FLEXIBLE NONUNIFORM FILTER BANKS USING ALLPASS TRANSFORMATION OF MULTIPLE ORDER

M. Kappelan, B. Strauß, P. Vary
Institute of Communication Systems and Data Processing (IND)
RWTH Aachen, University of Technology
D-52056 Aachen, Germany
Tel: +49 (0)241 80 6959; Fax: +49 (0)241 8888 186
e-mail: kiwi@ind.rwth-aachen.de

ABSTRACT

This paper deals with allpass frequency transformations of uniform filter banks to achieve nonuniform bandwidths. The known transformation with an allpass of first order [1] [4] [5] [6] is extended to an allpass transformation of order K. Thus the flexibility of the filter bank design can be increased significantly.

1 INTRODUCTION

In this contribution the polyphase realization of the uniform FIR filter bank is generalized towards a nonuniform frequency resolution. The proposed approach is an extension of the well known allpass transformation of first order [1] [4] [5] by introducing an allpass of order K. The frequency transformation of the filter characteristics is achieved by replacing the delay elements of the FIR filter bank by identical recursive allpasses. A causal allpass of order K would create a filter bank with multiple images of the original (bandpass) filters, which can of course be useful e.g. for the design of comb filters [3]. Here a new phase compensation technique is introduced which avoids the multiple mapping of the frequency axis and therefore gives an increased design flexibility by using an allpass of order K.

2 UNIFORM FILTER BANK

Filter banks with uniform frequency resolution can be implemented very efficiently using a polyphase network (PPN) and the Fast Fourier Transformation (e.g. [5]). There are two equivalent versions which can be described by using either complex modulators with uniformly spaced frequencies and identical lowpasses or modulated bandpass filters which have been derived from a common prototype lowpass. Here, the latter version will be considered. If the impulse response of the FIR prototype lowpass is denoted by \( w_0(n) \), the complex bandpass impulse responses with the center frequencies \( \Omega_\mu = 2\pi\mu/N \) (\( \mu = 0 \ldots N-1 \)) are given by

\[
 w_\mu(n) = w_0(n) e^{j 2\pi \mu n / N} \quad \mu = 0 \ldots N-1
\]  

The subband signal of the \( \mu \)th channel of the uniform filter bank can be described by

\[
 y_\mu(n) = x(n) * w_\mu(n)
\]

where \( x \) is the input signal and \( * \) denotes convolution (see fig. 1).

\[
 y_\mu(n) = \sum_{\nu=0}^{N-1} x(n-\nu) w_0(\nu) e^{j \frac{2\pi}{N} \mu \nu}
\]

\[
 = \sum_{\nu=0}^{N-1} x_\nu(n) e^{j \frac{2\pi}{N} \mu \nu}
\]

Thus the set of samples \( y_\mu(n) \) (\( n=\text{fixed} \), \( \mu = 0 \ldots N-1 \)) can be calculated efficiently using the Inverse Fast Fourier Transformation (IFFT). If we introduce the z-domain version of \( x_\nu(n) \) according to

\[
 X_\nu(z) = X(z) z^{-\nu} w_0(\nu)
\]

we obtain the subband signals in the z-domain

\[
 Y_\mu(z) = \sum_{\nu=0}^{N-1} X_\nu(z) e^{j \frac{2\pi}{N} \mu \nu}
\]

as shown in fig. 2.

\[\text{This approach can easily be extended to prototype filters with more than } N \text{ taps.}\]
The effective frequency responses are:

\[ H_\mu(z) = Y_\mu(z)/X(z) = \sum_{\nu=0}^{N-1} w_0(\nu) e^{-j\nu_\mu} \]  

(6)

On the unit circle i.e. for \( z = e^{j\Omega} \) we obtain the following transfer functions

\[ H_0(e^{j\Omega}) = \sum_{\nu=0}^{N-1} w_0(\nu) e^{-j\nu\Omega} \]  

(7)

\[ H_\mu(e^{j\Omega}) = H_0(e^{j(\Omega - \nu_\mu)}) \]  

(8)

The frequency responses of this uniform prototype filter bank given by (eq 8) are shown in fig. 3 by example.

Figure 3: Uniform filter bank with allpass transformation of order \( K = 1, \alpha = 0 \) (delay), \( N = 32, \mu = 0, 4, 8, 12, 16 \)

\[ \text{Figure 2: IFFT implementation of a uniform filter bank (N=8)} \]

The filter structure of fig. 2 is generalized as shown in fig. 4 by replacing the delay elements by identical allpass filters \( A(z) \) and by replacing the constant weights \( w_0(\nu) \) by \( B_\nu(z) \) \( (\nu = 0 \ldots N - 1) \).

\[ X(z) \rightarrow \frac{Y_\mu(z)}{X(z)} = \sum_{\nu=0}^{N-1} B_\nu(z)[A(z)]^\nu e^{+j\nu_\mu} \]  

(9)

\[ X(z) \rightarrow \frac{Y_\mu(z)}{X(z)} = \sum_{\nu=0}^{N-1} B_\nu(z)[A(z)]^\nu e^{+j\nu_\mu} \]  

(10)

The \( N \) outputs \( (\mu = 0 \ldots N - 1) \) of the filter bank in fig. 4 are now characterized by their transfer functions:

\[ H_\mu(z) = Y_\mu(z)/X(z) = \sum_{\nu=0}^{N-1} B_\nu(z)[A(z)]^\nu e^{+j\nu_\mu} \]  

(11)

3.1 Uniform Filter Bank As Special Case

A special case of (eq 11) is obviously the well known uniform polyphase filter bank (eq. 8) with the prototype filter (eq. 7) if we choose

\[ A(z) = z^{-1}, \quad B_\nu(z) = w_0(\nu) \]  

(12)

3.2 Allpass Transformation Of First Order

The substitution of delay elements \( z^{-1} \) in (eq 12) by the causal and stable allpass of order \( K = 1 \) with the complex parameter \( \alpha = ae^{j\alpha} \) with \( |\alpha| < 1 \) leads to:

\[ A(z) = A_1(z) = \frac{1 - \alpha^* z}{z - \alpha}, \quad B_\nu(z) = w_0(\nu) \]  

(13)

with

\[ A_1(e^{+j\Omega}) = e^{j\varphi_1(\Omega)} \]  

(14)

\[ \varphi_1(\Omega) = -\Omega - 2 \arctan \frac{a \sin(\Omega - \alpha)}{1 - a \cos(\Omega - \alpha)} \]  

(15)
which is the known nonuniform frequency transformation of the uniform filter bank of (eq. 8), describing the frequency responses:

\[
\tilde{H}_\mu(e^{j\Omega}) = \sum_{\nu=0}^{N-1} w_\nu(\nu)e^{+j\mu\nu}e^{+j\varphi_\nu(\Omega)} = H_\mu(e^{-j\varphi_\nu(\Omega)}) = H_0(e^{+j(-\varphi_\nu(\Omega)-\frac{\pi}{N}\nu)}) \tag{16}
\]

Generally speaking, the negative phase characteristic of the allpass describes the frequency transformation of the uniform prototype filter bank. In fig. 5 the negative phase of an allpass of first order \((a = 0.5)\) and in fig. 6 the corresponding nonuniform filter bank transfer functions are plotted.

### 3.3 Allpass Transformation Of Higher Order With Multiple Mapping

The substitution of delay elements \(z^{-1}\) in (eq. 12) by a causal and stable allpass of order \(K\) with the complex parameters \(a_k = a_k e^{j\alpha_k}\) with \(|a_k| < 1, \ k = 1 \ldots K\) and the prototype filter of (eq. 7) leads to:

\[
A(z) = A_K(z) = e^{j\alpha_0} \prod_{k=1}^{K} \frac{1 - a_k e^{-j\alpha_k} z}{z - a_k e^{-j\alpha_k}} \quad \varphi_K(\Omega) = \alpha_0 - K \Omega \quad B_\nu(z) = w_0(\nu) \tag{17}
\]

with the phase of this allpass:

\[
\varphi_K(\Omega) = \alpha_0 - K \Omega - 2 \sum_{k=1}^{K} \arctan \frac{a_k \sin(\Omega - \alpha_k)}{1 - a_k \cos(\Omega - \alpha_k)} \tag{18}
\]

It can be shown that the phase \(\varphi_K\) (eq. 18) of an causal stable allpass of order \(K\) of \(A_K(z)\) (eq. 17) is monotone decreasing [2]. Thus the (2\(\pi\) periodic) frequency interval \([-K\pi; K\pi]\) of the prototype filter is mapped to the interval \([-\pi; \pi]\). This is equivalent to multiple compressed mapping of \([-\pi; \pi]\) of the prototype filter to the target range \([-\pi; \pi]\) as proposed in [3] for the design of comb filters.

### 3.4 Allpass Transformation Of Higher Order With Single Mapping

In order to exploit the enhanced flexibility of the allpass of order \(K\) but to avoid multiple mappings, the phase has to be limited to \([-\pi; \pi]\). This can be achieved by reducing the linear term \(K\Omega\) in (eq. 18) to \(\Omega\), resulting in a non causal stable allpass \(\hat{A}_K(z)\) (eq. 19) with a phase \(\hat{\varphi}_K\) (eq. 20):

\[
\hat{\varphi}_K(\Omega) = \varphi_K(\Omega) + (K-1)\Omega \quad \hat{A}_K(z) = z^{+(K-1)}A_K(z) \tag{20}
\]

To avoid frequency reversions in the resulting filter structure, the phase \(\hat{\varphi}_K(\Omega)\) of the non causal allpass has to be monotone decreasing which leads to the group delay constraint (eq. 21) of the allpass \(A_K(z)\) (eq. 17).

\[
-\frac{d}{d\Omega} \varphi_K(\Omega) = \sum_{k=1}^{K} \frac{1 - a_k^2}{1 - 2a_k \cos(\Omega - \alpha_k) + a_k^2} \quad > K - 1 \tag{21}
\]

An example of allpass parameters which achieve a decreasing phase \(\hat{\varphi}_K(\Omega)\) is shown for \(K = 2\) in fig. 5 in comparison to the first order transformation characteristic \(\varphi_1(\Omega)\). This demonstrates the enhanced flexibility in designing the nonuniform frequency transformation. The allpass of order \(K = 2\) with conjugated complex parameters can be implemented by a network with real valued arithmetic.

### 3.5 Implementation

With (eq. 11) there are non causal implementations which have the same transfer functions:

\[
A(z) = \begin{cases} A_K(z) & \text{non causal} \\ \hat{A}_K(z) & \text{causal} \end{cases} \quad B_\nu(z) = w_0(\nu) \tag{22a}
\]

\[
A(z) = \begin{cases} A_K(z) & \text{non causal} \\ \hat{A}_K(z) & \text{causal} \end{cases} \quad B_\nu(z) = z^{+(K-1)}w_0(\nu) \tag{22b}
\]

If the non causal part is shifted to \(B_\nu(z)\) according to (eq. 22b), a causal implementation is possible by introducing an additional delay of \(z^{-((K-1)(N-1))}\) resulting in:

\[
A(z) = A_K(z) \quad \nu = 0, 1 \ldots N - 1 \quad B_\nu(z) = z^{-((K-1)(N-1))}w_0(\nu) \tag{23}
\]

The frequency response of this transformed causal transformed filter bank now reads:

\[
\tilde{H}_\mu(e^{j\Omega}) = e^{-j(N-1)(K-1)\Omega} \sum_{\nu=0}^{N-1} w_0(\nu)e^{+j\mu\nu}e^{+j\hat{\varphi}_K(\Omega)} = e^{-j(N-1)(K-1)\Omega}H_\mu(e^{-j\hat{\varphi}_K(\Omega)}) = e^{-j(N-1)(K-1)\Omega}H_0(e^{+j(-\hat{\varphi}_K(\Omega)-\hat{\varphi}_\mu(\Omega))}) \tag{24}
\]

The block diagram is given in fig. 4.
The frequency responses of the compensated nonuniform filter bank for e.g. $K = 2$ are shown in fig. 7 revealing the enhanced flexibility of the new approach. In comparison to the transformation of order $K = 1$ as shown in fig. 6 the frequency resolution can now be increased e.g. within a bandpass interval. Thus the filter structure allows more parameters to design nonuniform filter banks using a common prototype FIR filter.

**References**


