix of the sources steering vectors $\boldsymbol{a}_i$, which contains the information about the DOA of the sources.

The actual second and fourth order DF methods aim at finding the direction of the P sources from the information contained in the correlation matrix $R_x \stackrel{\Delta}{=} \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^{\dagger}]$ or in the quadricovariance Q_x (matrix of the 4-th order cumulants Cum[$x_i(t)$, $x_j(t)^*$, $x_k(t)^*$, $x_l(t)$]) of the data respectively where * means conjugation. In the presence of a gaussian spatially white noise, the matrix R_x and Q_x are defined respectively by

$$R_x = A R_m A^\dagger + \eta_2 \mathrm{I} \triangleq R_s + \eta_2 \mathrm{I} \tag{2.3}$$

$$Q_x = [A \otimes A^*] \, Q_m \, [A \otimes A^*]^\dagger \tag{2.4}$$

where η_2 is the mean power of the noise per sensor, R_m and Q_m are the correlation matrix and the quadricovariance of the vector $\boldsymbol{m}(t)$ respectively, $R_s \triangleq A R_m A^\dagger$ is the correlation matrix of the received sources and $\otimes$ is the Kronecker product.

3. THE BLIND-MAXCOR AND BLIND-MUSIC METHODS

The Blind-MAXCOR and Blind-MUSIC methods are two new DF methods aiming at finding the DOA of the *P* sources from a blind estimate of their steering vectors.

3.1 Blind identification of the sources steering vectors

Over the last decade, second and higher order blind sources separation methods have been strongly developed [1-2], [4-5] to perform spatial filtering when no information on the sources is a priori available. Some of these methods [1-2] are qualified of *indirect methods* since their implementation requires an intermediate stage of blind identification of the sources steering vectors before the sources separation stage. In most cases, the blind identification of the sources steering vectors requires a prewhitening of the datas which reduces the search to a unitary matrix *U*, simpler to handle. Then, the problem consists to find the matrix *U* which optimizes a blind criterion constructed from the outputs of the separator. For example, Cardoso and Souloumiac propose to find the unitary matrix *U* which minimizes the sum of the squared fourth-order cross-cumulants of the outputs with distinct first and second indices [2], whereas Comon proposes to find the matrix *U* which maximizes a contrast function [1].

Recently, it has been shown in [6-7] that Comon's and Cardoso-Souloumiac's methods are powerful tools for the blind separation of statistically independent sources in radiocommunications contexts. This result allows to think that these methods are also powerful tools for the blind identification of sources steering vectors and that it could be interesting to extract the information about the DOA of the sources from these blindly estimated vectors.

3.2 The Blind-MAXCOR method

After a first stage of blind identification of the *P* sources steering vectors $\boldsymbol{a}_i \triangleq \boldsymbol{a}(\theta_i)$, $1 \leq i \leq P$, the blind-MAXCOR method (MAXimum spatial CORrelation method after Blind identification of the sources steering vector) consists, for each detected sources *i*, to solve the one dimensional problem consisting to find the direction θ_i which maximizes the spatial correlation coefficient $\alpha_i(\theta_i)$ defined by

$$\alpha_i(\theta_i) \triangleq [\boldsymbol{a}(\theta_i)^\dagger \hat{\boldsymbol{a}}_i] \, / \, [\boldsymbol{a}(\theta_i)^\dagger \boldsymbol{a}(\theta_i)]^{1/2} \, [\hat{\boldsymbol{a}}_i^\dagger \hat{\boldsymbol{a}}_i]^{1/2} \tag{3.1}$$

where $\hat{\boldsymbol{a}}_i$ is a blind estimation of the steering vector $\boldsymbol{a}_i$ of the source *i*.

3.3 The Blind-MUSIC method

After a first stage of blind identification of the *P* sources steering vectors $\boldsymbol{a}_i$, $1 \leq i \leq P$, by the *P* vectors $\hat{\boldsymbol{a}}_i$, the blind-MUSIC method consists to find the *P* DOA θ_i which maximize the pseudo-spectrum $\mathrm{P}(\theta_i)$ defined by

$$\mathrm{P}(\theta_i) \triangleq 1 \, / \, \boldsymbol{a}(\theta_i)^\dagger \Pi_b \, \boldsymbol{a}(\theta_i) \tag{3.2}$$

where Π_b is the orthogonal projector on the space which is orthogonal to the space spanned by the vectors $\hat{\boldsymbol{a}}_i$. This projector is defined by

$$\Pi_b \triangleq \mathrm{I} - \hat{A} \, [\hat{A}^\dagger \hat{A}]^{-1} \hat{A}^\dagger \tag{3.3}$$

where $\hat{A}$ is the (N x P) matrix which columns are the vectors $\hat{\boldsymbol{a}}_i$, $1 \leq i \leq P$.

4. PERFORMANCE IN THE PRESENCE OF INDEPENDENT SOURCES

We analyse, in this section, the behaviour of the Blind-MAXCOR and Blind-MUSIC methods in the presence of multiple statistically independent sources and we compare the results to those obtained with the classical second order MUSIC method [3]. The steering vectors of the sources are assumed to be blindly estimated by the Cardoso-Souloumiac method of sources separation [2].

The performance of the Blind-MAXCOR and Blind-MUSIC DF methods are directly related to the quality of the sources steering vectors blind identification. This quality of blind identification has been indirectly studied recently in [6-7] for the Comon's and Souloumiac-Cardoso's methods of higher order blind sources separation, in terms of sources separation quality, with respect to a Signal to Interference plus Noise Ratio (SINR) criterion. The good results obtained with the Blind-MAXCOR and Blind-MUSIC methods for the DF of statistically independent sources confirm those obtained in [6-7] for sources separation.

More precisely, it can be shown that in the presence of multiple statistically independent sources which fourth-order normalized cumulant is not far from –1 (QPSK, FSK, MSK sources...) or –2 (BPSK, ASK sources..) and for a given number of independent snapshots, the performance of the Blind-MAXCOR and Blind-MUSIC methods are in most cases greater than or equal to that of MUSIC as long as the background noise is gaussian.

In particular, when all the sources are weak (Signal to Noise Ratio (SNR) weaker than or equal to 0 dB), the Blind-MAXCOR, MUSIC and Blind-MUSIC methods have approximaly the same performance. On the other hand, for sources which are not too weak, the Blind-MUSIC and MUSIC methods have almost the same performance whereas the Blind-MAXCOR method is the best one, especially for high value of the spatial correlation coefficient between the sources. The previous results are illustrated at figures 1 to 3 which show the sources DOA Mean Square Error (MSE), averaged over 10 realizations, obtained with the Blind-MAXCOR, Blind-MUSIC and MUSIC methods, for several values of the SNR, the angular separation of the sources and the number of independent snapshots *K*, when two statistically independent BPSK sources are impinging on a $\lambda/2$ Uniformaly spaced Linear Array (ULA) of 4 sensors, where λ is the wavelength.

Note that the previous results still hold in the presence of a gaussian source at most. Nevertheless, in the presence of a non gaussian noise or when the fourth-order normalized cumulant of the sources tends to zero, the performance of the Blind DF methods degrades with respect to that of MUSIC.

	MUSIC		Blind-MAXCOR		Blind-MUSIC	
K	s1	s2	s1	s2	s1	s2
50	3.0	2.2	1.0	1.8	3.1	2.0
200	0.6	0.4	0.5	0.4	0.7	0.5
500	0.1	0.2	0.1	0.2	0.1	0.2
1000	0.1	0.1	0.1	0.1	0.1	0.1

Fig.1 - *DOA MSE for a ULA, N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = 30°$, *SNR1 = SNR2 = 0 dB*

	MUSIC		Blind-MAXCOR		Blind-MUSIC	
K	s1	s2	s1	s2	s1	s2
50	0.2	0.2	0.1	0.1	0.2	0.2
200	0.0	0.0	0.0	0.0	0.0	0.0

Fig.2 - *DOA MSE for a ULA, N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = 30°$, *SNR1 = 10 dB, SNR2 = 10 dB*

	MUSIC		Blind-MAXCOR		Blind-MUSIC	
K	s1	s2	s1	s2	s1	s2
50	X	X	0.2	0.2	X	X
200	X	X	0.1	0.0	X	X
500	1.0	0.9	0.0	0.0	0.7	0.9
1000	0.3	0.3	0.0	0.0	0.3	0.3

Fig.3 - *DOA MSE for a ULA, N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = -23°$, *SNR1 = SNR2 = 10 dB*

5. PERFORMANCE IN THE PRESENCE OF NON INDEPENDENT SOURCES

In the presence of 2-th and 4-th order correlated sources, the previous results still approximately hold as long as the 2-th order temporal correlation coefficient between the sources remains low, and typically lower than 0.5.

However, for high values of the temporal correlation coefficient between the sources, as long as the noise is gaussian and the 4-th cumulant of the sources is not far from –1 or –2, the Blind-MUSIC and MUSIC methods give always better performance than the Blind-MAXCOR method. Besides, in these cases, the MUSIC and Blind-MUSIC methods perform similarly for low values of the spatial correlation coefficient between the sources whereas the performance of the Blind-MUSIC method become better than that of MUSIC for high spatial correlation between the sources.

	MUSIC		Blind-MAXCOR		Blind-MUSIC	
ρ	s1	s2	s1	s2	s1	s2
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0
0.8	0.1	0.1	0.3	0.3	0.1	0.1
0.9	0.1	0.2	0.3	0.4	0.1	0.2
1.0	X	X	0.0	0.0	X	X

Fig.5 - *DOA MSE for a ULA, N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = 30°$, *SNR = 10 dB*, $K_{noïse} = 200$

	MUSIC		Blind-MAXCOR		Blind-MUSIC	
ρ	s1	s2	s1	s2	s1	s2
0.0	0.4	0.5	0.2	0.1	0.4	0.5
0.5	0.9	0.6	2.1	1.4	0.4	1.0
0.8	X	X	X	X	2.0	0.9
0.9	X	X	X	X	6.0	6.8
1.0	X	X	X	X	X	X

Fig.6 - *DOA MSE for a ULA, N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = -23°$, *SNR = 10 dB*, $K_{noïse} = 500$

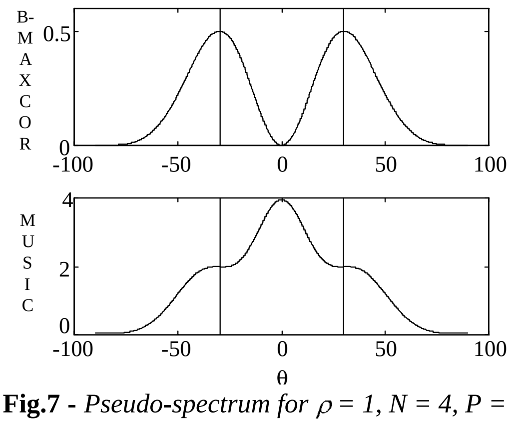

Fig.7 - *Pseudo-spectrum for* $\rho = 1$, *N = 4, P = 2*
$\theta_1 = -30°$, $\theta_2 = 30°$, *SNR = 10 dB*, $K_{noïse} = 200$

These results are illustrated at figure 5 and 6 which show, for several values of the temporal correlation coefficient ρ between the sources, the sources DOA Mean Square Error (MSE), averaged over 10 realizations, obtained with the three methods when two BPSK sources are impinging on the array.

Note finally that for coherent sources (ρ = 1), the Blind-MAXCOR method preserves the sources DOA information as long as the sources are well angularly separated, which is not the case for the MUSIC and Blind-MUSIC methods as it is illustrated at figure 7.

6.CONCLUSION

In this paper, a new approach of DF, based on a first step of blind identification of sources steering vectors, has been presented. Two new methods, corresponding to the Blind-MAXCOR and the Blind-MUSIC methods, have been developed. These methods give performance which are greater than or equal to that of MUSIC for low temporal correlation between the sources, whereas for high temporal correlation the Blind-MUSIC method seems to be the best one, except for coherent sources where the Blind-MAXCOR method seems to be much more powerful than the others.

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