DIFFERENTIAL CEPSTRUM DEFINED ON INTERPOLATED SEQUENCES

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ABSTRACT

The paper introduces a novel definition of the differential cepstrum. It is based on the interpolation sequences in the frequency domain and exists also for the singular signals with no spectral inverse. Besides, we showed analytically and statistically that such a differential cepstrum exhibits lower cepstral aliasing when calculated with the DFT comparing to the calculation without interpolation. On average, the improvement is 39 % in case of the interpolation to the half-intervals and 46 % in case of the quarter-intervals.

1 INTRODUCTION

Since the basics of generalised superposition and homomorphic systems have been established in [2], a variety of new approaches emerged in this non-linear signal processing framework. All of them rely upon the cepstra calculation, transforming non-linear operations of multiplication or convolution from the original time domain to a superposition in the cepstral domain [3]. The main advantage of such a transform is that simple linear filtering techniques become applicable in order to separate components originally joint by a non-linear operation. The separation works perfectly in a so-called convolutional model [9], which can be described typically by the problem of echo cancellation [3]. This feature has been utilised in several schemes in speech, geophysics, sonar, biomedicine, image processing, system identification, etc. [1, 4]. Recently, the properties of convolutional model have been used extended aiming at decomposing the superimposed signals [7, 8].

The basic type of the cepstrum used in all the aforementioned approaches is, because of its strictly defined inverse, the complex cepstrum. The easiest in the forward computation, however, is the differential cepstrum. Though, all the cepstra suffer from the drawback, like cepstral aliasing, singularities, and a necessity of the phase unwrapping. In the sequel, we are going to reveal a novel approach diminishing some difficulties of the differential cepstrum calculation. Section 2 defines the differential cepstrum on the interpolated sequences, Section 3 introduces a computer algorithm whose results are analysed with respect to cepstral aliasing, a short example is given in Section 4, while Section 5 concludes the paper.

2 DIFFERENTIAL CEPSTRUM WITH INTERPOLATED SEQUENCES

Suppose an exponential sequence $x(n)$ in the factorized z-transform presentation:

$$X(z) = \frac{Az^r \prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_o} (1 - b_k z)}{\prod_{k=1}^{N_i} (1 - c_k z^{-1}) \prod_{k=1}^{N_o} (1 - d_k z)}, \quad (1)$$

where A stands for a scale factor, $A = x(0) \prod_{k=1}^{M_o} (-\frac{1}{b_k}) \prod_{k=0}^{N_o} (-d_k)$, r for a delay, $r = N_o - M_o$, and a_k, b_k, c_k, and d_k for M_i inner and M_o outer zeros, and N_i inner and N_o outer poles, respectively ($|a_k|$, $|b_k|$, $|c_k|$, $|d_k|$ all less than 1). Referring to derivation in [5], the definition of the differential cepstrum yields:

$$x^{(d)}(n) \stackrel{\text{def}}{=} \mathcal{Z}^{-1} \left\{ \frac{\frac{d}{dz} X(z)}{X(z)} \right\}, \quad (2)$$

and after employing the DFT implementation:

$$x^{(d)}(n) = -\frac{j}{N} \sum_{k=0}^{N-1} \frac{X'(k)}{X(k)} W_N^{-k(n-1)}, \quad (3)$$

where $W_N^{-k(n-1)} = e^{j\frac{2\pi}{N}k(n-1)}$ and prime denotes differentiation.

Eq. 3 hides a potential danger: if transform $X(k)$ contains zero-valued samples, the expression becomes singular. On the other hand, $\frac{X'(k)}{X(k)}$ means a deconvolution that may be calculated even in case of a singular kernel if using the frequency-interpolated sequences [6]. As known [3], such an interpolation is obtained by padding the time-domain sequences of length N, $x(n)$, with N additional zeros. Actually, while only the interpolated sequences are of importance [6], they appear when transforming the sequence $x(n)W_{iN}^{n}$ to the frequency domain, with $\frac{1}{i}$ identifying a fraction of the frequency-domain sampling distance. Thus, the interpolation may be realised to any place inside the frequency sampling intervals. We will show the effects with interpolation to the half ($i = 2$) and to the quarter-intervals ($i = 4$).

Introduce a novel definition of the differential cepstrum with the interpolated sequences:

$$x_i^{(d)}(n) \stackrel{\text{def}}{=} \mathcal{Z}^{-1}\left\{\frac{\frac{d}{dz}X(W_{iN}^{-1}z)}{X(W_{iN}^{-1}z)}\right\}W_{iN}^{-n}. \qquad (4)$$

The properties of newly defined differential cepstrum remain the same as stated in [5], additionally, we can observe some new ones when computation is done using the DFT:

Property 1: For $i > 2$, $x_i^{(d)}(n)$ becomes a complex-valued sequence.

Property 2: For the principal, non-aliased version of the differential cepstra, the following is valid:
$x_i^{(d)}(n) = -x^{(d)}(n+1);\ n \geq 0$,
$x_i^{(d)}(n) = -x^{(d)}(n+1)W_i^{-1};\ n < 0$.

Property 3: $x_i^{(d)}(n)$ exists also in case of singular kernel $X(k)$.

Property 4: $x_i^{(d)}(n)$ exhibits lower cepstral aliasing comparing to the calculation without interpolation, on average.

3 COMPUTATIONAL ALGORITHM

The best computational performance is obtained with the DFT-based calculation:

$$x_i^{(d)}(n) = IDFT\left\{\frac{DFT[m \cdot x(m)W_{iN}^{m}]}{DFT[x(m)W_{iN}^{m}]}\right\}W_{iN}^{-n}. \qquad (5)$$

For the ease and clarity, calculation in Eq. 5 slightly modifies definition 4: namely, the resulting cepstrum is shifted anticausally by one sample, as well as it is multiplied by -1. Hence, with no attention paid to aliasing:

$$x_i^{(d)}(n) = \begin{cases} -\sum_{k=1}^{M_i} a_k^n + \sum_{k=1}^{N_i} c_k^n, & n > 0 \\ M_o - N_o, & n = 0 \\ \left(\sum_{k=1}^{M_o} b_k^{|n|} - \sum_{k=1}^{N_o} d_k^{|n|}\right) W_i^{-1}, & n < 0 \end{cases} \qquad (6)$$

where the notation comes from Eq. 1.

Unfortunately, the DFT-based computation corrupts the correct cepstral values from Eq. 6 with cepstral aliasing. The actual outcome is as follows:

$$\begin{aligned} &x_i^{(d)}(n) = \\ &(M_o - N_o)_{|n=0} + \sum_{\ell=0}^{\infty}\Bigg[-\sum_{k=1}^{M_i}(a_k W_{iN})^{\ell N+n}_{|(n>0 \text{ or } \ell>0)} + \\ &\sum_{k=1}^{N_i}(c_k W_{iN})^{\ell N+n}_{|(n>0 \text{ or } \ell>0)} + \sum_{k=1}^{M_o}(b_k W_{iN}^{-1})^{(\ell+1)N-n} - \\ &\sum_{k=1}^{N_o}(d_k W_{iN}^{-1})^{(\ell+1)N-n}\Bigg] W_{iN}^{-n},\ n = 0,\ldots,N-1. \end{aligned} \qquad (7)$$

The terms with ℓ in Eq. 7 depict the effect of cepstral aliasing.

3.1 Analysis of cepstral aliasing

The computation based on interpolated sequences eliminates threats of the singular kernel. At the same time, it influences the level of cepstral aliasing. Let's study it considering Eq. 7. Contributions of every individual zero and pole build up the final result in the same manner. Therefore, the extent of aliasing may be judged only referring to one of them. For example, the contribution of one of the inner zeros may be expressed in case of no interpolation as:

$$-\sum_{\ell=0}^{\infty} a_k^{\ell N+n} = -\frac{a_k^n}{1 - a_k^N},\ n = 1,\ldots,N-1 \qquad (8)$$

when aliasing is included, and as:

$$-a_k^n,\ n = 1,\ldots,N-1 \qquad (9)$$

with no aliasing.

A multiplicative error may be obtained as a ratio of expressions 8 and 9, i.e.

$$\frac{1}{1 - a_k^N}. \qquad (10)$$

It appears with the same value at any cepstral position.

In case of interpolation, the error ratio is computable in the same way. Thus, comparing the aliasing error in the interpolated case against the one calculated without interpolation:

$$\frac{1 - a_k^N}{1 + a_k^N}. \qquad (11)$$

Expression 11 is evidently in favour of the interpolated differential cepstrum if a_k is a real zero (supposedly, N even). In case of a conjugate complex pair of zeros $|a_k|e^{\pm j\phi_k}$, the interpolated solution gives strictly lower aliasing if:

$$-|a_k|^N < \cos N\phi_k, \qquad (12)$$

which is depicted with shaded area in Fig. 1.

With similar reasoning, we can analyse any other type of interpolation as well. The quarter-interval one produces complex-valued results, thus separating the even

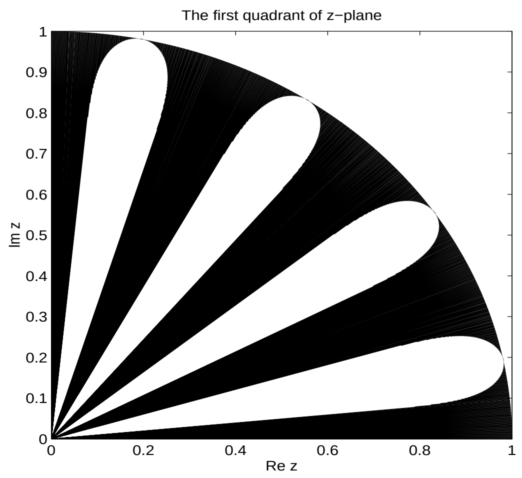

Figure 1: For the zeros and poles in the shaded area, the differential cepstrum with interpolation gives strictly lower aliasing

and odd aliasing periods into the real and imaginary part of the result, respectively. Besides, a condition similar to that one from Fig. 1 is valid.

We have evaluated the level of aliasing statistically. A sample of 28 real ECG signals of length 48 was collected randomly from the standard American Heart Association Database. Determining the spectral roots, we calculated the correct differential cepstra for all the signals. Then, we applied the DFT solution without interpolation, and with interpolation to half- and quarter-intervals. The aliasing error was estimated with the first norm between the correct cepstra and their aliased approximations (Table 1).

Type of calculation	First norm
No interpolation	258.14
Interpolation to half-intervals, $i = 2$	157.09
Interpolation to quarter-intervals, $i = 4$ (real part)	165.12
Interpolation to quarter-intervals, $i = 4$ (imaginary part)	139.09

Table 1: Comparison of the first norm of errors with different types of the differential cepstrum calculation

Table 1 shows that an average decrease of the aliasing error of 39 % is achieved in case of interpolation to half-intervals, and of 46 % in case of interpolation to quarter-intervals.

4 A SHORT EXAMPLE

To illustrate the performance of the interpolated differential cepstrum, an ECG signal of length 64 has been processed. The original signal is shown in Fig. 2 in parallel with its differential cepstrum. The cepstrum is plotted in solid line for the interpolated approach according to Eq. 5, $i = 2$, whereas dotted line depicts the exact cepstral values, and dashed line represents the values calculated via the DFT without interpolation.

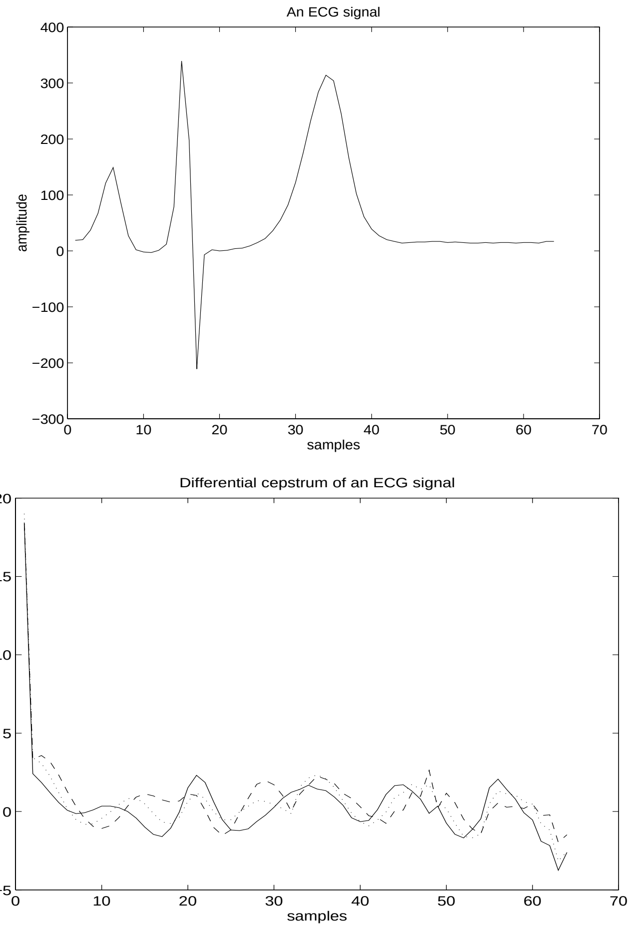

Figure 2: An ECG signal (top) and its differential cepstra (bottom): with interpolation – solid line, without interpolation – dashed line, exact values – dotted line.

Afterwards, the ECG signal was made zero-mean and, thus, singular for the definition of Eq. 3. However, Eq. 5 permits calculation even in case of singularity. The differential cepstrum obtained is depicted in Fig. 3 with solid line, whereas dotted line represent the exact cepstral values.

5 DISCUSSION AND CONCLUSIONS

The novel definition of the differential cepstrum brings twofold benefit: it eliminates problems with the singular signals (e.g. the zero-mean signals), at the same time, however, the cepstral aliasing is diminished on average.

Referring to the noise study in [6], the interpolation slightly improves the signal-to-noise ratios in the deconvolution procedure. It is, therefore, expected that this

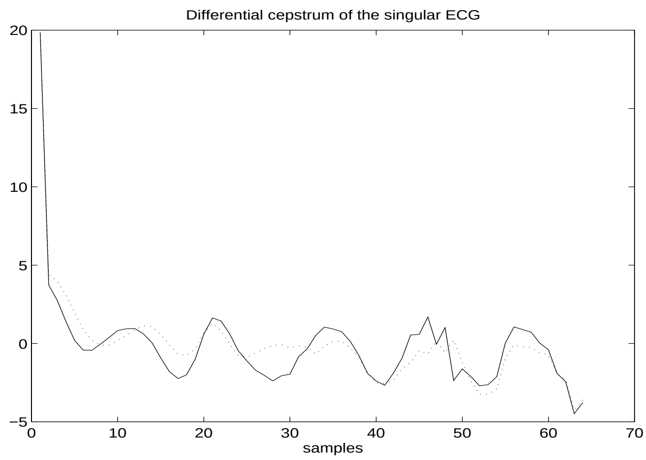

Figure 3: Differential cepstrum of a singular ECG signal: calculated with interpolation – solid line, exact values – dotted line.

fact must also contribute to a greater robustness of the differential cepstrum as defined here.

All the advantages encountered are also preserved with the polycepstra defined on interpolated sequences. The fact may be used with benefit in the system identification approach based on the higher-order cumulnats and polycepstra [4].

Finally, the separation of successive aliasing periods in case of interpolations with $i > 2$ into two sequences, i.e. a real and an imaginary one, suggests a possible further decrease of aliasing by introducing a scheme with more than two-component complex numbers.

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