

TIME VARYING WAVELET TRANSFORM FOR IMAGE CODING

Yangzhao Xiang, Ruwei Dai

Dept. of Intelligent Systems, Institute of Automation,
Chinese Academy of Sciences, Beijing 100080, P.R.China

Fax: +80 010 2545229

E-mail: student@ailab.ia.ac.cn

ABSTRACT

Wavelet with longer support length is used in the smooth area of image to integrate energy effectively, while wavelet with shorter support length is used in the vicinity of edges. These two sets of wavelet transform switch automatically according to the image. The main problem for the design of time-varying filter is how to reconstruct exactly the signal during the transition period. An exact reconstruction method between any two sets of the widely used biorthogonal wavelet bases was proposed in this paper. And even more, by feedback of quantized wavelet coefficients, side information is embedded in the coding bit stream.

1 INTRODUCTION

In sub-band or wavelet image coding, there exists amplitude oscillation in the vicinity of edges in the reconstructed image, which is also called Gibb's phenomena. To overcome these annoying phenomena, time-varying filter for image coding was first proposed in [1], in which filter's coefficient changed automatically to the input, the main problem for the design of time-varying filter is how to reconstruct exactly the signal during the transition period. On the basis of a specified simple filter bank, a simple exact reconstruction method was given. [2] developed this idea, and presented analysis of infinite impulse response (IIR) filter bank. The result of image coding simulation demonstrated the success of time-varying filter bank, the amplitude oscillation was reduced significantly and PSNR was improved a little compared to the conventional IIR filter bank. But at the same time the compression ratio decreased, this is because that this coding system needs to emit edge information as side information to tell the receiver when to switch filter bank.

This paper presents a reconstruction method for switching between any two sets of the widely used

biorthogonal wavelet bases. It is relatively a little complicated, but there is no limitation on the choice of biorthogonal wavelet. And even more, by feedback of quantized wavelet coefficients, side information is embedded in the coding bit stream.

2 TIME VARYING WAVELET TRANSFORM

The one level wavelet analysis of coding system is depicted in Fig.1.

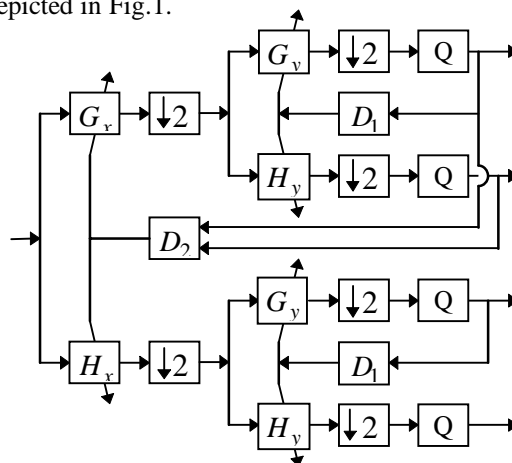


Fig.1 one level of time varying wavelet transform

Where Q means *quantize*, D_1 and D_2 are defined as :

$$D_1(x) = \begin{cases} 1, & |x| \geq T \\ 0, & |x| < T \end{cases}$$

$$D_2(x_1, x_2) = \begin{cases} 1, & \text{Max}(|x_1|, |x_2|) \geq T \\ 0, & \text{Max}(|x_1|, |x_2|) < T \end{cases}$$

If D_1 or D_2 is equal to 1, which implies that image edge is detected, we switch the corresponding filter bank to $\{(G'_x, H'_x), (G'_y, H'_y)\}$ with shorter support length, else $\{(G_x, H_x), (G_y, H_y)\}$ which

has longer support length is used.

3 EXACT RECONSTRUCTION

Here we only consider the exact reconstruction on switching with two biorthogonal wavelets. The symbols are defined as following:

(G,H) is the filter bank with longer support length;

The support length of G is $2N_G + 1$;

The support length of H is $2N_H + 1$;

$N = \text{Max}(N_G, N_H - 1)$;

(G',H') is the filter bank with shorter support length;

The support length of G' is $2N_{G'} + 1$;

The support length of H' is $2N_{H'} + 1$;

$N' = \text{Max}(N_{G'}, N_{H'} - 1)$;

If $c_{m,\tau}$ (m is the resolution level, τ is the time) -the output of high-pass filter G, is larger than threshold T, then from time $\tau + 1$, the filter bank is switched to (G,H). Generally, if $|c_{m,\tau}| > T$, pixel $a_{m-1,2\tau}$ is in the vicinity of image edge, the correlation between the pixels before $a_{m-1,2\tau}$ and those after $a_{m-1,2\tau}$ is very small. So after switching, we let $a_{m-1,2\tau}$ be the new starting point of wavelet analysis, and do border extension at $a_{m-1,2\tau}$. That is:

if $|c_{m,\tau}| > T$, then for $n > \tau$

$$c_{m,n} = \sum_k g_{2n-k+1}' a_{m-1,k}' , a_{m,n} = \sum_k h_{2n-k+1}' a_{m-1,k}'$$

where $a_{m-1,k}' = \begin{cases} a_{m-1,k} & , k \geq 2(\tau + 1) \\ a_{m-1,2(\tau+1)-k} & , k < 2(\tau + 1) \end{cases}$. This

will make the reconstruction of $a_{m-1,n}$ ($n \geq 2\tau + 2$) has no relation with $a_{m,n}$, $c_{m,n}$ ($n \leq \tau$).

For biorthogonal wavelet, the reconstruction is $a_{m-1,l} = \sum_n (\tilde{h}_{2n-l} a_{m,n} + \tilde{g}_{2n-l+1} c_{m,n})$, and

$g_n = (-1)^{n+1} \tilde{h}_{-n}$, $\tilde{g}_n = (-1)^{n+1} h_{-n}$. From the support length of G and H, we know that:

$$\tilde{g}_{2n-l+1} = 0, \quad \text{if } |2n-l+1| > N_H$$

$$\tilde{h}_{2n-l} = 0, \quad \text{if } |2n-l| > N_G$$

$$\tilde{g}_{2n-l+1} = \tilde{h}_{2n-l} = 0, \quad \text{if } l < 2n - N, .$$

so, for $l < 2(\tau + 1) - N$, the reconstruction of $a_{m-1,l}$ does not concern with $a_{m,n}$, $c_{m,n}$ ($\forall n > \tau$) which is obtained before switching. And for $l \geq 2(\tau + 1)$, the reconstruction of $a_{m-1,l}$ also does not concern with the wavelet coefficients obtained before switching because of border extension at $a_{m-1,2\tau}$.

What need to be reconstructed with a special way is $a_{m-1,l}$, $l \in [2\tau + 2 - N, 2\tau + 1]$.

Without loss of generality, we can assume that $N_H \geq N_G$ and N_H is odd number, then the part of wavelet analysis which is relative with $a_{m-1,l}$, $l \in [2\tau + 2 - N, 2\tau + 1]$ can be written in the following matrix form:

$$\begin{bmatrix} g_{N_G} & \cdots & g_1 & g_0 & g_{-1} & \cdots & g_{-N_G} \\ h_{N_H} & \cdots & \cdots & h_1 & h_0 & h_{-1} & \cdots & \cdots & h_{-N_H} \\ & & & \vdots & \vdots & \vdots & & & \\ & & & h_{N_H} & \cdots & h_1 & h_0 & h_{-1} & \\ & & & & g_{N_G} & \cdots & g_1 & g_0 & \cdots & g_{-N_G} \end{bmatrix}_{N \times (2N_H + N - 2)}$$

$$\times \begin{bmatrix} a_{m-1,2(\tau+1)-N-N_H+1} \\ a_{m-1,2(\tau+1)-N-N_H+2} \\ \vdots \\ \vdots \\ \vdots \\ a_{m-1,2\tau+1+N_H-1} \end{bmatrix}_{2N_H+N-2}$$

$$= \begin{bmatrix} c_{m,\tau+1-(N+1)/2+1} \\ a_{m,\tau+1-(N+1)/2+1} \\ c_{m,\tau+1-(N+1)/2+1} \\ a_{m,\tau+1-(N+1)/2+2} \\ \vdots \\ a_{m,\tau} \\ c_{m,\tau} \end{bmatrix}_{N \times 1}$$

the original reconstruction formula is invalid during the transition period. Through comparing coefficients of linear functions, we testify that exact reconstruction during transition period can be made for any biorthogonal wavelet with different coefficients of linear combination specified for different wavelet sets. Actually, this method can also be extended to any other cases with linear filter.

Another distinct advantage of the proposed coding system here is that through feedback of wavelet coefficients, there is no need to emit side information to decide when to switch filter bank, this information is embedded in the coding stream. So compared to that in [2], a large portion of bits is reduced.

Reference

- [1] K.Nayebi, T.P.Barnwell III and M.J.T.Smith, "Analysis-synthesis systems with time-varying filter bank structures", in Proc. IEEE ICASSP, Mar,1992, pp 617-620.
- [2] M.J.T.Smith and W.C.L.Chung, "Recursive time-varying filter banks for subband image coding", IEEE Trans. Image Processing, Vol. 4, No.7, July, 1995, pp 885-893.



Fig. 2 Original "Cameraman" image



Fig. 3 "Cameraman" coded at 0.1998 bpps with TVW , PSNR=24.96



Fig.4 "Cameraman" coded at 0.2012 bpps with CWT, PSNR=24.84

