LOCALLY ADAPTIVE TECHNIQUES FOR STACK FILTERING

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ABSTRACT

This paper introduces a new structure for stack filtering, where the filter adapts to the local characteristics encountered in data. Both supervised and unsupervised techniques for optimal design are investigated. We split the image into small regions and select the stack filter to process each region according to the spatial domain or threshold level domain characteristics of the input signal. This method provides a significant improvement potential over the global stack filtering approach. Some local statistics are computed, to build a reduced input space which efficiently describes the most important local characteristics of data. Vector quantization is used for clustering the reduced input space into a small number of regions, and then finding a mapping between reduced input space clusters and the filter space, will result in rules for selecting the best suited stack filter for a given region. The supervised clustering procedures are shown to surpass significantly the global filtering approach.

1 INTRODUCTION

Stack filters ([5]) are a wide class of nonlinear digital filters which includes a large number of filter classes with various types of behavior. Consequently, the successful design inside the stack filter class is a challenging task. Theoretical approaches to optimal stack filter design under the mean absolute error (MAE) criterion have been intensively investigated in the past[1],[4], encompassing different problem settings: model based approaches, training approaches, adaptive approaches, fulfilment of structural constraints. However, only a global type of filtering has been considered in all these approaches, where only one filter is designed to be used for all the filtering tasks in hand.

In this paper we introduce a new filtering architecture, locally adaptive stack filtering, and consider its applications to image restoration problems.

Since images are nonstationary signals, using highly different filters in different image regions may lead to better performance than the global filtering approach.

We propose a new filtering architecture, where the image is subdivided into small regions. The stack filter to be used for each region results by applying an optimal selection procedure (in the supervised case) or a sub-optimal selection procedure (in the unsupervised case) from a catalogue of stack filters, based on a few local statistics (forming the parameter space) computed for each region.

The process of building the catalogue of stack filters involves clustering the parameter space (the local statistics), clustering the stack filter space and finding a suitable mapping between them.

2 LOCALLY ADAPTIVE STACK FILTERING ARCHITECTURES

We denote in the sequel the desired image by D, and the noisy image by X, with integer pixel values in $\{0, 1, \ldots, M\}$, and indices $(i, j) \in \{0, \ldots, N\} \times \{0, \ldots, N\}$. The images are splitted into K small equally sized regions (e.g. square regions) defining the sets $I^{[k]}$ $(k = 1, \ldots, K)$, of indices (i, j) for the pixels belonging to each region.

2.1 $(D^{[k]},\ X^{[k]})$ Local Optimal Design

The optimal filter $f^{[k]^*}$ for region k can be obtained using the training approach for optimal design ([4]) for the training set $(D^{[k]}, X^{[k]})$, where we denote by $D^{[k]} = \{D(i,j)|(i,j) \in I^{[k]}\}$ and $X^{[k]} = \{X(i,j)|(i,j) \in I^{[k]}\}$

To illustrate the filtering performances when using the local filtering architecture, in Table 1 are presented the

Number of blocks Size of blocks	1 512×512	4 256×256	16 128×128	64 64×64	256 32×32	1024 16×16	4096 8×8
MAE	6.231	6.202	6.148	6.045	5.7819	5.282	4.475

Table 1: Mean absolute error (MAE) for locally optimal stack filtering image Airfield perturbed with zero-mean Gaussian contaminated noise (contamination $\lambda = 0.1$)[2] at SNR=9dB, MAE=10.329 for the corrupted image.

Size of arrays	16×16			
Number of filters	1024	256	64	1
MAE	5.282	5.798	6.025	6.231

Table 2: Mean absolute error (MAE) for locally stack filtering (512×512) image Airfield (1024 arrays) perturbed at SNR=9dB (MAE=10.329) with different number of filters.

Mean Absolute Error (MAE) values for seven different partitions of desired and corrupted images, for one contamination scenario of Airfield image. It can be seen that using 4096 filters instead of only one global filter will reduce the MAE value from 6.231 to 4.475. This potential of improvement is remarkable, but two problems are to be solved. The first problem is filter space clustering, and refers to reducing the number of necessary filters by grouping some regions to be (suboptimally) filtered by the same stack filter. The second problem is finding a classification rule, i.e. the stack filter which is most appropriate for a region must be selected based on the local statistics computed for that region. These two problems will be solved by different structures and with different performances, depending whether the design procedure is supervised or unsupervised.

In the supervised local optimal design, the feature vector is selected to be the vector of cost coefficients, computed using both $D^{[k]}$ and $X^{[k]}$. Regions are grouped into clusters using a vector quantization method for the cost coefficient vectors. In Table 2 four situations are considered, when the region size is kept to 16×16, but the number of clusters is decreased from 1024 (one region - one filter) to 1 (all regions - same filter). The results suggest that in order to obtain an important improvement in this locally adaptive approach a large number of different filters should be used.

2.2 ($X^{[k]}$) Features Based Locally Adaptive Filtering

Three solutions for the problem of finding a classification rule are presented, differing only in the selection of a local feature vector. For each region $X^{[k]}$, an "input feature" vector $\underline{s}^{[k]}$ is computed.

1. Motivated by the results obtained in the adaptation of the weighted median filters based on local statistics [3] we analyze first the use of mean and variance estimates as feature vector elements.

$$\begin{aligned} \underline{s}^{[k]} &= [m^{[k]}, \sigma^{[k]}] \\ m^{[k]} &= \frac{1}{card(I^{[k]})} \sum_{(i,j)\in I^{[k]}} X(i,j) \\ \sigma^{2[k]} &= \frac{1}{card(I^{[k]})} \sum_{(i,j)\in I^{[k]}} (X(i,j) - m^{[k]})^2 \end{aligned}$$

2. The optimal stack filter is completely defined by the cross level statistics of the binary (thresholded) images; therefore, the selection as elements in the feature vector of corrupted image binary window probabilities is more specific to stack filter design.

$$\begin{aligned} \mathcal{S}_l^{[k]} &= \{(i,j,m) | (i,j) \in I^{[k]}, \underline{X}^m(i,j) = \underline{v}_l\} \\ s_l^{[k]} &= card(\mathcal{S}_l^{[k]}) \end{aligned} \quad (1)$$

   where superscript m denotes the thresholding operation at level m, and the binary vector $\underline{v}_l$ belongs to $\{0,1\}^L$ (L is the number of pixels in the processing window).

3. Stack filter performances are mostly remarkable in dealing selectively with noise rejection and detail preservation, depending whether fine details, like edges, are present or not in the image. In order to use in the adaptation process the ability in changing the filtering behavior when the image contents is different, we use as feature vector elements the variables $s_1^{[k]}, \ldots, s_4^{[k]}$ introduced below. We consider the four border lines (three pixels each) from the 3×3 square neighborhood of the pixel (i,j). Denote V_{-1}, V_1 the median values of vertical lines and H_{-1}, H_1 the median values of the horizontal lines.

$$\begin{aligned} H_l(i,j) &= \text{median}\{X(i+l, j+k) | -1 \le k \le 1\} \\ V_l(i,j) &= \text{median}\{X(i+k, j+l) | -1 \le k \le 1\} \end{aligned}$$

where $l = -1, 1$. We distribute each pixel (i, j) of region k to one of the sets $P^{[k]^-}$, $P^{[k]^+}$, according to the sign of the difference $H_1(i,j) - H_{-1}(i,j)$. Then compute the average difference between median values of horizontal lines for each set

$$s_1^{[k]} = \sum_{(i,j)\in P^{[k]^-}} (H_{-1}(i,j) - H_1(i,j))$$

$$s_2^{[k]} = \sum_{(i,j)\in P^{[k]^+}} (H_1(i,j) - H_{-1}(i,j))$$

The values $s_1^{[k]}$, $s_2^{[k]}$ carry information about the dominance in the region of edges with horizontal direction. Taking into account the sign of the computed difference, we allow selectivity with respect to the sense of variation of gray level values across the horizontal edges. Similarly we define the values $s_3^{[k]}$, $s_4^{[k]}$ considering the vertical direction instead.

Based on the feature vectors, similar regions (with similar features) are clustered into a small number (Q) of classes ($Q \ll K$) using a vector quantization algorithm applied for the local feature vectors in the set $S = \{\underline{s}^{[1]}, \underline{s}^{[2]}, \ldots, \underline{s}^{[K]}\}$.

For each class, $\mathcal{C}_{[q]}$ ($q \in \{1, \ldots, Q\}$), the optimal filter $f^*_{[q]}$ is found by applying a fast optimal stack filter design procedure[4]. The training set for cluster $\mathcal{C}_{[q]}$ consists in the clean data set $\mathcal{D}_{[q]}$ and the noisy data set $\mathcal{X}_{[q]}$ computed as follows:

$$\mathcal{D}_{[q]} = \bigcup_{\underline{s}^{[k]}\in\mathcal{C}_{[q]}} D^{[k]}; \quad \mathcal{X}_{[q]} = \bigcup_{\underline{s}^{[k]}\in\mathcal{C}_{[q]}} X^{[k]}. \tag{2}$$

The results obtained in the above *supervised* approach can be used to build a catalogue of stack filters. The more interesting case, of *unsupervised local adaptive stack filtering*, consists in using for each region in the corrupted image a filter selected from the catalogue, previously designed in the supervised procedure. The selection is done according to a mapping between the local statistics vector and the filters in the catalogue. This mapping is established in the training procedure, when the parameter space is vector quantized.

3 EXPERIMENTAL RESULTS

In our experiments we compare the different possible choices of the local feature vector. In all the simulations a 3×3 processing window is used. In Table 3, some of these results are presented. They include the filtering error (MAE) obtained in the supervised and unsupervised procedures, when each of the three vector parameters enumerated above are separately used for clustering. The same number of classes, $Q = 49$ have been imposed during vector quantization. The filters obtained in the supervised procedure are used for unsupervised filtering. For both supervised and unsupervised procedures 2048 (8×8) regions were used from image Airfield perturbed with zero-mean noise at SNR=9dB (MAE=10.329) from TUT noisy image database [2]. The values of MAE performance obtained in the supervised and unsupervised procedures indicate that features based on local edge information have the best potential for designing locally adaptive stack filters.

The results presented in the experimental section show that a good library of stack filters may be obtained using appropriate choices of local features and large enough training sets.

4 CONCLUSIONS

Locally adaptive stack filtering provides a significant improvement in performance compared to global stack filtering. We introduce here adaptive techniques, using various local features and various mapping rules for selecting the best filter for a given region. The complexity of the adaptive structures, when libraries of hundreds of filters may be required, is compensated by the major reduction of MAE criterion and by the possibility to use these structures in an unsupervised mode.

References

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Supervised filtering					
	No clustering		Clustering		
			Local mean and var.	Edge parameters	Binary windows prob.
Number of filters	1	2048	49	49	49
MAE	6.210	4.455	6.337	6.130	6.093
Unsupervised filtering					
	No clustering		Clustering		
			Local mean and var.	Edge parameters	Binary windows prob.
Number of filters	1		49	49	49
MAE	6.650		6.728	6.206	6.336

Table 3: Supervised and unsupervised filtering. Performance of different types of clustering using parameters computed from the noisy image arrays.

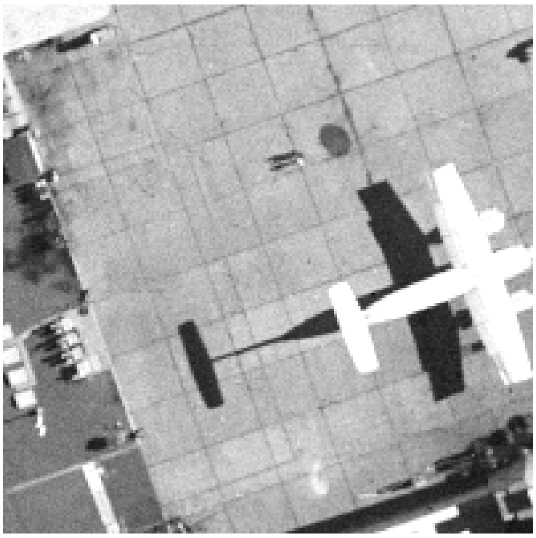

Figure 1: Original image

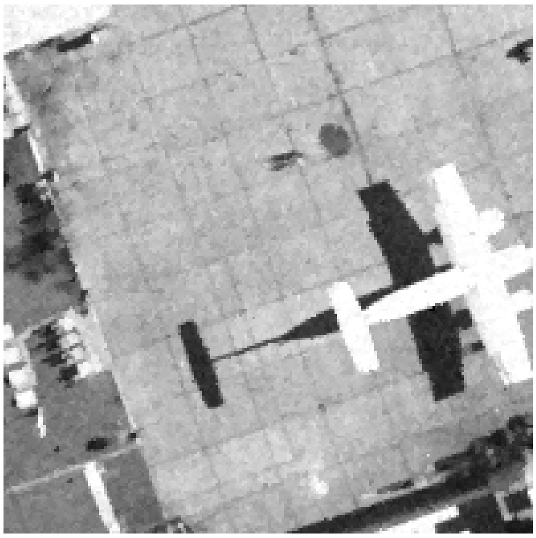

Figure 3: Noisy image filtered with one stack filter

Figure 2: Noisy image

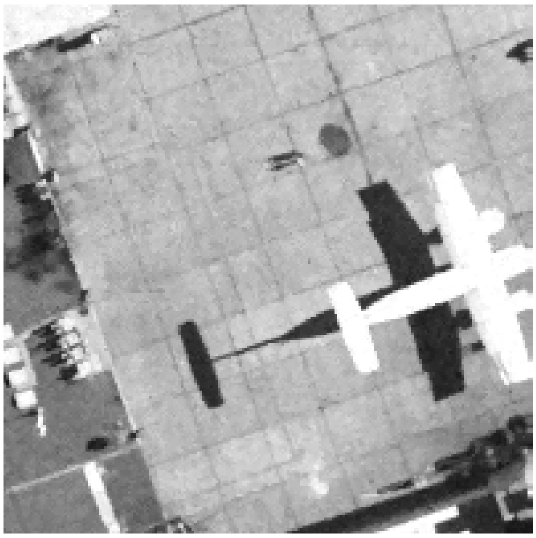

Figure 4: Noisy image filtered with 1024 locally optimal stack filters