TIME DELAY AND MOTION ESTIMATORS BASED ON DIGITAL FAST TIME-SCALING OF RANDOM SIGNALS

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ABSTRACT

The estimation of time-delay and time-scaling is required in many signal processing applications. A parabolic approximation was recently suggested for fine estimation of time delay from sampled signals. The method directly extends to scaling estimation by a parallel multi-rate sampling of the analog received signal. Such rescaling can be implemented by digital techniques and two efficient algorithms are here devised and analysed.

1. INTRODUCTION

Time delay and Doppler shift estimation is an important issue in many signal processing areas [1-2]. These include the direction of arrival and trajectory in underwater acoustics, sonar and radar range and speed estimation in a multisensor environment, mobile communications, inter-satellite communications, timing acquisition in a spread spectrum communication system, motion detection and compensation in moving images, stereo vision, etc..

A model for Doppler-shifted narrow-band signals consists of a frequency shift of the spectrum [3]. This simple model does not apply in the presence of wide-band signals, where Doppler effect acts as an instantaneous compression or expansion of the time scale (or, for duality, an expansion or compression of the frequency scale).

Minimization of the ambiguity function based on generalized cross-correlation allows to find a Maximum Likelihood (ML) optimal estimate of the unknown parameters [1]. Efficient estimation methods are based on a sequential algorithm [4]: a *coarse* estimate is obtained from an unambiguous smoothed ambiguity function; a *fine* estimate then works on a wide-band ambiguity function starting from the coarse estimate. This allows to estimate the absolute minimum of the ambiguity function avoiding the wrong convergence on a relative one. In a recent paper [5], a parabolic approximation was employed for fine (sub-sample) estimation of time delay from sampled signals. The method was extended in [6] to Doppler estimation by a parallel multi-rate sampling of the analog received signal. In this work, such a Doppler multi-compensation is conversely implemented by digital techniques and two fast algorithms are here devised and analysed.

2. SIGNAL MODEL

One preliminary question arises, namely whether it is possible to separate the estimation procedures of time delay and Doppler shift. The answer provided by the theory is that one may use two separate estimators if the relative estimation errors are not correlated. In fact, this fact actually depends on the particular model chosen to represent a Doppler-shifted signal. As a consequence, we have employed a model [7] for which the above condition applies.

We assume to know the reference signal x(t) and a given number of delayed and Doppler-shifted versions $y(t;d_i,f_j)$ of the received signal r(t) for a given observation time window, i.e.:

$$x(t) = s(t) + n_1(t)$$

$$r(t) = a\ s\left(\frac{t - D}{1 + F}\right) + n_2(t) \qquad (1)$$

$$y(t;d_i,f_j) = r[(1+f_j)t+d_i]$$

where s(t) is the information signal while $n_1(t)$ and $n_2(t)$ are uncorrelated additive noises. In eq. (1), D and F are the unknown time delay and the Doppler velocity, respectively, to be estimated.

3. THE ESTIMATION PROCEDURE

3.1. Using a discrete ambiguity function

The received sampled signal has to be processed to obtain a set of time-shifted and time-scaled versions of the same signal in order to evaluate a two-dimensional ambiguity function, quantized on a proper grid both in time and Doppler domains. The resolution of the quantization grid basically depends on the observation time window.

Let us define $A(d_i,f_j)$ as the estimated ambiguity function over a set of discrete values of time delays (d_i) and Doppler shifts (f_j). The coarse estimate is implemented by searching for its minimum value, say $A(d_I,f_J)$. In order to find a fine (sub-sample) estimate of (D,F), we interpolate A(d,f) around $A(d_I,f_J)$ by a local two-dimensional Taylor expansion, after retaining the only terms up to the second order. Since the estimation errors are not correlated for the assumed model, such interpolation reduces to two separable one-dimensional ones.

In other words, two distinct parabolic interpolations, based on other four measurements $A(d_i,f_j)$ placed at the vertices of a cross around $A(d_I,f_J)$, need to estimate the time delay and the Doppler shift:

$$d = d_I - \frac{\Delta d}{2} \cdot \frac{A(d_I+\Delta d,\ f_J) - A(d_I-\Delta d,\ f_J)}{A(d_I+\Delta d,\ f_J) - 2\,A(d_I,\ f_J) + A(d_I-\Delta d,\ f_J)} \quad (2)$$

$$f = f_J - \frac{\Delta f}{2} \cdot \frac{A(d_I,\ f_J+\Delta f) - A(d_I,\ f_J-\Delta f)}{A(d_I,\ f_J+\Delta f) - 2\,A(d_I,\ f_J) + A(d_I,\ f_J-\Delta f)} \quad (3)$$

where D_d and D_f are the resolution quanta of the time delay and the Doppler coefficients.

3.2. Doppler-compensation and correlation

For our purposes, at least three measurements of the Doppler-shifted signal are needed, other than the reference one. The actual Doppler must belong to an interval determined by the lowest and the highest values.

As we are usually dealing with band-limited signals, discrete-time tecniques can be employed. The Doppler-compensation can be directly implemented as suggested by eq. (1) by collecting a parallel grid of samplers, tuned at several rates, i.e.:

$$y_{mr}(kT;d_i,f_j) = r[k(1+f_j)T+d_i] \quad (4)$$

Such a multi-rate method (discussed also in [6]) is equivalent to sample the ambiguity function in the Doppler domain.

The presence of a large observation window for an effective speed estimation suggests the use of fast digital algorithms to collect the whole set. For this purpose, the input samples ideally need an infinite-length time-varying interpolation to obtain the Doppler-shifted signal samples.

Because of its complexity, such computation can be approximated by a zero-order or a first-order time-varying interpolator, i.e.:

$$y_0(kT;d_i,f_j) = r\{\mathrm{nint}[k(1+f_j)]T+d_i\} \quad (5)$$

$$\begin{aligned} y_1(kT;d_i,f_j) = \{1-k(1+f_j)+\mathrm{int}[k(1+f_j)]\} \cdot \\ \cdot\, r\{\mathrm{int}[k(1+f_j)]T+d_i\} + \{k(1+f_j)-\mathrm{int}[k(1+f_j)]\} \cdot \\ \cdot\, r\{\mathrm{int}[k(1+f_j)+1]T+d_i\} \end{aligned} \quad (6)$$

where int(•) is the lower integer value of the real argument and nint(•) denotes the nearest integer one.

In practice, the interpolator (5) consists of deleting (or doubling) one sample from time to time; this is equivalent to discretizing in the Doppler domain the analog Betz method [8]. The function (6) just performs a linear interpolation whose coefficients depend on the current time.

4. NUMERICAL RESULTS

The results have been obtained for random Gaussian signals with a Gaussian-shaped auto-correlation function, i.e.:

$$R_{ss}(\tau) = e^{-\frac{\tau^2}{2\sigma^2}} \quad (7)$$

corrupted by two uncorrelated Gaussian white noises with several Signal-to-Noise Ratios (SNRs).

The direct, ASDF, and AMDF discrete-time correlators, respectively defined as [5]:

$$A_{direct}(d_i,f_j) = \frac{1}{N}\sum_{k=1}^{N} x(kT)\ y(kT;\ d_i,f_j) \quad (8)$$

$$A_{ASDF}(d_i,f_j) = \frac{1}{N}\sum_{k=1}^{N} \left[\ x(kT) - \frac{1}{a}\ y(kT;\ d_i,f_j)\right]^2 \quad (9)$$

$$A^{1/2}_{AMDF}(d_i, f_j) = \frac{1}{N} \sum_{k=1}^{N} \left| x(kT) - \frac{1}{a} y(kT; d_i, f_j) \right| \quad (10)$$

have been used for estimating the sampled ambiguity function $A(d_i,f_j)$. The correlators (8)-(10) work on the sampled reference signal x(kT) and the samples of a perfectly Doppler-compensated received signal $y_{mr}(kT;d_i,f_j)$, or the approximated ones $y_0(kT;d_i,f_j)$ or $y_1(kT;d_i,f_j)$ defined in eqs. (4)-(6).

The practical example considered here refers (just like in [2]) to an underwater object moving with a radial speed of 7 knots (corresponding to a time scaling factor on the order of $F=2.4\ 10^{-3}$) and uses an observation window of 0.1 sec to obtain the timing and Doppler estimates. A unitary gain factor (a=1) has been here considered for sake of simplicity. The central sampling time is $T=5\ 10^{-5}$ sec, while the autocorrelation standard deviation in eq. (7) has been assumed s=T. The quantized grid resolution is one sampling period for the timing and 2 knots for the speed.

The time delay error is usually much less relevant than the Doppler error. In practice [2], 30-60 Nyquist samples are enough for a good estimation of time delay, while at least 2000 samples need to be employed to achieve a comparable performance for the Doppler shift.

As a consequence, working on the largest window, we are focusing on the estimation of the Doppler coefficient. The performed simulated analysis therefore refers to the case of a perfect sampling of time-delay (which can be well estimated) and an intermediate wrong sampling of the speed (namely, a central speed of 7±0.5 knots with 2 knots of resolution quantum).

In practice, a systematical error affects all the estimates, but such bias results strongly dependent on the signal autocorrelation [5]; as a consequence, it can be neglected if the statistical properties of the signals are known.

The variances of the speed estimator (3) using the three analysed interpolators (4)-(6) have been evaluated by 1000 independent runs of computer simulations and are reported in the figs. 1-3 versus the SNR in the range [0,30] dB. In particular, the three figures respectively refer to the three discrete-time correlators (8)-(10).

As expected, the analogically-compensated multi-rate estimator (4) shows the best accuracy for all the correlators employed here. Nevertheless, the linear interpolator (6) has near efficient performance for intermediate values of SNR (10-20 dB), but it becomes poorer than a zero-order interpolator (5) for low SNR values (5 dB or less).

5. CONCLUSION

Digital methods based on fast time-scaling of signals and discrete-time correlation have been devised for the simultaneous estimation of time-delay and Doppler speed. Their performance has been investigated by computer simulations for some typical values of the parameters. Their accuracy has been shown as a function of the actual SNR value, and finally matched to the reference one of an ideally Doppler-compensated estimator.

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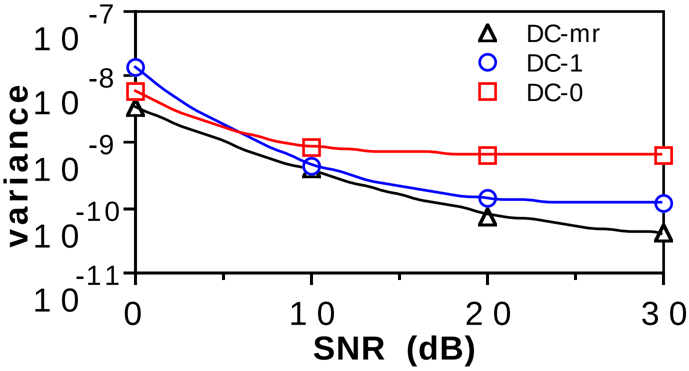

Fig. 1. Variance of the speed (F= 2.4 10^{-3}) estimator based on multi-rate, 0^{th}-order, and 1^{st}-order interpolators versus SNR, employing the direct discrete-time correlator, in the case of a perfect sampling of time-delay and an intermediate wrong sampling of speed.

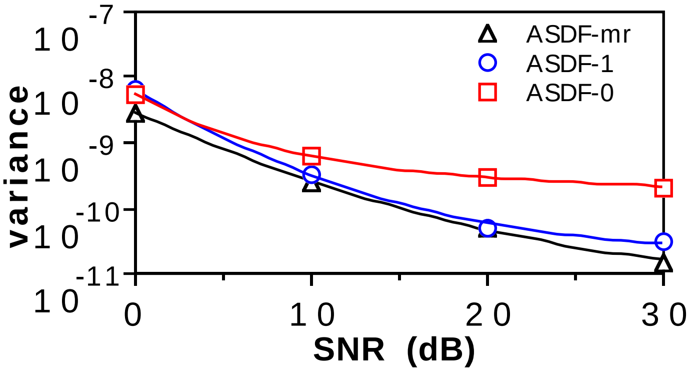

Fig. 2. Variance of the speed (F= 2.4 10^{-3}) estimator based on multi-rate, 0^{th}-order, and 1^{st}-order interpolators versus SNR, employing the ASDF discrete-time correlator, in the case of a perfect sampling of time-delay and an intermediate wrong sampling of speed.

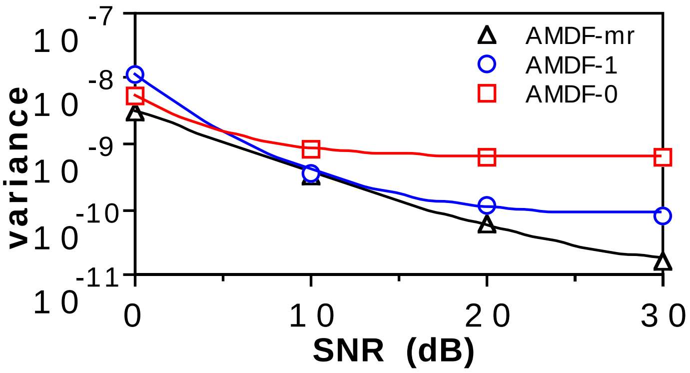

Fig. 3. Variance of the speed (F= 2.4 10^{-3}) estimator based on multi-rate, 0^{th}-order, and 1^{st}-order interpolators versus SNR, employing the AMDF discrete-time correlator, in the case of a perfect sampling of time-delay and an intermediate wrong sampling of speed.