Fractal Coding of Subbands using an Oriented Partition

Kamel Belloulata, Atilla Baskurt, Hugues Benoit-Cattin and Rémy Prost

CREATIS, Research Unit - CNRS (#C5515), affiliated to INSERM

INSA 502, 69621 Villeurbanne cedex, France.

e-mail : belloulata @ creatis.insa-lyon.fr

ABSTRACT

In this paper, we propose a new image coding scheme based on fractal coding of the coefficients of a wavelet transform, in order to take into account the self-similarity observed in each subband. The original image is first decomposed into subbands containing information in different spatial directions and at different scales, using Finite Impulse Response filters. Subbands are encoded using Local Iterated Function Systems (LIFS), with range and domain blocks presenting horizontal or vertical directionalities. Their sizes are defined according to the correlation lengths in each subband. The proposed method is applied on standard test images, distortion vs rate is compared with the algorithm proposed in [1] for fractal coding of the whole image. We also discuss about the distortion vs rate improvement obtained on high frequency subbands when using fractal coding instead of pyramidal vector quantization [2,3].

1 INTRODUCTION

Fractal image coding based on the self-similarity has been proposed by Barnsley [4,5] which has introduced the Iterated Function System (IFS). The first fractal coding algorithm based on Local Iterated Function Systems (LIFS) has been developed by Jacquin [1]. This approach using self-affine transformations has been found powerful at low bit rates by Fisher [6] and Beaumont [7], original partitions have been proposed in [6],[8] for searching the self-similarity and recent studies have implemented LIFS in the DCT domain [9-10]. However, these approaches do not avoid some blocking effects, especially at low bit rates and the search of the self-similarity is generally very time consuming. The use of the subband decomposition is one way of overcoming these two problems :

- it reduces the blocking effect in the decoded image (Fig.2).
- it allows to separate the directional information contents into different subbands, which leads to a "natural classification". This significantly speeds up the search of the self-similarity.

2 LIFS THEORY

The IFS (Iterated Function System), is a powerful tool used in the image compression field. A LIFS is an extension of the IFS concept [5] in which the maps ($w_1,...,w_N$) are not applied to the whole image but to restricted domains. The contractive mapping fixed point theorem and the collage theorem still hold.

Let A be the image to be coded, $R=\{R_1,...,R_N\}$ be the set of N range blocks which partitions A, and $\mathbf{D}=\{D_1,...,D_M\}$ be the collection of domain blocks forming another partition of A. We have to find for each R_i both the best D_j in the set $\mathbf{D}$ and the transformation w_i which minimizes the distance $d(A\cap R_i,\ w_i(A\cap D_j))$, where d(.,.) is a measure of a *dissimilarity*. From the collage theorem, once $W=\{w_1,....,w_N\}$ is known, we can compute an approximation $\hat{A}$ of the original image by iterating W on any initial image B [11] ($W^{ok}(B)=\hat{A}$ with $k\in\mathbf{N}$, where the index **'ok'** is the iteration number of **W**). A sufficient contractivity requirement is the eventual contractivity of W [6], if there exists a positive integer m called the exponent of eventual contractivity such that W^{om} is contractive. The generalized theorem of collage is given by

$$d(A,\hat{A})\leq\frac{1}{1-s}\frac{1-\sigma^m}{1-\sigma}d(A,W(A))$$

with

$$\hat{\mathbf{A}}=W(\hat{\mathbf{A}})=\lim_{k\to\infty} W^{ok}(B) \quad (2$$

where s is the contractivity of W^{om} and σ is the Lipschitz factor of W. The collage theorem allows to bound the distance between the original image and the attractors. This bound depends on three factors: the Lipschitz factor σ, the contractivity factor s, and the quality of the collage W(A). If the quality of W(A) can be enhanced by slightly increasing σ, we might in fact obtain an eventual contractivity.

The compression is achieved by coding W, which serves as representation of the fixed point in the fractal domain and which can be expressed with fewer bits than the original image itself. The compression ratio will depend on the description of the image partitions, the quantization and coding of the parameters of each local transformation w_i.

3 FRACTAL CODING OF SUBBANDS

The previous studies on fractal coding have been implemented using the original image domain. The self-similarity is searched on the whole image although the physical images do not usually have fractal properties. But, some local self-similarity exists in the neighbourhood of the range block R_i.

In this study, we develop the Fractal Coding of Subband using an Oriented Partition which allows:

- to quantize and code subbands presenting an increased self-similarity, using adapted directional partitions [12] (Fig.1);
- to reduce the computation time: the searching area (one subband) is four times reduced using an octave decomposition and equivalent block sizes (4x4, 2x8, or 8x2 for example). Note that the computation time gain is highly greater than the time required for both the decomposition and the bit allocation.

The partition pool R contains vectors belonging to different subbands corresponding to the resolution and orientation under consideration. We choose the domain blocks D_j sizes twice larger than blocks R_i and a spatial contraction (2x2 averaging) is done on D_j in order to match R_i. The shape of vectors R_i and D_j is adjusted, taking into account the detail orientation of each subband and the correlation length in each direction.

Methods using similar partitions have been applied in Vector Quantization [12]. Here we use the oriented partition (R_i and D_j) as part of a Local Iterated Transform algorithm.

In our work, we use an octave subband decomposition (Fig. 1) using the orthonormal wavelet 8 taps filters of Daubechies [13]. These filters are orthonormal and provide a perfect reconstruction.

3.1 Adaptive partition of subbands

The subband decomposition of an image allows to assemble the directional information contents in different subbands. It leads to a "natural classification" and to an increasing self-similarity in each subband, especially in high frequency subbands.

The sizes of R_i and D_j blocks are determined by computing the correlation length of both rows and columns in each subband, in order to take into account the directionality. Note that the correlation in the horizontal (respectively vertical) subband should be preponderant in the horizontal (respectively vertical) direction. This remark leads us to choose :

- horizontal mxn rectangles (m<n), for the ranges and domains blocks in horizontal detail subbands;
- vertical mxn rectangles (m>n), for the ranges and domains blocks in vertical detail subbands.

In order to confirm this choice, we measure the correlation length given by:

$$C_i(k) = \frac{\sum_{j=1}^{N-k} x(i,j).\,x(i,j+k)}{\sum_{j=1}^{N} x^2(i,j)} \qquad k = 1,\dots,N.$$

for a row i and where x(i,j) is the grey level.

Clearly, $C_i(0) = 1$ and $C_i(k) < 1$. The same formula is applied for column by changing $\sum_j$ by $\sum_i$ and x(i,j+k) by x(i+k,j). For each row (respectively column), the value of k giving $C(k) > 0.5$ is determined. The correlation lengths r on rows and c on columns, are computed as the mean of the parameter k for all the rows and columns, respectively.

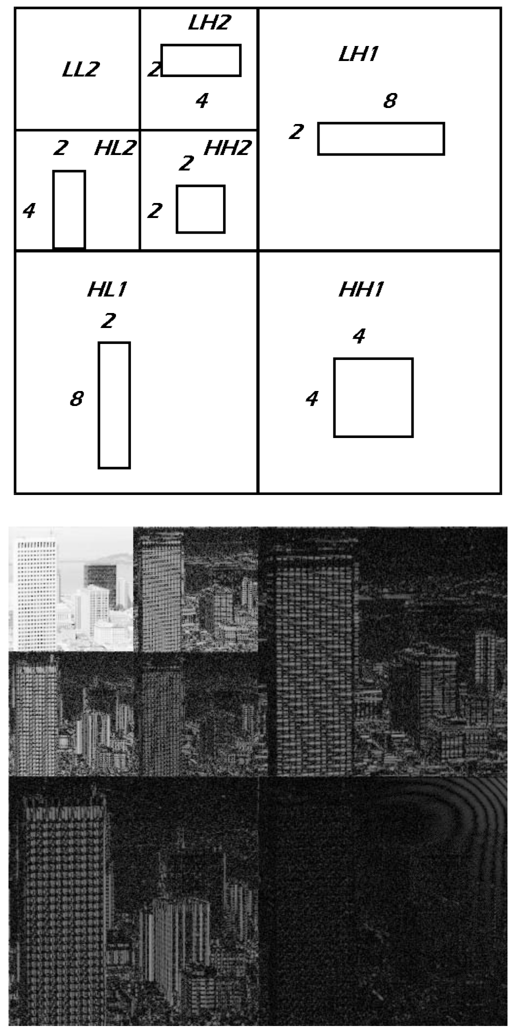

Fig. 1 *Octave decomposition of an image until resolution '2^{-2}'. LLi, LHi, HLi, HHi are the low frequency, the horizontal high frequency, the vertical high frequency and the angular high frequency subband at resolution 2^{-i} respectively; the size of the range blocks in each subband is also presented.*

Computing r and c of the "Building" image, at the resolution "2^{-2}" in the horizontal detail subband, we find that r=3.89 and c=1.20. In the vertical detail subband and at the same resolution, we find r=1.15 and c=4.19. These results confirm our previous hypothesis on the shape of the blocks.

In the diagonal detail subband, at resolution "2^{-2}" of the "Building" image, we calculate r and c and find that r=2.49 and c=2.10. Thus, r and c are approximately equal for the diagonal subbands.

We adjust the size of blocks to the resolution of the subbands. If the block size is mxn at resolution "2^{-2}", the corresponding block size at resolution "2^{-1}" is 2mxn or mx2n (Fig. 1). For example, if R_i size is (2x4) in LH2, it is (2x8) in LH1, at a lower resolution. Only the size of the privileged direction is changed.

3.2 Optimal bit Allocation and coding scheme

Given a total bit budget, the subbands bit rate and the number of decompositions are allocated using a similar algorithm to the one proposed by Ramchandran [14]. LL2 is quantized with DPCM and the other subbands using LIFS and a directional partition.

In order to take into account the local statistics, to reduce the computation time and to increase the compression ratio, R_i blocks are classified into two classes : R_i with high variance are encoded with the appropriate w_i. The uniform R_i (low variance) are represented only with their average grey level without searching a similarity. So, we allocate a number of bits function of the variance of the range blocks. This allows to vary the compression ratio by changing a decision level (a threshold for the variance).

For a global bit rate of 0.8 bpp, Tab. 1 shows the enhancement of the reconstructed image quality when applying the fractal coding on subbands (square and rectangular blocks) instead of on the original image. Note that for the "Building" image when using the directionality, our gain is about 3 dB (Tab. 1 and Fig. 2).

PPSNR (dB) for 0.8 bpp	Lena	Peppers	Building
Jacquin algorithm	32.1	30.2	31.8
Square Partition of Subband	32.87	31.9	33.9
Rectangular Partition of subband	33.2	32.2	34.9

Tab. 1 Decompressed image quality comparing fractal coding of subbands and Jacquin algorithm.

Fig. 2 Decompressed image quality when using the Fractal Coding of Subbands FCS with directionality
SNR=34.9 dB. Bit Rate = 0.8 bpp.

We also present the distortion vs rate obtained on high frequency subbands using the Fractal Coding of Subbands (FCS) or the Pyramidal Vector Quantization (PVQ) [3]. Fig. 3 shows the PPSNR/bpp increases we have with FCS comparing to the PVQ. For the PVQ, we use a concentric pyramidal structure and a product code to encode the radius of the pyramids and the cubic lattice points lying on each pyramid. We remark that FCS using directional blocks is particularly efficient between 0.3 bpp and 1.1 bpp. These results confirm that the fractal approach is a good candidate for coding subbands.

4 CONCLUSION

A new coding approach has been presented based on the combination of subband decomposition and LIFS with oriented blocks. Maintaining a high quality (Fig. 2), these results show that fractal coding is very efficient for coding high frequency subbands (Fig. 3). At low encoding rates, the subband coding with LIFS using oriented partitions yields a PPSNR slightly larger than the PPSNR obtained using PVQ subband coding and using LIFS with square block (Table.1). At larger rates (say, larger than 1 bpp), PVQ subband coding is better (Fig. 3).

Our approach is different from the studies proposed in [15,16] which use the explicit link between a local fractal transform and a multiresolution transform and perform a block prediction between subbands of different resolution without using LIFS. Indeed, our study is based on Jacquin algorithm and searches the self-similarity in each subband with directional partitions.

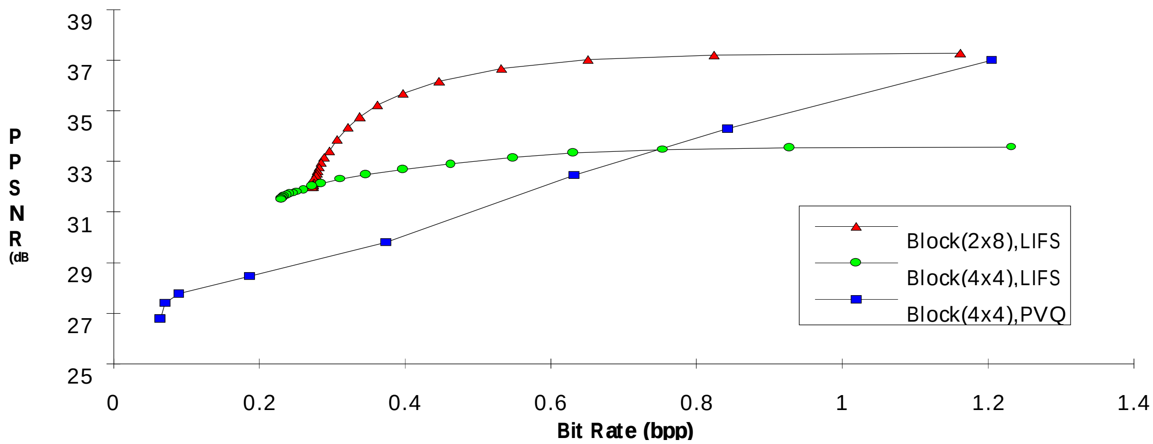

Fig. 3 PPSNR/bpp curves obtained with the horizontal high frequency subband LH1 of the image Building 512x512x8 bits, using 4x4 and 2x8 range blocks for fractal coding of subbands (LIFS) and 4x4 blocks for pyramidal vector quantization (PVQ).

4 REFERENCES

[1] JACQUIN A.E., "Image coding based on a fractal theory of iterated contractive image transformation", *IEEE Trans. on Image Processing*, Vol. IP-1, N° 1, pp. 18-30, January 1992 .

[2] FISCHER T. R., "A Pyramid Vector Quantizer", *IEEE Trans. on Information Theory*, Vol. IT-32, N° 4, pp. 568-583, July 1986.

[3] BARLAUD M., SOLE P., GAIDON T., ANTONINI M, MATTHIEU P., "Pyramidal lattice vector quantization for multiscale image coding"', *IEEE Trans. on Image Processing*, Vol. IP-3, N° 4, pp. 367-381, July 1994.

[4] BARNSLEY, M.F.: "Fractal everywhere", *Academic Press Inc., London*, 1988.

[5] BARNSLEY, M.F. and HURD, L.: "Fractal Image Compression'", Wellesley, MA, AK Peters, 1993.

[6] FISHER, Y., ed., "Fractal Image Compression: *Theory and Applications to Digital Images", Springer-Verlag, New York, 1995.*

[7] BEAUMONT, J.M.: "Image data compression using fractal techniques", *BT Technol. J*ournal , Vol. 9, N° 4, pp.93-109, 1991.

[8] DAVOINE F., SVENSSON J., CHASSERY J.M., "A mixed triangular and quadrilateral partition for fractal image coding", in Proc. IEEE ICIP-95, Vol. III, pp. 284-287, Oct. 23-26, 1995.

[9] BARTHEL, K.U. and VOYE, T., "Adaptive Fractal Image in the Frequency Domain", Proceedings of International Workshop on Image Processing, Budapest, Journal on Communication, vol.XLV, pp.33-37, May-June 1994.

[10] ZHAO, Y. and YUAN, B., "Image Compression Using Fractals and Discrete Cosine Transform", Electronics Letters, Vol.30, n^0.6, March.1994, pp.474-475.

[11] KREYSZIG, E.: "Introduction to Function Analysis with Applications", *Robert E. Krieger Publishing Company, Malabar, Florida*, (reprint edition), p.524, 1989.

[12] AKROUT N.M., CHAOUKI D., PROST R. and GOUTTE R., "Codeword orientation: an improved subband coding/vector quantization scheme for image coding", *Optical Engineering*, Vol. 33, N°7, pp. 2394-2399, July 1994.

[13] DAUBECHIES I., "Ten lectures on wavelets", SIAM Philadelphia, Pennsylvania, 1992.

[14] RAMCHANDRAN K. and VETTERLI M., "Best wavelet packet bases in rate-distortion sense", *IEEE Trans. on Image Processing*, Vol. IP-2, N° 4, pp. 160-175, April 1993.

[15] RINALDO R. and CALVAGNO G., "Image coding by block prediction of multiresolution subimages", *IEEE Trans. on Image Processing*, Vol. IP-4, N° 7, pp. 909-920, July 1995.

[16] SIMON B., "Explicit link between local fractal transform and multiresolution transform", in Proc. IEEE ICIP-95, Vol. I, pp. 278-281, Oct. 23-26, 1995.