MATCHED BLOCK TRANSFORM DESIGN TECHNIQUES

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ABSTRACT

In this work, two new design techniques for matched (adaptive) orthogonal block transforms (BT) based partly on Vector Quantization (VQ) are presented. Both techniques start from reference vectors that are adapted to the characteristics of the signal to be coded. Then the corresponding orthogonal block transform is obtained in the first technique via signed permutations of the reference vector, while in the second technique an optimization search in the null space of the reference vector is executed. The resulting transforms represent a signal coding tool that stands between a pure VQ scheme on one extreme and signal independent fixed block transformation like DCT on the other.

1 INTRODUCTION

This study has provided further insight into the two well known techniques, BT and VQ [1-2]. It has been shown that better compression performance in the sense of less coding artifacts can be achieved by employing nice features of BT and VQ together. Our proposed coding technique can be interpreted as a bridge between these two techniques.

Since image signals are nonstationary, it is conjectured that a block transform for which the basis functions adapt to the local statistics of the image will have superior compression performance. The adaptation of the transform bases is instrumented by constructing a transform book, which is similar in concept to a VQ codebook. In the transform book, each block transform corresponds to some typical image structure or waveform. In each segment of the image the appropriate transform block should be chosen based on some simple feature of the regional image statistics or simply using the mean square error criterion.

However the transform book contains much fewer (typically 4-8) transforms as compared to the number of vectors in a conventional VQ codebook. Notice that if some bit assignment rule were to single out the first row only, then the proposed coder would reduce to a scheme similar to the VQ coder. On the other hand in our VQ-based block transform the rows other than the first one can be interpreted to encode the residual error of the VQ. Selection of the VQ vectors, which will act like seeds in our proposed block transform designs, is outlined in [3] both for 1-D and 2-D cases.

In this paper, we intend to design "VQ-adaptive block transforms", such that the basis vectors of the transform are expected to match the statistics of the input process. This can be effected by having the rows of the transform block to resemble waveform portions most typically encountered in the process to be coded. An adaptive matched block transform can be constructed first by obtaining a reference vector which also forms, let us say, the first row, while the other rows of the transform block are generated by operations on this reference vector. A set of three such vectors are illustrated in Fig.4

These two new methods to obtain an orthogonal block transform from a given reference vector will be referred to as Vector Quantization-Permutation Based Block Transform (VQ-PBT) and VQ based Optimization Based Block Transform (VQ-OBT). Fig.1 shows the block diagram of the proposed VQ-adaptive block transform techniques. The performance of the proposed matched BT technique vis-a-vis DCT in the case of 1-D signal is illustrated in Fig.2, where a sample line from the Lena image is employed. It can be observed that VQ-PBT can handle abrupt changes better than DCT.

2 PERMUTATION BASED ORTHOGONAL BLOCK TRANSFORM

One method to generate an orthogonal block transform matrix from a given reference vector is to use systematic permutations and sign changes on the elements of the reference vector [3-4]. Thus, while the first row of the transform matrix is constituted of the reference vector itself, the other rows are simply given by permutation and sign change operations on the elements of the reference vector. The end result is a codebook of orthogonal block transform matrices, each matched to a dominant structure in the image.

Consider now generically one such vector in the codebook, denoted as $\mathbf{h}_0$, whose any codebook index has been omitted for simplicty. This vector will also constitute the first row of the transform. The row vector $\mathbf{h}_0$ has N components

$$\mathbf{h}_0 = [h_0(0)\ h_0(1)\ ...\ h_0(N-1)]$$

while the other rows are denoted as $\mathbf{h}_i$, $i = 1, ..., N-1$. Furthermore rows of the transform matrices form orthonormal sets, that is

$$\mathbf{h}_i \mathbf{h}_j^T = \delta_{i-j} \tag{1}$$

The construction of an orthogonal matrix given a reference vector $\mathbf{h}_0$ based on the permutations and sign changes of its components is outlined below:
Define **permutation** functions

$$\beta_0(j), \beta_1(j), ..., \beta_{N-1}(j), \quad j = 0, 1, ..., N-1. \tag{2}$$

and **sign change** functions

$$\alpha_0(j), \alpha_1(j), ..., \alpha_{N-1}(j), \quad j = 0, 1, ..., N-1 \qquad \alpha_i(j) \in \{-1, 1\} \tag{3}$$

where the subscript denotes the row number, and the argument denotes the component position in a row. Then $\mathbf{h}_i$ is defined as

$$h_i[j] = \alpha_i(j) h_0[\beta_i(j)] \quad j = 1, ..., N-1 \tag{4}$$

This definition says that the j'th element of the i'th row of the transform matrix is the same as the element of $\mathbf{h}_0$ on the $\beta_i(j)$ 'th position except with a sign change given by $\alpha_i(j)$.

Such permutation and sign change operations result in pairwise orthogonality if only if they satisfy the relationship given as [3-4]

$$\alpha_i(k)\alpha_j(k) + \alpha_i(t)\alpha_j(t) = 2\delta_{i-j} \tag{5}$$
$$0 \leq i, j, k \leq N-1$$
$$\beta_i^{-1}(\beta_j(k)) = \beta_j^{-1}(\beta_i(k)) \tag{6}$$
$$0 \leq i, j, k \leq N-1$$

where $t = \beta_i^{-1}(\beta_j(k))$.

This technique, a variation of Hadamard modulation of a reference vector, is in fact a generalization of the Walsh-Hadamard Transform since the low-pass function $\mathbf{h}_0$ will have in general components different than 1's. The only restrictions on the reference vectors are their size, which must be integer power of 2.

In permutation based orthogonal block coder designed using the algorithm described above, all basis functions use the same set of coefficients (in different shift and sign positions). Notice also that the orthogonality of the rows does not depend on the word length of the coefficients, but on their relative positioning. Hence, multiplier free structures can be obtained by simply choosing the coefficients to be integer, let us say the first two digit taken. It has been observed that truncating the coefficients to two digits has negligible effect on the compaction performance of the transform.

3 OPTIMIZATION BASED ORTHOGONAL BLOCK TRANSFORM

If, on the other hand, maximization of coding gain is a more important consideration than efficient implementation, it is possible to generate all the rows of the transform matrix out of an optimization scheme, thereby, achieving an even higher compaction. This process leads to orthogonal block transforms that no longer share the same coefficient values. For given first basis function and input statistics, this technique aims to construct $N-1$ orthogonal basis functions such that overall transform maximizes the compaction performance. Again starting from $\mathbf{h}_0$'s as obtained from the VQ classification algorithm, one proceeds to select any $\mathbf{D}$ matrix of dimensions $(N-1 \times N)$, in the null space of $\mathbf{h}_0$ that is

$$\mathbf{D}\mathbf{h}_0 = 0$$

Since $\mathbf{D}\mathbf{D}^T > 0$ is a positive definite matrix, one can introduce a $\mathbf{Q}$ matrix such that

$$\mathbf{Q} = \mathbf{D}^T(\mathbf{D}\mathbf{D}^T)^{-\frac{1}{2}} \tag{7}$$

and thus one can construct a transform matrix $\mathbf{Q}$ such that

$$\mathbf{A} = [\mathbf{h}_0 \ \mid\ \mathbf{D}^T(\mathbf{D}\mathbf{D}^T)^{-\frac{1}{2}}]^T \tag{8}$$

The resulting matrix $\mathbf{A}$ is an orthonormal matrix for any given reference vector $\mathbf{h}_0$ satisfying, $\mathbf{A}\mathbf{A}^T = \mathbf{A}^T\mathbf{A} = \mathbf{I}_{N \times N}$. There are several ways to span the null space of $\mathbf{h}_0$, i.e., several possibilities for $\mathbf{D}$. Since our objective is to design an orthonormal transform having good compaction performance, we set an optimization procedure in this direction:
Recall that optimum compaction performance is obtained when geometric mean of the transform coefficient variances is minimum. Thus, we set the optimization problem as

$$min\{J\} = \prod_{i=1}^{N} diag_i\{\mathbf{A}\mathbf{R}_{xx}\mathbf{A}^T\} \tag{9}$$

with constraint

$$\mathbf{D}\mathbf{h}_0 = 0$$

for any given unit-norm $\mathbf{h}_0$ vector. Several block transform coders are designed for reference vectors obtained from VQ algorithm.

4 CONCLUSIONS

Two methods for signal adaptive orthogonal block transform design have been advanced. This adaptive nature is due to the fact that these block transforms are generated from a reduced size VQ basis obtained from the signal process itself. In addition in the VQ-OBT technique, the autocorrelation sequence of the signal is employed in the optimization. It has been found that the VQ-PBT technique is more practical in wider range of applications and leads to multiplierless transformations, while the VQ-OBT technique has a slight compaction advantage.

The adaptive transforms have superior performance especially in the highly textured and patterned regions, while the DCT bases remain superior in smooth regions. The proposed hybrid coder then tests each segment of the signal or block of the image and decides

- to use the DCT bases rather than the corresponding VQ-PBT (VQ-OBT) bases if the segment is DC-like.
- to use the corresponding VQ-PBT (VQ-OBT) bases if the segment is not DC-like, i.e., with high variance.

The proposed coder has than superior compaction performance as compared to DCT both in the rendition of details in the image and in the PSNR figures. Fig.3 gives quantative comparison of proposed techniques and DCT in terms of PSNR values for sample images taken from motion compansated frame differences of Miss America sequence and the Lena image. It can be observed that the hybrid scheme (DCT in smaller regions and VQ-PBT in rough regions) has a PSNR advantage of 0.5-0.8 dB over the conventional DCT. We believe thus that the coder forms a serious competitor to DCT for both still and video images in the low bit rate range.

References

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[4] H. Çağlar, S. Güntürk, E. Anarım, B. Sankur *"VQ Based Adaptive Block Transform Coding of Images,"* submitted to IEEE Trans. on Image Processing, August, 1995.

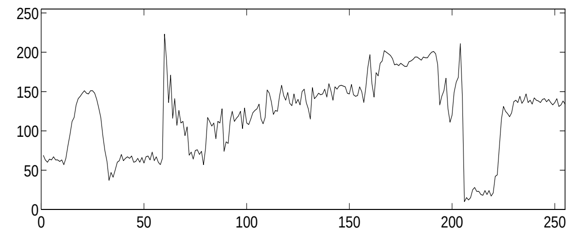

Fig. 2-a) A sample line from Lena image

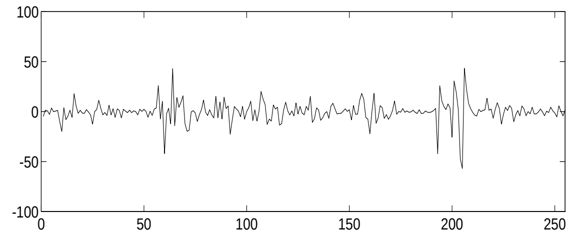

Fig. 2-b) Error signal for adaptive 8x8 VQ-PBT with transform codebook of size 8 and reconstructed from 2 coefficients in each block

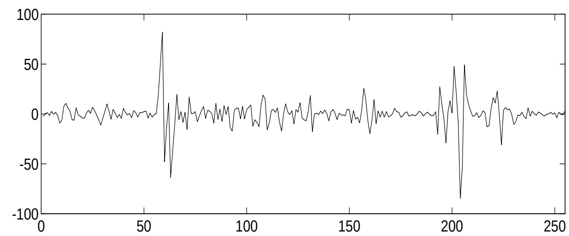

Fig. 2-c) Error signal for DCT with the same compression rate above

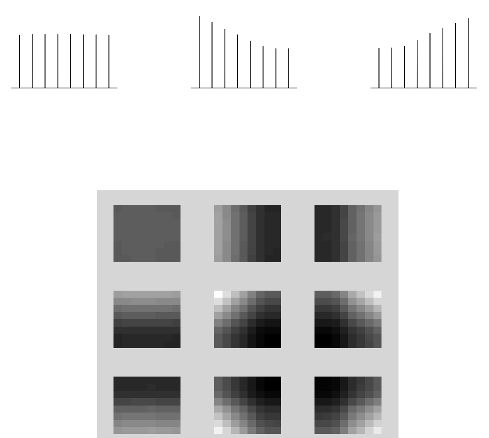

Fig.4. A reference codebook of size 3 and the 9 image patterns formed with the outer product of these vectors.

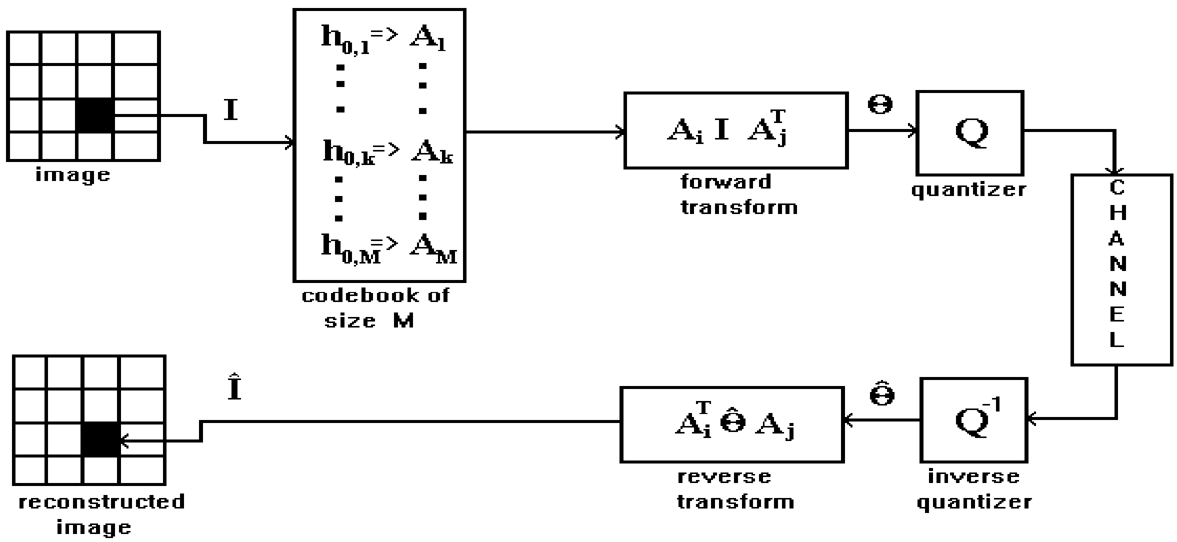

Fig.1 The block diagram of the proposed matched block transform technique.

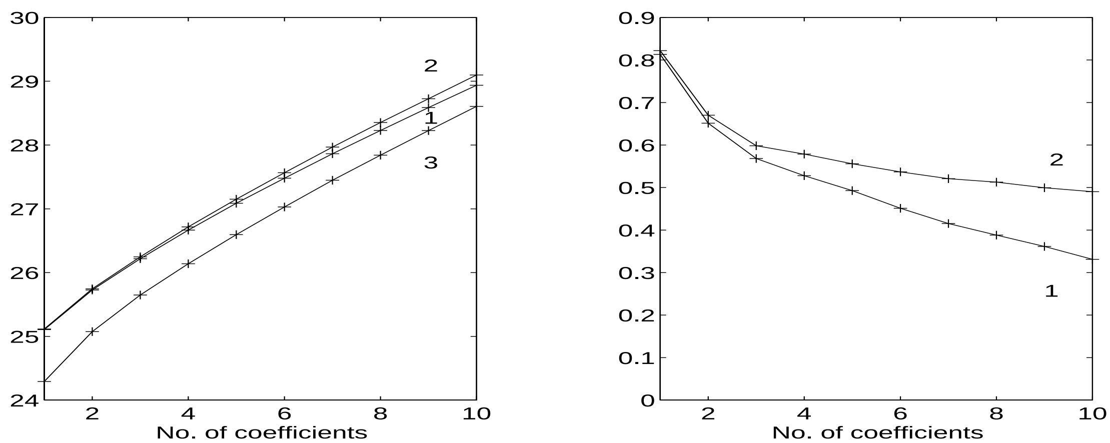

Fig. 3a) PSNR results on Miss America for 1:VQ-PBT, 2:Hybrid VQ-PBT, 3:DCT techniques
3b) Difference PSNR results for 1:PSNR(VQ-PBT)-PSNR(DCT),
2:PSNR(Hybrid VQ-PBT)-PSNR(DCT)

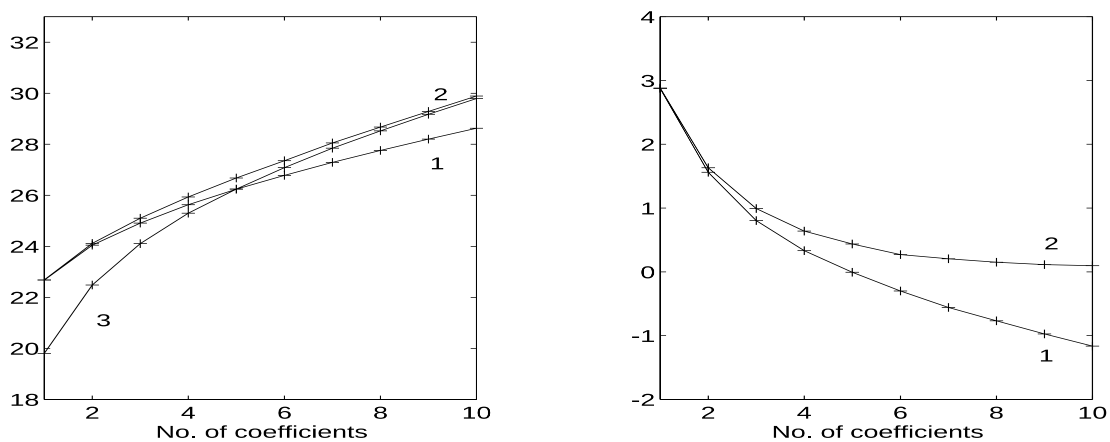

Fig. 3c) PSNR results on Lena image for 1:VQ-PBT, 2:Hybrid VQ-PBT, 3:DCT techniques
3d) Difference PSNR results for 1:PSNR(VQ-PBT)-PSNR(DCT),
2:PSNR(Hybrid VQ-PBT)-PSNR(DCT)