HOS BASED DETECTORS FOR PERIODIC SIGNALS

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Abstract

This paper discusses algorithms for the detection of periodic pulse-like signals. Such signals exhibit phase as well as frequency coupling and are thus suitable for detection using HOS. The algorithm presented herein can be regarded as an extension to an existing second order spectral algorithm, to include third order terms. The results of simulation studies are presented which demonstrate the performance advantage offered by this new algorithm.

1. INTRODUCTION

In this paper we discuss the detection of periodic signals in noise. Such a model is appropriate for several applications including speech [1], acoustic detection of helicopters [2] and torpedo detection [3]. This problem is a subset of the more general harmonic retrieval problem and can be seen as a more complex version of the single tone detection problem.

It has long been recognised that the optimal detector for a single complex tone in white Gaussian noise is the periodogram, and for coloured Gaussian noise one need only pre-whiten the data prior to the periodogram to restore optimality. Whether one can obtain information about the noise spectrum to perform the pre-whitening is application dependent. In the case of a periodic signal in white Gaussian noise then a relatively simple modification to the periodogram yields a detector whose performance is close to optimal. However the optimality in this discussion is based on the premise that we have no *a priori* information concerning the phase structure of the periodic signal.

Conventionally the bispectrum has been exploited for detecting quadratic phase coupling (QPC) which arises when a signal is passed through a quadratic-type non-linear system [4]. If the signals involved are narrow band random processes then this problem becomes very similar to that of detecting a periodic signal. Specifically the input signal is often assumed to be a sinusoid with a random phase. In which case conventional bispectral estimates are only consistent if the phase within each segment is random: an assumption which is false for single record problems.

It is to overcome the fact that such signals are non-ergodic with respect to conventional bispectral estimates that alternative estimators for such mixed signals (*i.e.* signals with deterministic and random components), have been proposed [5].

In this paper we discuss the optimal second order detectors for periodic signals, with phase coupling, in white Gaussian noise; the case of coloured noise can be included assuming that the noise spectrum can be accurately estimated. These detectors can be considered as extensions to the existing second order algorithms but incorporating additional information supplied by HOS.

2. PROBLEM STATEMENT

We seek to produce detectors for signals, *x(t)*, of the form:

$$x(t) = \sum_{k=1}^{M} A_k \cos(2\pi_o k + \phi_k) \tag{1}$$

where f_o is the fundamental frequency $f_o = 1/T_p$ and where T_p is the period of the signals. The amplitudes A_k and phases ϕ_k are fixed within a single record. Further, it is assumed that f_o»$1/T$ (*T* is the record duration), this ensures that all the frequency components are well separated.

We obtain the sequence $x(n)$ by sampling the complex form of $x(t)$ at $t=n\Delta$ with Δ representing the sampling interval. The Fourier transform of $x(n)$ is then

$$X(f) = \sum_{k=1}^{K_2} A_k \, \sigma(f - kf_o) \, e^{ik\phi_k}$$

where $\sigma(f)$ is the real valued function $\sin(2\pi N\Delta)/\sin(2\pi\Delta)$ so that at the points $f=kf_o$ we obtain $X(kf_o) = A_k N e^{i\phi_k}$. By exploiting the assumption that $f_o \gg 1/T$ then one can neglect the effect of the negative frequency components which arise when one considers a real valued sequence.

This signal is measured in the presence of an additive noise $n(t)$ so that the received signal is

$$y(t) = x(t) + n(t)$$

2i) Phase and Frequency Coupling

A periodic signal can be said to be phase coupled if the phases, ϕ_k, in (1) satisfy :

$$\phi_k = k\,\phi_1$$

This restriction is stricter than the requirement of frequency coupling, which merely requires that a component be at a frequency which is a sum of the frequencies associated with other components. All non-degenerate periodic signals can be said to possess frequency coupling.

Physically the effect of phase coupling is to ensure that all the frequency components are “in phase” at some point during the signal’s cycle. The resulting waveforms tend to have a pulse-like structure. Applications where such pulse-like periodic signals can be anticipated include condition monitoring, where one is searching for an impacting type fault in rotating machinery [7].

2ii) Ergodicity

Assuming the noise free signal $x(t)$ has phase coupling then it can only be considered as random if the phase term ϕ_1 is random. However from a single record, which we assume is all that is available for this algorithm, then it is impossible to determine the *distribution* of the ϕ_1. One can estimate this phase factor for the one available record but can say nothing about how it may vary across records. For the familiar power spectrum and auto-correlation functions this presents no problem since they are invariant to phase (*i.e.* they are phase blind). Hence one can reliably estimate these second order properties. However the High Order Spectra (HOS) are not necessarily phase blind, with the result that one cannot rely on ergodicity when estimating them from single records.

3. SECOND ORDER DETECTORS

Two optimal (maximum likelihood) estimators of the fundamental frequency of periodic signals have been derived in the literature [1,3]. From these estimators one can devise detectors, the first is that based on the correlation function

$$\lambda_{cor}(\tau) = \sum_{l=1}^{K_1} r_{yy}(k\tau) \qquad (2)$$

where $r_{yy}(\tau)$ $\{= E[y(t)y(t+\tau)]\}$ is the auto-correlation function of the measured data and K_1 is the assumed number of periods in T seconds. In practice the auto-correlation function is estimated using

$$\hat{r}_{yy}(m) = \frac{1}{N} \sum_{k=o}^{N-1-M} y(k)y(k+m) \qquad (3)$$

The performance of the detector improves if the function $r_{yy}(m)$ is interpolated. This is efficiently achieved by exploiting the FFT to evaluate (3) and inserting zeros appropriately in the frequency domain.

The second detector is $\lambda_{spec}(f)$ defined as

$$\lambda_{spec}(f) = \sum_{k=1}^{K_2} S_{yy}(kf) \qquad (4)$$

where $S_{yy}(f)$ is the spectrum of $y(t)$ the spectrum should be estimated by a periodogram, in spite of the fact that a periodogram is an inconsistent estimate of $S_{yy}(f)$. The maximal resolution of the periodogram yields an optimal detector. K_2 is the assumed number of harmonics. In practice an FFT routine can be employed to estimate $S_{yy}(f)$, but one should ensure that the data length is zero padded to a length of K_2L.

This ensures that the factor $S_{yy}(K_2 f)$ retains a resolution of $1/T$.

These second order detectors in fact detect frequency coupling. The algorithms developed herein, intend to improve the performance of these detectors in scenarios where phase, as well as frequency, coupling occurs. In order to obtain phase information we resort to using HOS.

4. HOS BASED DETECTORS

If we consider a signal containing only *3* (phase coupled) components then the following quantities are all invariant to the phase, ϕ_1 :

$$c_{2,p}(f_o) = X(pf_o)\, X(pf_o)^* \qquad p=1,2,3$$
$$c_{3,1}(f_o) = X(f_o)\, X(f_o)\, X(2f_o)^*$$
$$c_{3,2}(f_o) = X(f_o)\, X(2f_o)\, X(3f_o)^*$$
$$c_{4,1}(f_o) = X(f_o)\, X(f_o)\, X(f_o)\, X(3f_o)^*$$

Along with these functions their conjugates are also phase invariant. The above quantities are all slices through HOS as estimated using the concepts discussed in [5]. In the notation above the first argument of $c_{p,q}(f)$ reflects the order of the spectrum involved, so that the $c_{2,q}(f)$ are power spectra.

The test statistic we discuss will involve summing the $c_{p,q}(f)$. However simply summing these values tends to cause problems which arise as a consequence of the fact that each term tends to have a very different magnitude. If $|X(f)|$ is generally bigger than 1 then $c_{2,q}(f)$ will be significantly smaller than $c_{4,1}(f)$. As a heuristic solution to this problem we work with a normalised Fourier transform, specifically we form $\tilde{X}(f) = X(f)/\overline{X}(f)$ where $\overline{X}(f) = E[\,|X(f)|\,]$ which is calculated in practice by taking the average of the absolute values of the Fourier coefficients.

The test statistic we consider is thus :

$$\lambda_{mix}(f) = \sum_{p,q} c_{p,q}(f)$$

where the $c_{i,j}(f)$ are calculated using the measured data $y(t)$ and use the normalised spectrum, $\tilde{Y}(f)$.

The generalisation of this to the case where one assumes that there are K harmonics involves considering all the terms of the form :

$$c_{p,q}(f) = \tilde{Y}(pf)^* \prod_r Y(k_r f)$$

in which $p \le K$ and where the k_r's are the q^{th} set of integers such that $k_1+k_2+..+k_r = p$.

The function $\lambda_{mix}(f)$ will peak at the fundamental frequency, *i.e.* $f=f_o$ and so can be considered for use as an estimator as well as a detector.

Care must be taken when evaluating $\lambda_{mix}(f)$ since the highest order term, *i.e.* the term $Y(kf)$ in which k is largest, should have a resolution of at least $1/T$. This can be achieved by zero padding the input sequence to a total length of KN, where K is the assumed number of harmonics.

5. SIMULATIONS

To compare the performance of the HOS based detector and the second spectral detector as series of simulations were performed. Each consisted of *1000* Monte Carlo runs, simulating *1024* samples of a periodic signal in i.i.d. background noise. The periodic signal in each case contained a full set of harmonics up to the folding frequency. The amplitudes were taken in the form $A_k = k^{-½}$. The fundamental frequency in each run was randomly generated within a frequency bin centred on *57* Hz, with assumed sampling rate of *1000* Hz. Similarly the initial phase ϕ_1 was also randomly selected using a uniform distribution. In all cases the number of harmonics was under-estimated and taken to be *3*.

From these *1000* data sets the test statistics $\lambda_{mix}(f)$ and $\lambda_{spec}(f)$ were calculated and various threshold levels were then applied, from which probabilities of detection and false alarm can be calculated to finally yield the Receiver Operating Characteristic (ROC) curves.

6. RESULTS

Figure 1 shows a typical time series used in the simulations. The upper trace shows a portion of the noise free time history, with its associated pulse-like periodic structure, whilst the lower trace shows the same signal corrupted by noise. The SNR in this example is -13.5 dB and at this level the periodic signal is visually swamped by the noise.

In Figure 2 we see the test statistic on which a detection decision is based. The upper frame shows the results of computing spectral test statistic (4), and the lower frame shows the result of the HOS based detection statistic. Both statistics show peaks at around 57 Hz and 114 Hz, (the fundamental frequency in this example being close to 57 Hz). In both cases the most prominent peak is the one at 114 Hz. For detection purposes this represents no problem since the second peak is signal related. Examination of these plots is encouraging. The peak in the HOS based detector protrudes a greater distance above the noise floor than the peak in the spectral detector.

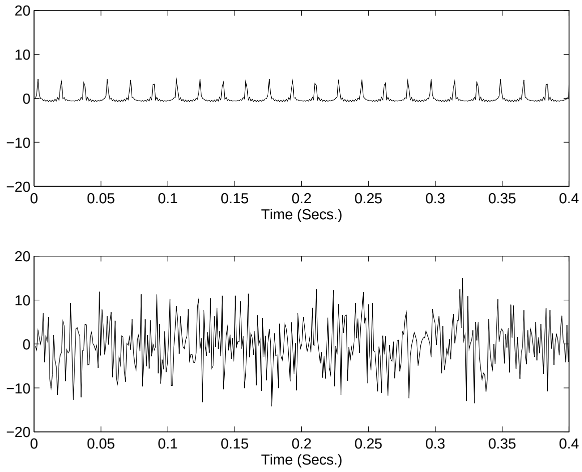

Figure 1 : *Typical Time Series.*
Upper Frame : Noise Free Data, Lower Frame : Data at an SNR of -13.5 dB.

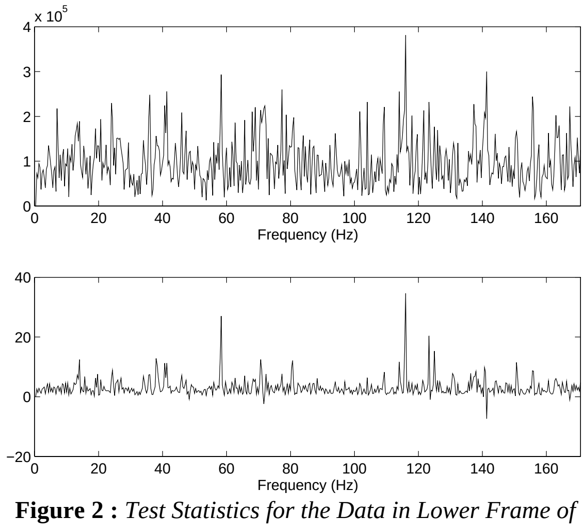

Figure 2 : *Test Statistics for the Data in Lower Frame of Fig. 1. Top Frame : Output of Second Order Spectral Detector. Lower Frame : Output of HOS Detector.*

To fully assess the performance of these detection algorithms then their ROC curves were calculated. These curves are shown in Figure 3 for a SNR of -14.4 dB. Whilst the HOS algorithm still out performs its spectral counterpart the level of improvement is not as pronounced as one might anticipate from the results in Figure 2.

7. CONCLUSIONS

This paper has presented an algorithm for detecting phase coupled harmonic signals based on HOS estimators for mixed processes. The algorithm is simple to implement and produces measurable performance improvements over existing second order algorithms.

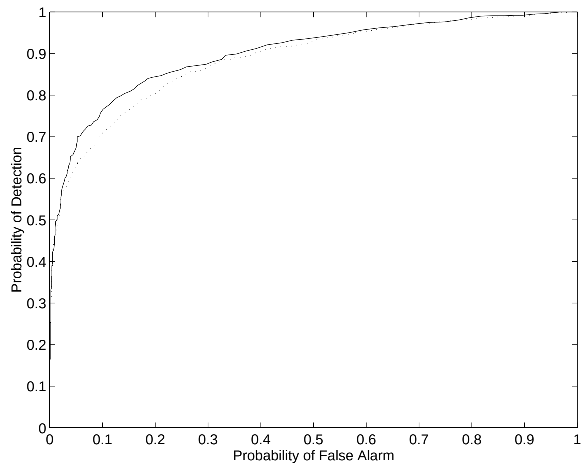

Figure 3 : *ROC Curves for -14.4 dB Case*
Solid : HOS Algorithm, Dotted : Equation (4)

8. REFERENCES

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