CEPSTRAL SYNTHESIS OF MINIMUM-PHASE FIR AND IIR DIGITAL FILTERS

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ABSTRACT

A new technique for designing causal and minimum-phase FIR and IIR digital filters is presented. Here, the deviation from a desired quefrency response is minimised using the Fletcher-Powell algorithm. As a consequence, this leads to an optimisation of both log-magnitude response and phase response. Therefore, the method is of special interest for both equalisers and allpasses. It works with real parameters which represent the poles and zeros of the system.

1 INTRODUCTION

Digital Filters can be designed in three different domains: in the time domain, in the frequency domain, or in the quefrency domain. While in the time domain, we approximate a desired impulse response $h_d(k)$, in the quefrency domain we approximate a desired quefrency response $\hat{h}_d(k)$. Generally, the quefrency response $\hat{h}(k)$ of a system is defined as the cepstrum of the impulse response of the system,

$$\hat{h}(k) := Z^{-1}\{\log(Z\{h(k)\})\}. \qquad (1.1)$$

In the above equation, Z denotes the Z-transform and Z^{-1} denotes the inverse Z-transform, respectively. It is clear that (1.1) can be approximately carried out by using FFT. Moreover, if we write the transfer function $H(z)$ of a system in terms of its poles $z_P(i)$ and zeros $z_0(i)$,

$$H(z) = Z\{h(k)\} = C\,\frac{\prod_{i=1}^{m}(1 - z_0(i)\,z^{-1})}{\prod_{i=1}^{n}(1 - z_P(i)\,z^{-1})}, \qquad (1.2)$$

where C is a real constant, we can express the quefrency response of the system as follows:

$$\hat{h}(k) = \begin{cases} \log C & \text{for } k = 0 \\ -\sum_{i=1}^{m}\frac{z_0^k(i)}{k} + \sum_{i=1}^{n}\frac{z_P^k(i)}{k} & \text{for } k \geq 1 \\ 0 & \text{else} \end{cases}. \qquad (1.3)$$

All this is well-known [8]. However, the last equation is only valid if $|z_0(i)| \leq 1$ and $|z_P(i)| < 1$ for all i, that is, if the system is causal, stable and minimum-phase. Easily we see that the quefrency response of such systems is causal. Now, from (1.1) and (1.2) it follows that for $z = e^{j\Omega}$,

$$Z\{\hat{h}(k)\} = \log\{H(e^{j\Omega})\} = \log|H(e^{j\Omega})| - jb(\Omega), \qquad (1.4)$$

where $H(e^{j\Omega})$ denotes the frequency response of the system and $b(\Omega)$ denotes its unwrapped phase response. Inserting this into Parseval's theorem for causal sequences [9] we get

$$\sum_{k=0}^{\infty} \left|\hat{h}_d(k) - \hat{h}(k)\right|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (\log\left|H_d(e^{j\Omega})\right| - \log\left|H(e^{j\Omega})\right|)^2 + (b_d(\Omega) - b(\Omega))^2 d\Omega \quad , \quad (1.5)$$

where all of the quantities indexed d are desired quantities. From (1.5) we see that optimising the quefrency response of a system means also optimising its log-magnitude response and its phase response. Therefore, this is of special interest for equalisers, because the log-magnitude response of the cascade of the equaliser and the system to be equalised is equal to the log-magnitude response error of the equaliser. While well-known design techniques exist, which optimise the frequency response of a system [1] or the impulse response of a system [10], filters obtained until now from quefrency domain design techniques are of a special class [3], [4], and are suitable for fast design [5], [6], [7], but are not optimal with respect to the number of multipliers. Therefore, in this paper we present a synthesis procedure which minimises an error measure in the quefrency domain. Filters obtained employ a minimum number of multipliers, under the condition that their transfer functions may be expressed in the form (1.2). Moreover, we assume that both complex poles and zeros occur in conjugate-complex pairs. Apart from this restriction they can take any position inside the unit circle, and so $m+n+1$ real multipliers are required for the filter realisation. It should be clear that halfband filters, for instance, do not fit into this category. Further, both m and n can be chosen arbitrarily, and so also FIR filters and all-pole filters can be designed.

2 SYNTHESIS PROCEDURE

First we introduce the above-mentioned error measure,

$$F = \sqrt{\sum_{k=1}^{L} (\hat{h}(k) - \hat{h}_d(k))^2} \quad . \qquad (2.1)$$

L is an upper limit, which must be finite in order to be able to calculate (2.1) on computers. Theoretically, it should be infinite. In practise, we should choose L great enough, such that F will no more change its value decisively if we increase L.
We want to minimise F by using the Fletcher-Powell algorithm [2]. Towards this end, we express (1.2) as a function of the real parameters $x(v) \in \{p, q, a(i), b(i), c(i), d(i)\}$,

$$H(z) = C\,\frac{(1 - p\,z^{-1}) \prod_{i=1}^{\lceil m/2 \rceil} (1 + a(i)\,z^{-1} + b(i)\,z^{-2})}{(1 - q\,z^{-1}) \prod_{i=1}^{\lceil n/2 \rceil} (1 + c(i)\,z^{-1} + d(i)\,z^{-2})} \qquad (2.2)$$

where $\lceil y \rceil$ denotes the integer part of y, and $p = 0$ $(q = 0)$, if the numerator degree (denominator degree) is even. Clearly, for $p \neq 0$, p is a zero of the filter, and the other zeros can be expressed as

$$z_0(v) = -\frac{a(i)}{2} \pm \sqrt{\frac{a^2(i)}{4} - b(i)} \quad . \qquad (2.3)$$

We now need the partial derivatives of F with respect to the parameters $x(i)$. First, from (2.1) we get

$$\frac{\partial F}{\partial x(i)} = \frac{1}{F} \sum_{k=1}^{L} (\hat{h}(k) - \hat{h}_d(k))\,\frac{\partial \hat{h}(k)}{\partial x(i)} \quad . \qquad (2.4)$$

Next, we insert (2.3) into (1.3). Hereby, noting that a pair of parameters $a(i)/b(i)$ corresponds to a pair of zeros, which are either both real or conjugate-complex. From this, with

$$Y := \sqrt{\frac{a^2(i)}{4} - b(i)}$$

we get

$$\frac{\partial \hat{h}(k)}{\partial a(i)} = \frac{1}{2}\left[\left(-\frac{a(i)}{2} + Y\right)^{k-1}\left(1 - \frac{a(i)}{2Y}\right) + \left(-\frac{a(i)}{2} - Y\right)^{k-1}\left(1 + \frac{a(i)}{2Y}\right)\right] \quad . \qquad (2.5.1)$$

If the two zeros considered are a conjugate-complex pair, then Y becomes purely imaginary, and (2.5.1) can also be expressed as

$$\frac{\partial \hat{h}(k)}{\partial a(i)} = \mathrm{Re}\{(-\frac{a(i)}{2} + Y)^{k-1}(1 - \frac{a(i)}{2Y})\} \quad . \tag{2.5.2}$$

Moreover, we obtain

$$\frac{\partial \hat{h}(k)}{\partial b(i)} = \frac{1}{2Y}[(-\frac{a(i)}{2} + Y)^{k-1} - (-\frac{a(i)}{2} - Y)^{k-1}] \quad , \tag{2.6.1}$$

or, if Y is purely imaginary,

$$\frac{\partial \hat{h}(k)}{\partial b(i)} = \frac{j}{Y}\mathrm{Im}\{(-\frac{a(i)}{2} + Y)^{k-1}\} \quad . \tag{2.6.2}$$

Similarly, we obtain for the real zero p

$$\frac{\partial \hat{h}(k)}{\partial p} = -p^{k-1} \quad . \tag{2.7}$$

With (1.3), (2.2) and (2.3) it is easy to see that the last five equations also hold for $\frac{\partial \hat{h}(k)}{\partial c(i)}, \frac{\partial \hat{h}(k)}{\partial d(i)}$, and $\frac{\partial \hat{h}(k)}{\partial q}$, respectively, if $a(i)$ is replaced by $c(i), b(i)$ by $d(i)$, and p by q, respectively, and if all of the expressions on the right-hand side of the last five equations are multiplied by -1. Further, from (1.3) we see with (1.2) and (2.2), that we have to choose

$$C = \exp(\hat{h}_d(0)) \tag{2.8}$$

for the gain level, thus getting the element $\hat{h}_d(0)$ of the desired quefrency response exactly. Further, the parameters $x(\nu)$ we have chosen above for the Fletcher-Powell algorithm represent the poles and zeros of the system, and therefore, they have no influence on $\hat{h}_d(0)$. For this reason, we have chosen 1 for the lower limit of the sum in (2.1) (instead of 0).

In the numerous design examples worked out, the filters obtained, in general, turned out to be causal, stable, and minimum-phase, with the upper limit in (2.1), L, usually in the order of 50 to 500. In rare cases with L chosen too small, poles and/or zeros may be located outside (but near) the unit circle, which, however, may be remedied by increasing the value of L.

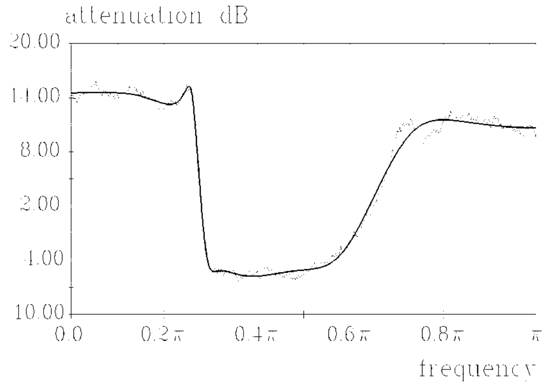

Fig. 1: Desired log-magnitude response (thin line) and approximating one (thick, smooth Line) for a 6 th order IIR filter

One of the examples is shown in Fig. 1. Here, the value of F for $H(z) \equiv 1$ (and, consequently, $\hat{h}(k) \equiv 0$) has been 1.485 This value has been reduced to 0.108 using a filter with both numerator and denominator being of order 6, hence requiring 13 multipliers. We see that the desired log-magnitude responses must not necessarily be smooth curves. They may rather be complicated ones, as it is the case, for instance, with those of loudspeakers in living rooms.

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