DIRECTIONAL COMPOSITE MORPHOLOGICAL FILTER IN IMAGE PROCESSING

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ABSTRACT

In this paper, two pairs of dual morphological filters, the composite morphological filters (CMFs) are introduced. They have distinctive properties comparing with other similar morphological operations. CMFs are used to construct a directional morphological filter in impulsive noise removal procedures. It is proven that it has better noise-removal and detail-preserving abilities than classical morphological filters and other non-linear filters such as median or centre weighted median based filters. A threshold scheme is added to improve the final filtering performances.

1 INTRODUCTION

In image processing, the most famous nonlinear filters are rank-order filters[1], stack filters[2], morphological filters[3] and their derivations. Median or more generally weighted median filters, especially center weighted median filters[4~5] can remove impulsive noise or small image components while preserving important image features such as edges. The morphological filters[3] analyse the geometrical structure of a signal by locally comparing it with a predefined structuring element. By properly choosing the different forms of structuring elements, we can select or eliminate local signal structures or modify local signal geometry. For example, open-closing[6] is used to remove impulse noise; alternating sequential filters[7] are taken to detect edges in noisy images.

In this paper, we construct a detail-preserving algorithm based on directional composite morphological filters (CMFs). First the composite morphological filters (CMFs) and their main properties are introduced. Next a series of structuring elements are taken to implement a directional CMF in the processing of impulsive noise corrupted images. Then, comparisons are made in noise suppression and detail preservation abilities between the directional CMF, median filter and center weighted median based filters. In final experiments, a threshold scheme is added to get even better filtering result.

2 COMPOSITE MORPHOLOGICAL FILTERS

Let us denote γ, φ opening and closing respectively, $\wedge$ as infimum and $\vee$ as supremum transformation between two sets or functions respectively, I as the identical transform, i.e., $I(X)=X$. The result Y of a set or function X filtered by an open-closing is denoted as: $Y=\varphi\gamma(X)$. We define the following group of transformations as composite morphological filters (CMFs):

(1) $\varphi\gamma \wedge I$, (2) $\gamma\varphi\gamma \wedge I$, (3) $\gamma\varphi \vee I$, (4) $\varphi\gamma\varphi \vee I$.

The important properties of these four transformations are as follows:

(a) They are increasing transformations.

(b) They are idempotent transformations.

(c) $\varphi\gamma \wedge I$ and $\gamma\varphi\gamma \wedge I$ are antiextensive transformations; $\gamma\varphi \vee I$ and $\varphi\gamma\varphi \vee I$ are extensive transformations.

(d) $\gamma \le \gamma\varphi\gamma \wedge I \le \varphi\gamma \wedge I \le I \le \gamma\varphi \vee I \le \varphi\gamma\varphi \vee I \le \varphi$.

Thus $\varphi\gamma \wedge I$ and $\gamma\varphi\gamma \wedge I$ belong to algebraic opening, and $\gamma\varphi \vee I$ and $\varphi\gamma\varphi \vee I$ belong to algebraic closing, and clearly $\varphi\gamma \wedge I$, $\gamma\varphi \vee I$ and $\gamma\varphi\gamma \wedge I$, $\varphi\gamma\varphi \vee I$ are two couples of dual operators. From the last property, it can be seen that CMFs, like opening and closing, are always comparable with identical transform (which corresponds to original image).

The introduced new algebraic openings and closings bear much resemblance with operations proposed in [8~9], but they have different and distinguishable properties. In [8], $\gamma\varphi \wedge I$ and $\varphi\gamma \vee I$ are used in image contrast extraction, but it is easy to find examples to show that these two operations are not idempotent, thus they are not morphological filters. In [9], $\wedge_{I=1}^{N}\varphi_I\gamma_I$ and $\vee_{I=1}^{N}\gamma_I\varphi_I$ are constructed and named connected filters by reconstruction, they belong to the strong filter class and have been used to create flat zones during image segmentation procedure. But here φ_I and γ_I are respectively restricted to closing by reconstruction and opening by reconstruction, so they are more time-consuming.

3 DIRECTIONAL COMPOSITE MORPHOLOGICAL FILTER

In impulsive noise suppression procedures, the classical morphological filter $\varphi\gamma$ using square SE are not well adapted. Firstly, the usually used two-dimensional SE is relatively too large even for simple morphological operations when images have many smaller structures. Secondly, opening and closing usually modify local image values so much that many image details are lost. To overcome these drawbacks, we choose linear structuring elements in different orientations and use two dual CMFs $\varphi\gamma \wedge I$ and $\gamma\varphi \vee I$ as noise removal operators.

The directional CMFs is described as follows: linear structuring elements, of length p, in several directions u_1, u_2, ..., u_n are used separately in $\varphi\gamma \wedge I$ (resp. $\gamma\varphi \vee I$). Then we select the maximum (resp. minimum) between the different outputs of these CMFs. Thus we can construct the following algebraic opening γ_p and algebraic closing φ_p:

$$\gamma_p = \mathrm{Max}\ \{(\varphi\gamma \wedge I)_{(u_1, p)}, (\varphi\gamma \wedge I)_{(u_2, p)}, ..., (\varphi\gamma \wedge I)_{(u_n, p)}\}$$
$$\varphi_p = \mathrm{Min}\ \{(\gamma\varphi \vee I)_{(u_1, p)}, (\gamma\varphi \vee I)_{(u_2, p)}, ..., (\gamma\varphi \vee I)_{(u_n, p)}\}$$

The dual combination of γ_p and φ_p acts even more softly than $(\varphi\gamma \wedge I)_{(u_i, p)}$ and $(\gamma\varphi \vee I)_{(u_i, p)}$. The more directions taken, the more details of images will be preserved, but at the same time the noise removal ability will be degraded.

4 DIRECTIONAL CMF IN IMPULSIVE NOISE SUPPRESSION

The experimental performances of the proposed directional morphological filter ($\varphi_p\gamma_p$) compared to some other well known non-linear filters are investigated. Both mean square error (MSE) and mean absolute error (MAE) criterion are considered.

As we know, a filter with a smaller size structuring element or mask keeps more image details. When the structuring element becomes larger, the corresponding filter will be more efficient in noise suppression but at the same time image details will be blurred. It is a compromise between noise removal and detail preservation. Our experiments on natural and rather detailed pictures have shown that the appropriate size for a mask or a structuring element is 3 or 5.

A real image (Figure 1(a)) has been chosen for more thorough experimentation. We have investigated the differences between outputs of this image filtered respectively by several filters and its original. This procedure without adding noise is to compare the detail smoothing effects of those filters on the tested image. These filters are: (1) $\varphi_p\gamma_p$, (2) $\beta_p\alpha_p$, (where α_p=Max $\{\gamma_{(\theta_1, p)}, \gamma_{(\theta_2, p)}, ..., \gamma_{(\theta_n, p)}\}$, β_p = Min $\{\varphi_{(\theta_1, p)}, \varphi_{(\theta_2, p)}, ..., \varphi_{(\theta_n, p)}\}$), (3) median filter, (4) separable median (SM) filter, (5) center weighted median (CWM) filter and (6) separable center weighted median (SCWM) filter successively. We have chosen $\beta_p\alpha_p$ for the purpose of comparison of the dual filters $\varphi\gamma \wedge I$, $\gamma\varphi \vee I$ with traditional opening γ and closing φ. We compare the directional CMF with median filter and center weighted median based filters (i.e. SM, CWM, SCWM filters) because median filter and center weighted median based filters are regarded as very efficient non-linear filters for removal of impulsive noise from images. In $\varphi_p\gamma_p$ and $\beta_p\alpha_p$, linear structuring elements are taken in four directions: horizontal, vertical, diagonal 45°, diagonal 135°. The MSE and MAE between the original image and the filtered ones are calculated and the results are shown in Table 1. From this table we can see that $\varphi_p\gamma_p$ and $\beta_p\alpha_p$ filters have the best performance in image detail preservation, next comes center weighted based filters.

Table 1. Results of detail smoothing effects. Image: Port, size 256×256, 256 graylevels (p, h, v, SE denote respectively point, horizontal, vertical, structuring elements).

Filter	MAE	MSE
median filter(masque: 3×3 square)	6.002	130.719
median filter(masque: 5×5 square)	9.585	308.019
SM filter(3-p line, h+v)	5.531	118.413
SM filter(5-p line, h+v)	8.945	287.196
CWM filter(3×3 square, center weight=3)	3.600	70.481
CWM filter(3×3 square, center weight=5)	1.634	22.133
SCWM filter(5-p line, h+v, center weight=3)	4.132	104.355
$\beta_p\alpha_p$ filter(4-directional linear SE, 3-p line)	1.512	16.269
$\varphi_p\gamma_p$ filter(4-directional linear SE, 3-p line)	1.511	16.268

Now the performances of our directional CMF on real noisy images are studied. The same image is chosen as reference, then it is corrupted with 5% random impulsive noise to form a noisy image (Figure 1(b)). After filtering the noisy image with the different filters mentioned above, MAE and MSE are calculated and the results obtained are listed in Table 2. From this table, we can see that, for this image, $\varphi_p\gamma_p$ with 3-point linear structuring elements acts best for removal of noise, it has both the smallest MSE and MAE. We have tried with several other kinds of images using the same experimental method, these images were corrupted with different percentages of impulsive noise from 1% to 6%, and almost the same result appeared. From property (d) of section 2, we can see that $\varphi\gamma \wedge I$ and $\gamma\varphi \vee I$ are the closest filters to identical transformation, i.e. noisy images filtered by $\varphi\gamma \wedge I$ or $\gamma\varphi \vee I$ change less their forms comparing with other filters such as opening or closing. The best filter for image restoration is one that has globally better noise removal and detail preserving abilities, and $\varphi_p\gamma_p$ is an example.

Table 2. Results of impulsive noise removal abilities. Image: Port, size 256×256 with 256 graylevels. Image corrupted with 5% randomly distributed impulsive noise(p, h, v, SE denote respectively point, horizontal, vertical, structuring elements).

Method	MAE	MSE
median filter(masque: 3×3 square)	6.345	144.917
median filter(masque: 5×5 square)	9.784	317.778
SM filter(3-p line, h+v)	5.918	133.958
SM filter(5-p line, h+v)	9.239	299.166
CWM filter(3×3 square, center weight=3)	3.959	90.627
CWM filter(3×3 square, center weight=5)	2.334	71.669
CWM filter(3×3 square, center weight=7)	2.354	219.504
SCWM filter(5-p line, h+v, center weight=3)	4.443	120.547
$\beta_p\alpha_p$ filter(4-directional linear SE, 3-p line)	2.280	67.493
$\varphi_p\gamma_p$ filter(4-directional linear SE, 3-p line)	2.278	67.492
$\beta_p\alpha_p$ filter(4-directional linear SE, 5-p line)	4.386	106.042
$\varphi_p\gamma_p$ filter(4-directional linear SE, 5-p line)	4.400	108.978
noisy image(5% impulse noise)	6.169	1023.000

A filter which preserves good image details sometimes does not remove noise efficiently. To avoid this possible drawback, a compromise can be found by introducing a threshold scheme to obtain a more adaptive filtering effect. The idea is to compare two filtered images, one filtered by our directional CMF, and another by a stronger filter (i.e. being efficient for noise suppression but at the same time not destroying too many image details). In order to better remove the remaining impulsive noise at these points, we can search for a suitable threshold *T*. if $y_{\varphi_n\gamma_n}$ denotes the output of filtered image by directional CMF, y_S the output of filtered image by a stronger filter, and *Y* the final output, the following scheme is proposed:

$$Y = \begin{cases} y_{\varphi_n\gamma_n}, & \text{if} \quad \left| y_{\varphi_n\gamma_n} - y_S \right| < T, \\ y_S, & \text{else} \ . \end{cases}$$

The threshold value *T* should be pre-defined between 0 and 255, if *T*=0, the final image is the one filtered by the stronger filter; the greater *T* is, the more image details and more noisy points will be kept. How to choose the value of *T* depends on the used stronger filter and the image pattern. For example, for most of texture images, if we use a median filter with a square mask of size 3×3 as the stronger filter, the suitable *T* is between 50 and 70.

The threshold scheme has been tested on the same noise corrupted image. In this algorithm, we have chosen a median filter with a 3×3 square mask as the stronger filter because this median filter structure is efficient in impulsive noise suppression and does not eliminate image details too much. When the threshold value is properly chosen, it can be seen that MAE and MSE are both smaller than those obtained by using only directional CMF. In our case the smallest MAE is 2.226 when *T* is 90 and the smallest MSE is 56.064 when *T* is 85, both MSE and MAE are smaller than using only $\varphi_p\gamma_p$ when *T*>65 (Figure 2). The final filtering result can be seen clearly in Figure 1(e). We have done the same experiments on other kinds of images, very good results in image restoration are obtained.

5 CONCLUSION

We construct two pairs of new algebraic openings and closings called composite morphological filters (CMFs). To illustrate their attractive properties in noise suppression, the dual combination of directional CMF gives very efficient impulsive noise removal and detail preservation abilities. In most cases it outperforms the median, and center weighted median based filters. Moreover, this combination keeps the remarkable idempotent property. This filter application can be viewed as a pre-processing step before other more complex image processing procedures (e.g. image structure decomposition, segmentation or coding).

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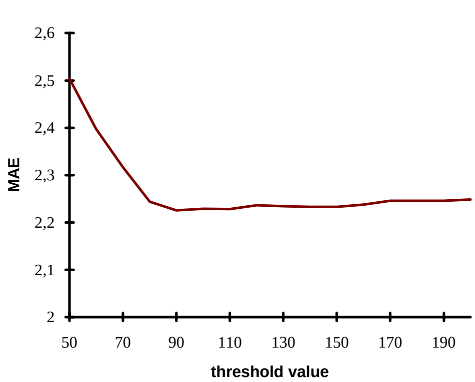
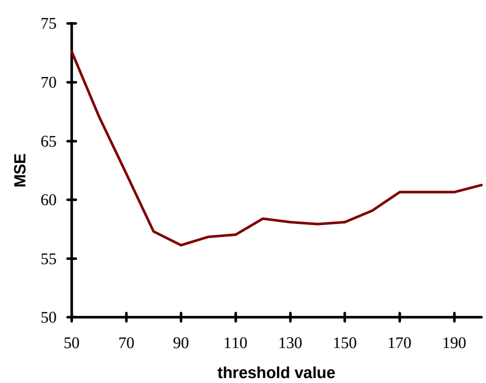
a	c	e
b	d	

Figure 1. (a) original image "port", (b) noisy "port": original image corrupted with 5% random impulsive noise, (c) noisy "port" filtered by median filter (3×3 square mask), (d) noisy "port" filtered by directional CMF: $\varphi_p\gamma_p$ (3-p linear structuring element, 4 direction), (e) filtered by $\varphi_p\gamma_p$ when threshold scheme is added (T = 85).



(a)



(b)

Figure 2. (a) MAE and (b) MSE values as T changes from 50 to 200