TEXTURE ANALYSIS: COMPARISON OF AUTOCORRELATION-BASED WITH CUMULANT-BASED APPROACHES

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ABSTRACT

In this paper the use of 3rd-order cumulants, i.e. triple correlations, is proposed for texture analysis. Properties of such features are derived, with particular attention to insensitivity to symmetrically distributed noises and statistical estimate stability. Experimental evaluation of 3rd-order cumulants as descriptive features for textures is carried out in comparison with autocorrelation-based approaches.

1 INTRODUCTION

A texture analysis scheme based on 3rd-order statistics (3rd-order cumulants) is proposed, and its performances in texture segmentation are tested and compared with those of autocorrelation features [1].

The use of statistical features is motivated by the conjecture of Julesz [2] that a 2nd-order probability distribution suffices for human discrimination of different patterns. More recent theories have demonstrated that such statistics are suited to capturing basic statistical dependences in natural images but are blind to very strong and anisotropic dependences. In [3], it is shown that many natural textures, considered as 2D stochastic fields, are non Gaussian processes; therefore, they can be characterized more completely by Higher Order Statistics (HOS) than 2nd-order statistics, sufficient for Gaussian processes.

Another important characteristic of HOS, that can significantly improve the performances of a texture classification or segmentation scheme is the insensitivity to Gaussian noise.

Since for the analysis of real images (even non-optical), the presence of noise may seriously compromise performances of well-assessed texture processing methods, use of HOS is suggested as an alternative robust approach. In the paper it is also shown that HOS insensitivity to Gaussian noise can be extended to many classes of symmetrically distributed noises, if 3rd-order cumulants are considered.

On the other hand, stable estimation of cumulants requires the availability of many data samples, that generally does not match the analysis resolution required in segmentation problems.

In the paper, an evaluation of obtainable estimation accurancy is performed and some solutions to obtain effective features are addressed. Then, analysis of textured images is carried out, showing limits and possibilities of cumulants versus autocorrelation-based features for classification and segmentation problems.

2 THIRD-ORDER CUMULANT ESTIMATION

The 3rd-order cumulant of a zero-mean discrete 2D signal x(i,j) is defined as follows:

$$\hat{c}_3^x(\tau_{11},\tau_{12};\tau_{21},\tau_{22}) = \frac{1}{N^2}\sum_{i,\,j\,\in\,N_S} x(i,j)\cdot x(i+\tau_{11},\,j+\tau_{12})\cdot x(i+\tau_{21},\,j+\tau_{22}) \qquad (1)$$

where $(\tau_{11},\tau_{12};\tau_{21},\tau_{22})$ correspond to the spatial lags on the two image directions, N is the image block (16×16 pixel for segmentation) and N_S is a set that can be expressed as:

$$N_S = \left\{ \begin{array}{l} (i,j): i=1,...,N-\tau_{11};\; j=1,...,N-\tau_{12}; \\ 0>\tau_{11}>\tau_{21};\, 0>\tau_{12}>\tau_{22} \end{array} \right\} \qquad (2)$$

One of the main drawback that arises using cumulants concerns the computational effort necessary to obtain stable and consistent estimates of HOS functions. Generally, it is observed that HOS functions require more data samples than, for example, autocorrelation function to make estimated values converge to expected values.

Thus, when analysis resolution requires window size to become smaller and smaller, estimate of cumulants as discriminanting features can be critical in terms of estimate variance.

The influence of *N* on the estimated stability was evaluated on different kinds of natural textures [4]. Experimental tests demonstrated that 3rd-order cumulants estimate converges when *N* > 16, i.e., for window sizes of practical interest for segmentation purposes. In Fig. 1, the behaviour of the 3rd-order cumulants arranged in a vector of 150 elements is shown for the "snake" texture, by varying the value of N from 7 to 32. Here, K=1 and F=5 are taken fixed, being K the number of blocks considered to average the cumulant vector and F the size of the mask in which the lags can vary (e.g., F=5 means that τ_{ij} can range in the interval [-2,2]). One can notice how the stability of the profiles increases as soon as N becomes larger.

In Fig. 2, for the same texture, convergence was made faster by averaging results obtained on different N×N image blocks (K=15 blocks were considered in this test).

A further problem concerns the number of computable cumulants, which remains very large and requires considerable computation time with respect to approaches as autocorrelation, although symmetry properties allow the number of obtainable lags to be reduced. In this case, computation of cumulants can be limited to cumulant slices, i.e. two spatial lags can be fixed a-priori, while the other two can vary in the N×N window. The choice of which projection to insert in the feature vector can be related to texture geometry. For example, some slices may be more suitable than others to describe periodical textures characterised by elements located along preferred directions.

Furthermore, in supervised texture classification problems, an a-priori sample selection, based on typical feature selection criteria used in pattern recognition, may also be introduced to reduce the number of computable lags without limiting discrimination power [5].

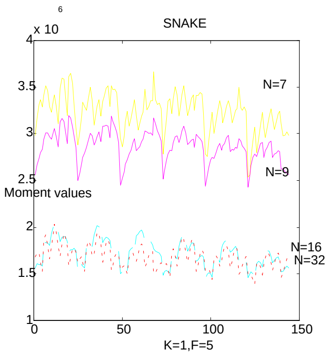

Figure 1: Estimated 3rd-order cumulants/moments versus N (N=7, 9, 16, 32) ranked in a feature vector. Estimate is computed on a single (K=1) image block N×N; spatial lags $(\tau_{11},\tau_{12};\tau_{21},\tau_{22})$ vary in [-2,2].

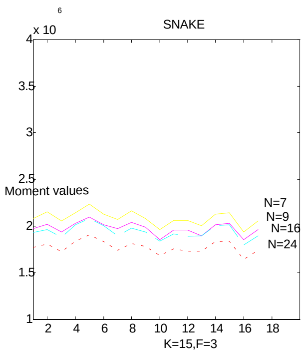

Figure 2: Estimated 3rd-order cumulants/moments versus N (N=7, 9, 16, 24) ranked in a feature vector. Estimates are computed for K=15 image blocks N×N; spatial lags $(\tau_{11},\tau_{12};\tau_{21},\tau_{22})$ vary in [-1,1].

3 NOISE ROBUSTNESS

It is well-known that when signals are corrupted by additive Gaussian noise $n(i)$, i.e., $x(i) = s(i) + n(i)$, the use of HOS makes it possible to reduce the effect of such a noise [6]. In case of non-Gaussian noise, 3rd-order cumulants estimation shows the following property. Denoting $n(i)$ as white non-Gaussian noise, cumulant estimation gives:

$$c_3^x(\tau_1,\tau_2) = c_3^s(\tau_1,\tau_2) + \gamma_3^n\delta(\tau_1)\delta(\tau_2) \tag{3}$$

Similarly, for coloured noise, resulting from a linear filter $h(i)$ driven by i.i.d. (independent identically distributed) noise, we obtain:

$$c_3^x(\tau_1,\tau_2) = c_3^s(\tau_1,\tau_2) + \gamma_3^n \sum_{i=0,\infty} h(i)h(i+\tau_1)h(i+\tau_2) \tag{4}$$

If such noises have symmetric distributions (e.g., Laplace, uniform), i.e., skewness γ_3^n=0, cumulants still provide insensitivity to noise. Differently, for correlation (i.e., second-order statistics), relations (3) and (4) give:

$$c_2^x(\tau) = c_2^s(\tau) + \gamma_2^n\delta(\tau) \tag{5}$$

and

$$c_2^x(\tau) = c_2^s(\tau) + \gamma_2^n \sum_{i=0,\infty} h(i)h(i+\tau) \tag{6}$$

with variance $\gamma_2^n \neq 0$, in both cases. Thus, for images corrupted by noise, the use of cumulant features may lead to robust classification schemes.

4 EXPERIMENTAL RESULTS

Some experiments were carried out to test cumulant vs. autocorrelation performances in texture analysis. Two sets of textures were considered: synthetic textures and natural textures from Brodatz's [4].

The use of synthetic textures was aimed at verifying that, in some cases, 2nd-order statistics are completely unable to discriminate, even when textures are significantly different to the human eye (Figs. 3 and 4). It is worth noting that performances were evaluated by using the same number of samples, i.e., lags, for each type of feature, computed on sub-blocks of 16×16 pixels. The simple K-means segmentation algorithm was used to produce the final image partition.

Concerning the examined natural textures, autocorrelation generally presented powerful and efficient discrimination properties on noise-free images, but, when the presence of symmetric noise was not negligible, 3rd-order statistics yielded better results.

In particular, this was verified in the presence of coloured symmetrically distributed noise. Figure 5 describes the performances obtained by a classifier using $c_2^x(\tau)$ and $c_3^x(\tau_1,\tau_2)$ as discriminating features in the presence of coloured noises. By decreasing SNR, probability of correct classification (P_d) also decreases, but, generally, cumulants outperform autocorrelation.

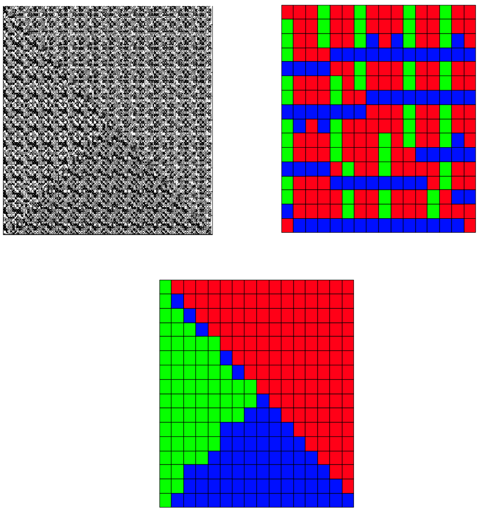

Figure 3: Artificially generated image containing three different textures (top left). Image partitions using $c_2^x(\tau)$(top right) and $c_3^x(\tau_1,\tau_2)$ (down), as discriminating features.

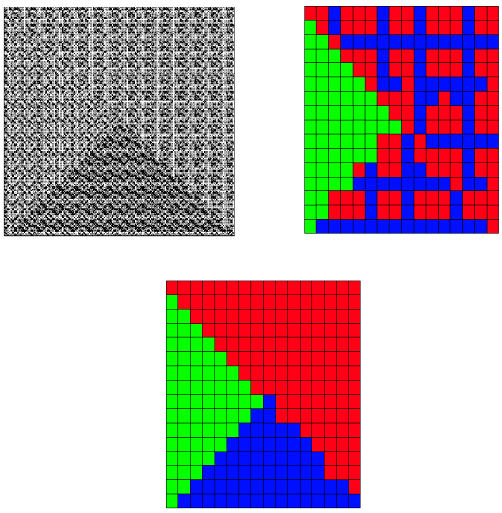

Figure 4: Artificially generated image containing three different textures (top left). Image partitions using $c_2^x(\tau)$(top right) and $c_3^x(\tau_1,\tau_2)$ (down), as discriminating features.

Further improvements are expected by applying efficiency weights to cumulant samples and selecting the most discriminating to be included in the feature vector [5].

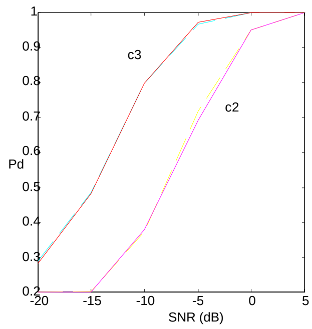

Figure 5: Probability of correct classification of $c_2^x(\tau)$ and $c_3^x(\tau_1,\tau_2)$ vs. SNR. Dashed line: performances in case of coloured Gaussian noise. Solid line: performances in case of coloured uniform noise. Five classes of natural textures were considered: coffee, cloth, wall, naphta, tweed.

5 CONCLUSIONS

A statistical approach for texture analysis has been presented. Despite the computational effort and the statistical instability due to cumulant estimates, HOS-based method demonstrated suitable for noisy environements, such as Gaussian (possibly coloured) additive noise as well as symmetrical distributed additive noises. With respect to well-assessed 2nd-order statistical features, HOS features may provide a more complete characterization of non-Gaussian stationary textures and be complementary with those ones for texture analysis.

6 REFERENCES

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