LOSSLESS IMAGE COMPRESSION WITH WAVELET TRANSFORM

Jianmin Jiang
Department of Computer Studies, Loughborough University
Loughborough LE11 3TU United Kingdom
Tel: +44 01509 222698; Fax: +44 01509 211586
e-mail: j.jiang@lut.ac.uk

ABSTRACT

The research work presented in this paper explores new alternatives for lossless image compression where the entropy coding is applied to the wavelet transform coefficients rather than pixels. The advantage of using wavelet transform prior to entropy coding is that the statistical properties of the resulting coefficients can be analysed and exploited before the model is established for arithmetic coding. Experiments show that the proposed algorithm achieves competitive performances to that of JPEG.

1. INTRODUCTION

Lossless image compression provides a practical technology for compact representation of image data without losing any information in its reconstruction, i.e. the decoded image can be reconstructed exactly the same as its original. The technology is developed in the main direction that predictive coding is laid down as the foundation for nearly all the variations of the algorithm development. Since JPEG published the standard in lossless compression mode, many new approaches have been developed with the claim that all the new algorithms outperform JPEG. In summary, the improvement resulted come from the two major modifications: (i) optimisation of predictive coding; and (ii) better entropy coding. The first direction is represented by vector quantizer based predictive coding [1-2] and linear predictive coding(LPC)[2-3]. The second is typically represented by Laplacian distribution based arithmetic coding[4]. The vector quantizer based predictive coding adopts vector quantization to pick up those contexts which share the same controid with the context of the pixel to be encoded. The predictive value is then produced by those pixels predicted by all the contexts picked up by the vector quantizer[1]. LPC technology is developed to minimise the distance between the predictive value and the pixel predicted:

$$\min_{x,y \in image} E\{y\text{-}y'(x)\}^2$$

In practice, the summation of errors within a fixed size window in the image is used instead which can be given as follows:

$$\sum_{i,j \in window} \{y - y'(x)\}^2$$

where y is the pixel to be encoded and $y'(x)$ is the predictive value based on the context x.

The research work reported is to explore other new alternatives in lossless image compression by using wavelet transform as the preprocessing stage prior to entropy coding. The structure of the paper is designed into three sections. Section 2 describes the algorithm design; section 3 reports experiments and simulation results. In addition, conclusions are drawn for the pioneering work described in the paper in lossless image compression and further research potentials are also identified in this section.

2. ALGORITHM DESIGN

For an image of size N×N with 8 bits per pixel, the normal lossless image compression using arithmetic coding is to build up an adaptive statistical model with 256 different grey levels altogether. Each image has N×N pixels to be compressed. After a wavelet transform, the multiresolution decomposition gives us the same number of WT coefficients and the lowest resolution subimage to be encoded by arithmetic coding. The overall wavelet transform can be illustrated in Fig. 1 and the internal structure of PE in Fig. 2 [5-6]. The reconstruction at this stage is lossless subject to the precision of representation in this process.

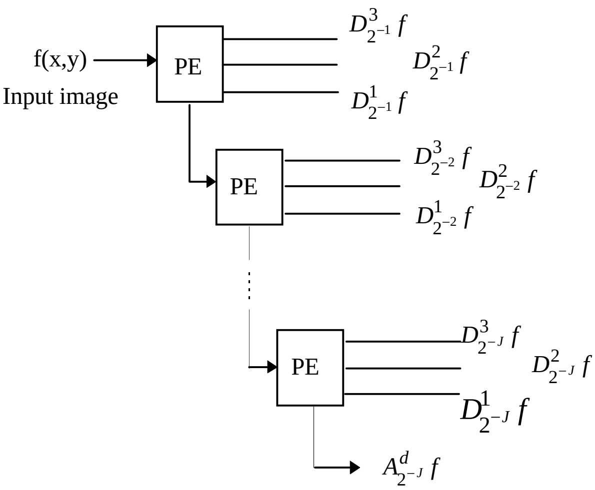

Figure 1 *Overall wavelet transform system*

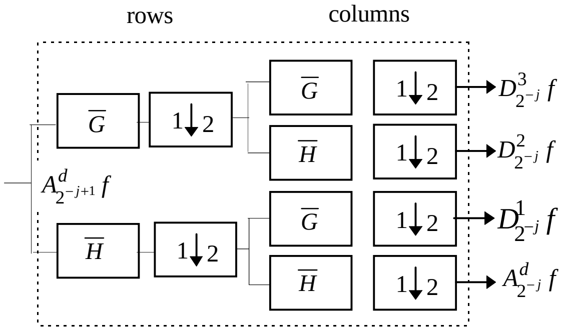

Figure 2 *Internal structure of a PE*

The design of both high pass filter, g(n), and low pass filter, h(n) are related to each other as well as the wavelets, $\psi(xy) = \{\psi^H(xy) = \Phi(x)\ \psi(y)$, $\psi^V(xy) = \psi(x)\ \Phi(y)$, $\psi^D(xy) = \psi(x)\ \psi(y)\}$, and the scaling function $\Phi(x)$ [5-6]. When biorthogonal bases of wavelets are used for both decomposition of input image and its reconstruction, the filters can be constructed from the following equation.

$$H(\xi)\overline{H(\xi)} = \cos(\xi/2)^{2l} \times \left\{\sum_{p=0}^{l-1}\binom{l-1+p}{p}\sin(\xi/2)^{2p} + \sin(\xi/2)^{2l}R(\xi)\right\} \quad (1)$$

where $R(\xi)$ is an odd polynomial in $\cos(\xi)$, and $2l = k + \overline{k}$.

Three types of filters [6], $H(\xi)$, are designed by Antonini et al. in reference[6]. Namely spline filters, spline variant with less dissimilar lengths and filters close to orthonormal filters. We use the latter filter to complete the image decomposition by wavelet transform.

Statistical distribution of wavelet coefficients can be modelled by the following equation[5].

$$p(x) = ke^{-(|x|/\alpha)^{\beta}} \quad (2)$$

where constant k is used to adjust the function so that the following condition can be satisfied:

$$\int_{-\infty}^{+\infty} p(x)dx = M - \frac{M}{2^J} \quad (3)$$

in which $M = N \times N$ is the total number of pixels and J is the number of decomposition levels. Since the model is used to approximate the probability distribution of wavelet coefficients, the total number of the coefficients is $M - M/2^J$ exclusive of those pixels inside the subimage with the lowest resolution.

To obtain a practical simulation for the statistical property, we can further use a generalised Gaussian law to represent the function given in [6]:

$$p_{md}(x) = a_{md}e^{-(|b_{md}x|^{r_{md}})} \quad (4)$$

where:

$$a_{md} = \frac{r_{md}\Gamma\left(\frac{3}{r_{md}}\right)^{\frac{1}{2}}}{2\sigma_{md}\Gamma\left(\frac{1}{r_{md}}\right)^{\frac{3}{2}}}$$

$$b_{md} = \frac{1}{\sigma_{md}} \frac{\Gamma\left(\frac{3}{r_{md}}\right)^{\frac{1}{2}}}{\Gamma\left(\frac{1}{r_{md}}\right)^{\frac{1}{2}}}$$

and *m* stands for the resolution scale of the subimage concerned and *d* for the oriented directions in the coefficients; σ_{md} is the standard deviation of the subimage at (*m*, *d*) and $\Gamma(x) = \int_0^{\infty} e^{-u} u^{x-1} du$ is the normal Gamma function.

Experiments carried out in [6] show that when the parameter r_{md} is selected to be 0.7 the generalised Gaussian law can best approximate the probability distribution function. For other subimages represented by wavelet coefficients at different resolution scales and directions, the results are very similar. Henceforth, a few values of r_{md} around 0.7 can be tested to get multiple general models for the statistic property of those wavelet coefficients corresponding to $\psi^H(xy)$, $\psi^V(xy)$ and $\psi^D(xy)$. Since the subimage at the lowest resolution normally has totally different probability distribution from those represented by wavelet coefficients, we consider a fixed length coding scheme to encode the subimage with the lowest resolution. In other words, 8 bits per pixel is maintained in this particular case. For this part of entropy coding, no compression can be achieved. The number of pixels involved, however, is only $M/2^J$. When the level of image decomposition is designed to be relatively large, the effect of no data compression for that part of entropy coding can be ignored.

By putting the model represented by equation (4) into the integral (3), the standard deviation σ_{md} can be determined from:

$$\frac{1}{\sigma_{md}} \int_{-\infty}^{\infty} A e^{-\left(\frac{Bx}{\sigma_{md}}\right)^{r_{md}}} dx = M - \frac{M}{2^J} \qquad (5)$$

where constant A and B can be obtained from Gamma functions explained above.

Two options are available at this stage for the specific algorithm design. One is to establish a model for each subimage refined by the probability approximation, and the other is to establish one model for all the subimages regardless of their resolution scale and orientation directions. The latter is based on the fact that all the subimages represented by WT coefficients virtually have very similar statistical property with r_{md} around 0.7.

To obtain adaptive statistical models for arithmetic coding, initial standard deviation is estimated from all the possible 8 bit pixel values by tabulating equation (5). Probability distribution for each D value at the first level decomposition can then be produced from the resulted table and equation (4). After that, further estimation of σ_{md} is carried out from these D values and the table of standard deviation is also updated. Hence the probability distribution for the next level D values are obtained on this basis. The adaptive scheme of producing statistical model proceeds until the *J*th level decomposition is reached.

3. EXPERIMENTS AND CONCLUSIONS

Software simulation of the above algorithm is implemented and tested on a number of image samples in comparison with JPEG. The experimental results are summarised in Table I in which figures given are compression ratios. The image samples are illustrated in Fig. 3.

Table I Experimental Results

Image Samples	JPEG	Proposed Algorithm
Pepper	0.43	0.41
Bridge	0.58	0.59
Clown	0.56	0.54
Truck	0.49	0.48

(a) *Truck* **(b)** *Clown* **(c)** *Bridge*

Figure 3 *image samples*

Although the probability distribution varies from image to image, the theoretical analysis and a priori knowledge can be used as an approximation scale to produce initial statistical models for arithmetic coding of those possible wavelet coefficient entries. The variation of statistical property regarding each specific input image can also be acquired by updating the model with the occurrence of each coefficient. The arithmetic coding is a modified version of PPPM in reference[4]. For the convenience of decoding, we sent the fixed length code for the lowest resolution subimage first. As the number of pixels is fixed at $M/2^J$, all the codes can be easily decoded without adding any overhead bits for their identification.

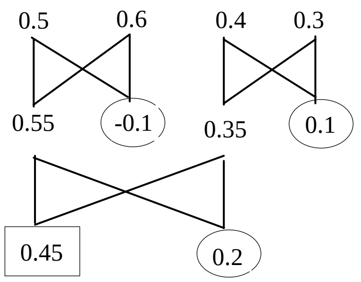

Figure 4 A simple example

In conclusion, the initial efforts in incorporating wavlets into lossless image compression algorithm design does provide competitive performance compared with JPEG. Further analysis shows that the wavelets based algorithm plays a role similar to that of predictive coding. A typical example is given in Fig. 4, where one dimensional sequence of 4 values are to be encoded. If we use the simple average as the $\overline{H}$ filter and the difference of its two inputs as the $\overline{G}$ filter, the wavelet transform coefficients can be produced as shown in Fig. 4, where the figures in circus are D values and the figure in square is the A value. According to the proposed scheme, only those values either in circus or in square are encoded for correct decoding of the original sequence.

Therefore, the simplest wavelet transform gives us 3 D values and one A value. From Fig. 4, it can be seen that all the D values are produced by prediction in which the context is simply its preceding data. The key to the success of the proposed algorithm relies on the filters adopted and how well they can produce coefficients close to zero. Hence further work can be concentrated on the following directions:

- Designing better wavelet transform filters in such a way that the majority of resulted coefficients are close to zero.

- Investigating systematic analysis of probability distribution of those coefficients so that better statistical model can be derived for arithmetic coding.

As a direct result, further work is undergoing to improve the performance of the algorithm in line with other existing technologies.

REFERENCES

1. Slyt M.J. and Neuhoff D.L. ‘A nonlinear VQ-based predictive lossless image coder’, Proceedings of Data Compression Conference, March 29-31, 1994, USA, pp 166-175.

2. Jiang, J. ‘A neural network based algorithm for lossless image compression’ Proceedings of Visual’96: International Conference on Visual Information Systems, 5-6 February 1996, Australia, pp 192-200, ISBN: 1-875-33852-7.

3. Kuroki, N. etc. ‘Lossless image compression by two-dimensional linear prediction with variable coefficients’ IEICE Trans. Fundamentals, Vol E75-A, No 7, July, 1992, pp 882-889.

4. Howard P. and Vitter J.S. ‘New methods for lossless image compression using arithmetic coding’ Information Processing and Management, Vol 28, No 5, pp765-779. 1992

5. Mallat S.G.’A theory for multiresolution signal decomposition: The wavelet representation’ IEEE Trans. Pattern Analysis and Machine Intelligence, Vol. 11, No.7, July 1989, pp 674-693.

6. Antonini, M.etc. ‘Image coding using wavelet transform’ IEEE Trans. Image Processing, Vol 1, No 2, April 1992, pp 205-220.