DESIGN OF PULSE SHAPING FILTERS AND THEIR APPLICATIONS IN RADIO SYSTEMS

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ABSTRACT

Partial-response signaling is known as correlative level coding wherein the constraint on waveforms is relaxed so as to allow a controlled amount of ISI. In this paper, the Lagrange multiplier approach, which is easy to incorporate both time- and frequency-domain constraints by minimizing a quadratic measure of the error in the design bands, is applied to design a large class of such digital filters for communication in this paper. Also, the iterative Lagrange multiplier approach combining the Lagrange multiplier approach and a tree search algorithm is proposed for designing discrete coefficient pulse shaping FIR digital filters. System experiments such as an SSB radio system using partial response signaling are demonstrated to present the usefulness of the proposed algorithm.

1 INTRODUCTION

In communication systems, the channel is always bandlimited. A bandlimited channel disperses a pulse waveform passing through it. When the channel bandwidth is close to the signal bandwidth, the spreading will exceed a symbol duration and cause signal pulses to overlapping is called intersymbol interference (ISI) [1,2]. Like any other source of interference, ISI cause system degradation. In practice, the characteristic is usually specified, and the problem remained is to determine a transmitting filter and a receiving filter, such that the ISI of the pulses are minimized at the output of the receiving filter. To design such filters, some constraints in both time- and frequency-domains must be considered.

In this paper, the Lagrange multiplier approach is applied to design such digital filters for communication, which is proposed by minimizing a quadratic measure of the error in the design bands. Among several quadratic programming methods, the Lagrange multiplier approach is easy to incorporate both time- and frequency-domain constraints [3]. In this paper, several digital filters for communication will be designed such as Nyquist filters, M-th band filters, and partial response filters. In Section II, we will use the Lagrange multiplier approach to design communication filters.

Multiplication, in particular, is extremely time consuming. So if a multiplication operation could be replaced by only a few shift operations, then the complexity of the entire system could be reduced dramatically, such that a fast real-time system becomes feasible. In Section II, a new method is also proposed for the design of pulse shaping filters. The method bases on the iterative procedures combining the Lagrange multiplier approach and a tree search algorithm. For each branch, the conventional Lagrange multiplier approach is used to optimize the remaining unquantized coefficients of the designed filter in the least-squares sense when one or more of the coefficients takes on discrete values. And for each node, one more appropriate coefficient is chosen to quantize. These procedures are repeated until all filter coefficients are quantized.

Moreover, the designed pulse shaping filters are applied in radio systems such as SSB radio systems. The simulations are based on LabVIEW software package, which provides excellent visibility of the designed filters in communication systems. Through several experiments, good performances of filtering and ISI elimination can be obtained.

2 DESIGN OF PULSE SHAPING FIR FILTERS AND SYSTEM SIMULATIONS

A typical FIR digital filter can be characterized by the transfer function

$$\hat{H}(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \tag{1}$$

where N is the filter length of the impulse response $h(n)$, and its frequency response is represented by

$$\hat{H}(e^{j\omega}) = H(\omega) e^{j[\frac{L\pi}{2} - \frac{N-1}{2}\omega]} \tag{2}$$

where $H(\omega)$ is the amplitude response which is a real-valued function and L is equal to 0 or 1. According to the symmetric properties of the impulse response and the filter length, there are four cases to be considered [4].

The least-squares approach to linear-phase filter designs is to formulate an objective error function as below

$$E = \int_R W(\omega)[D(\omega) - H(\omega)]^2 dw \qquad (3)$$

where $D(\omega)$ is the desired amplitude response, $W(\omega)$ is the weighting function and the design bands R is $[0, \frac{\pi}{M}] \cup [\frac{\pi}{M}, \pi]$. It is general that the amplitude response $H(\omega)$ can be represented in vector product form as below :

$$H(\omega) = \mathbf{A}^t\mathbf{C}(\omega) = \mathbf{C}^t(\omega)\mathbf{A} \qquad (4)$$

where t denotes the transpose operation, $\mathbf{A}$ is the coefficient vector and $\mathbf{C}$ is the kernel vector. Substituting Eq.(4) into Eq.(3)

$$E = s + \mathbf{P}^t\mathbf{A} + \mathbf{A}^t\mathbf{Q}\mathbf{A} \qquad (5)$$

where

$$s = \int_R W(\omega)D^2(\omega)d\omega, \qquad (6)$$

$$\mathbf{P} = \int_R -2W(\omega)D(\omega)\mathbf{C}(\omega)d\omega \qquad (7)$$

and

$$\mathbf{Q} = \int_R W(\omega)\mathbf{C}(\omega)\mathbf{C}^t(\omega)d\omega. \qquad (8)$$

For designing pulse shaping filters, it is general to incorporate certain time-domain and/or frequency-domain constraints. It is noted that some of the filter coefficients are fixed which can be represented in a constrained equation. For example, the coefficients $a(0), a(2)$ and $a(4)$ in $\mathbf{A}$ are required to fix in $\frac{1}{2}, 0$ and 0 respectively, the constrained equation is given by

$$\mathbf{B}^t\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots \end{bmatrix} \mathbf{A} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}. \qquad (9)$$

Hence, the design of pulse shaping filters can be formulated as a quadratic programming problem :

$$\begin{array}{ll} \text{Minimize} & E = s + \mathbf{P}^t\mathbf{A} + \mathbf{A}^t\mathbf{Q}\mathbf{A} \\ \text{subject to} & \mathbf{B}^t\mathbf{A} = \mathbf{G}. \end{array} \qquad (10)$$

So, the Lagrange multiplier approach can be applied to design arbitrary pulse shaping filters, and the closed-form solution is given by [3]

$$\begin{aligned} \mathbf{A} &= \mathbf{Q}^{-1}\mathbf{B}(\mathbf{B}^t\mathbf{Q}^{-1}\mathbf{B})^{-1}\mathbf{G} \\ &+ \frac{1}{2}\mathbf{Q}^{-1}[\mathbf{B}(\mathbf{B}^t\mathbf{Q}^{-1}\mathbf{B})^{-1}\mathbf{B}^t\mathbf{Q}^{-1} - \mathbf{I}]\mathbf{P} \end{aligned} \qquad (11)$$

where $\mathbf{I}$ is an identity matrix with proper dimensions.

In this paper, we only take the design of Class 4 pulse shaping filters as examples.

The desired amplitude response of Class 4 pulse shaping filters are :

$$D_4(\omega) = \begin{cases} sin(M\omega), & |\omega| < \frac{\pi}{M}, \\ 0, & \text{otherwise,} \end{cases} \qquad (12)$$

where M is the intersymbol duration, so we can use **Case 3** linear-phase FIR filters to design such filters. For **Case 3** design,

$$\mathbf{A} = \begin{bmatrix} a(1) & a(2) & a(3) & \ldots & a(\frac{N-1}{2}) \end{bmatrix}^t \qquad (13)$$

and

$$\begin{aligned} \mathbf{C}(\omega) &= [\sin(\omega) \quad \sin(2\omega) \\ &\qquad \sin(3\omega)\ldots\sin(\frac{N-1}{2}\omega)]^t, \end{aligned} \qquad (14)$$

where

$$a(n) = 2h(\frac{N-1}{2} - n), \quad n = 1, 2, \ldots, \frac{N-1}{2}. \qquad (15)$$

For eliminating ISI, the following constraints should be considered :

$$\begin{cases} a(M) = \frac{1}{M}, \\ a(iM) = 0, \quad i \geq 2, \end{cases} \qquad (16)$$

and the corresponding constrained equation is same as Eq.(9), where for example $M = 4$,

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 1 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 1 & \cdots \\ 0 & 0 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}_{\frac{N-1}{2} \times \lfloor \frac{N-1}{2M} \rfloor} \qquad (17)$$

and

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{M} & 0 & 0 & \cdots \end{bmatrix}^t_{\lfloor \frac{N-1}{2M} \rfloor \times 1}. \qquad (18)$$

So, the Lagrange multiplier approach can be applied to design such filters.

Before designing a finite-precision filters, an infinite-precision coefficient prototype should be obtained in advance. In this paper, the prototype is designed by the Lagrange multiplier approach in the previous section which is given by Eq.(11) with coefficient constraints as

$$\mathbf{B}^t\mathbf{A} = \mathbf{G}. \qquad (19)$$

Once the continuous coefficient filter is obtained, the key operation in the discrete optimization algorithm is to optimize the unquantized coefficients, except the constrained coefficients, when some of the coefficients take on discrete values. Notice that the constraints of some coefficients to discrete values can be represented in matrix form likes Eq.(19). For example, the coefficients $a(2), a(5)$ and $a(3)$ should be constrained to discrete values $a_d(2), a_d(5)$ and $a_d(3)$ respectively, which can be represented by the constrained equation

$$\hat{\mathbf{B}}^t \mathbf{A} = \hat{\mathbf{G}} \tag{20}$$

or

$$\begin{bmatrix} & & & & \mathbf{B^t} & & & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \end{bmatrix} \mathbf{A} = \begin{bmatrix} \mathbf{G} \\ a_d(2) \\ a_d(5) \\ a_d(3) \end{bmatrix}. \tag{21}$$

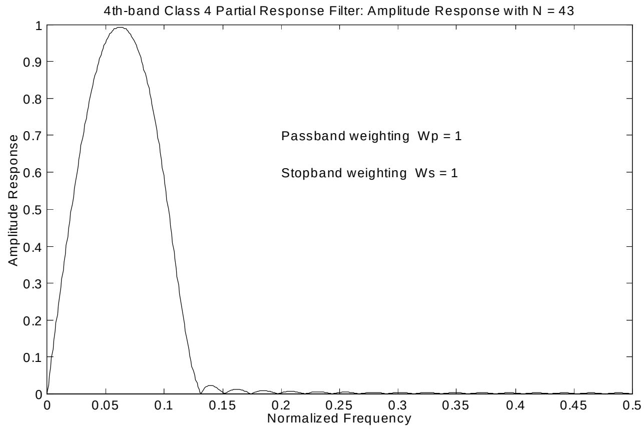
where the $\hat{\mathbf{B}}$ and $\hat{\mathbf{G}}$ are the extensions of $\mathbf{B}$ and $\mathbf{G}$, respectively.

So the Lagrange multiplier approach can be applied iteratively as an effective discrete optimization algorithm [5,6].

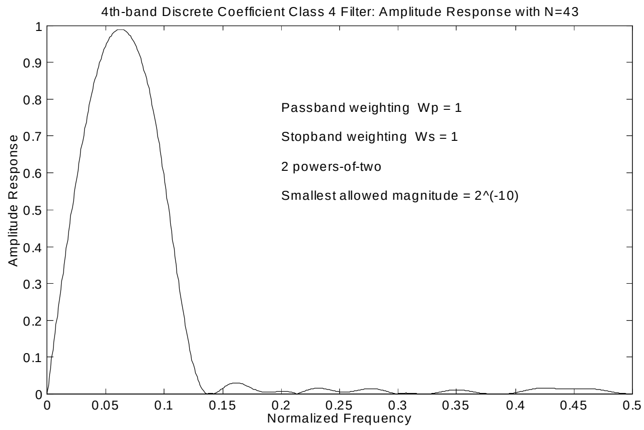
For demonstration, when $N = 43$, $M = 4$, $W(\omega) = 1$ for $\omega \in R$, the amplitude responses of the obtained continuous and discrete coefficient Class 4 partial response filters are shown in Fig.1 (a) and (b), respectively. In Fig.2 the block diagram of an experimental system is illustrated, and the corresponding block diagram in LabVIEW is shown in Fig.3.

3 CONCLUSIONS

In this paper, an effective method has been proposed for designing infinite-wordlength and finite-wordlength Nyquist filters, M-th band filters and partial response filters with additional time- or frequency-domain constraints. The method associates successfully the Lagrange multiplier approach and a tree search algorithm.



(a)



(b)

Figure 1: The amplitude response of Class 4 partial response filters. (a) Continuous coefficient version. (b) Discrete coefficient version.

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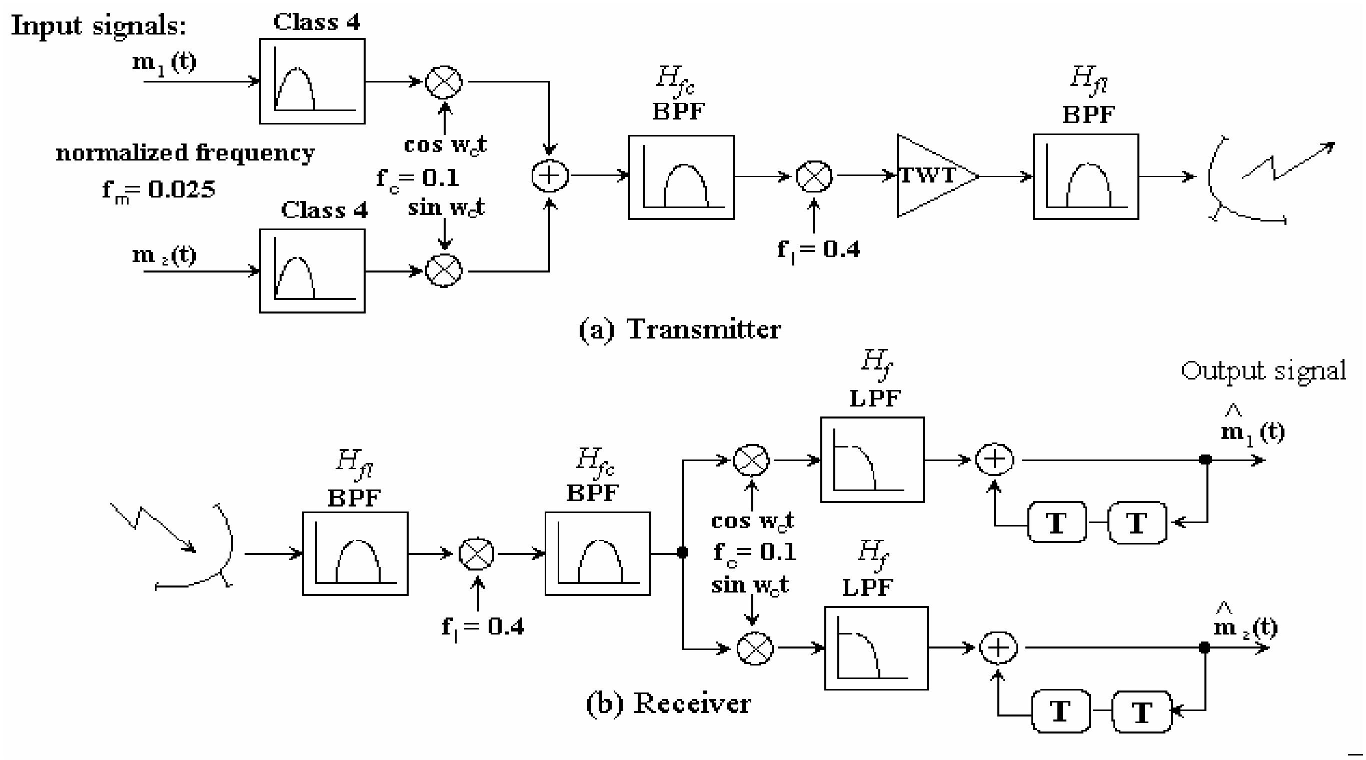

Figure 2: Block diagram of an SSB radio system using partial response signaling

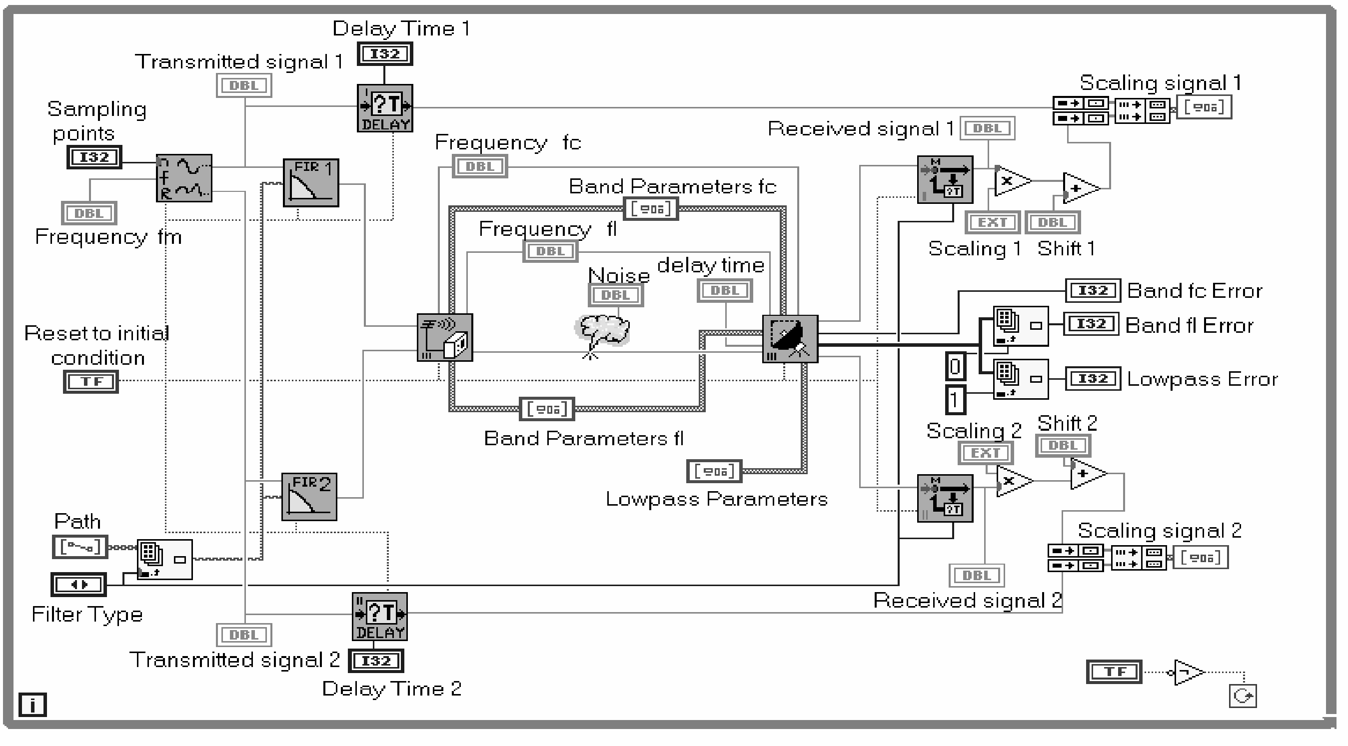

Figure 3: Block diagram of an SSB radio system in LabVIEW using partial response signaling