DESIGN OF 3-D OPTIMAL FIR FILTERS WHICH EXTRACT OBJECTS MOVING ALONG LINEAR TRAJECTORY

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ABSTRACT

We propose a design method of optimal FIR filter which selectively extracts the particular moving object from other moving objects and noise. Stochastic approach is applied to the problem using the information of signals and the probability distribution of velocity vectors. In the method, the frequency response of the proposed Linear Trajectory Filter (LTF) specified by a priori information of the moving object's shape and its velocity vector. In addition, we derive a general formulation of the problem for optimal filter design and its solution for any signal and noise. Through some examples, it is shown that the target object is effectively enhanced in the noisy environment.

1 INTRODUCTION

In the study of computer vision, detection of moving objects with specific trajectory vector (velocity and direction) from sequences of images is very important. Especially, in the dynamic image study, an analysis of signal representing moving objects with fixed trajectory vector, which is refered as linear trajectory signal (abbreviated as LTS), is basic for designing detection filter. Very important contribution to the design problems for LTS detection filter is made by L.T.Bruton and others[1]-[3]. These studies address the design of frequency selective filter whose passband contains the specific plane including the origin. This is based on the fact that the spectrum of LTS is contained in the plane whose normal vector is exactly the velocity vector of LTS. The present paper investigates the optimal filter design for separating the target LTSs from other signals using the information of object signals and the probability distributuion of their velocity vectors. Random process approach applied to the problem gives the optimal frequency characteristics.[4] Therefore, filtering process realized in the Fourier transform domain. On the other hand, this paper proposes an optimal FIR filter realized in the 3-D spatio/temporal domain.

2 PROBLEM FORMULATION AND ITS SOLUTION

2.1 Analysis of Linear Trajectory Signals

A point LTS moving in the 2-D plane with velocity V and azimuth angle θ draws a linear trajectory in the spatio/temporal (x-y/t) space. A point LTS $f(x,y,t)$ can be written as

$$f(x,y,t) = \begin{cases} A & \left(\dfrac{x-x_0}{\cos\theta\cos\phi} = \dfrac{y-y_0}{\sin\theta\cos\phi} = \dfrac{t}{\sin\phi}\right) \\ 0 & \text{(otherwise)} \end{cases} \tag{1}$$

where A is a real number (nonzero), (x_0, y_0) is the initial position of LTS at $t=0$ and $V = \cot\phi$.

Therefore, LTS is specified uniquely by the parameters of (x_0, y_0), velocity V and direction θ. We can treat signal $f(x,y,t)$ as a random field for given probability distribution of θ and V. That is, the random signal of LTS $f(x,y,t)$ is a set of LTS whose sample is a deterministic signal $f(x,y,t;x_0,y_0,\theta,V)$. Thus we will discuss the input stochastic process as an infinite set of deterministic signals.

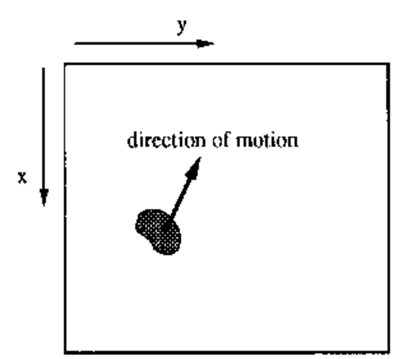


Figure 1: Moving object with arbitrary 2-D signal

Now we consider a moving object which is represented by $s(x,y,t)$, as shown in Fig.1. As we assume the shape and signal value of the moving object are time-invariant, then

$$s(x,y,t) \quad = \quad s(x-ut, y-vt, 0)$$

$$\triangleq \quad w(x-ut, y-vt) \tag{2}$$

, where (u,v) represents the velocity vector $(V\cos\theta, V\sin\theta)$.

We can treat a 2-D object in the same way as a point LTS. The reason is founded by the following fact: the spectrum of a point LTS is nonzero and has constant value only in the particular plane written by $u\omega_x + v\omega_y + \omega_t = 0$, and both a point LTS and other moving objects with same velocity vector have their spectrum on a plane. We merely notice the difference of magnitude characteristics on the spectrum plane between the spectrum of a point LTS and 2-D object.

Now, we can formulate an optimal filtering problem for the detection of LTS. It is assumed that input random sequence $u(x,y,t)$ is degraded by additive noise sequence $n(x,y,t)$, that is,

$$u(x,y,t) = s(x,y,t) + n(x,y,t) \tag{3}$$

where $s(x,y,t)$ represents the LTS desired to be extracted and $n(x,y,t)$ represents the signal-independent additive noise which includes LTSs with different velocity and/or different direction from the signal $s(x,y,t)$.

2.2 Design of Optimal FIR Filter

The zero phase FIR filter is used as a restricted class of optimal filter whose output is the optimal estimate $\hat{s}(x,y,t)$, as shown in Fig.2. This is solved by the zero phase smoothing Wiener filter, that is, the optimal estimate is represented as the following form

$$\hat{s}(x,y,t) = \sum_{i=-M_1}^{M_1} \sum_{j=-M_2}^{M_2} \sum_{k=-M_3}^{M_3} h(i,j,k)u(x-i, y-j, t-k) \tag{4}$$

, where the filter impulse response $h(i,j,k)$ is determined such that the performance index

$$e = \int_{\omega} \boldsymbol{E}[|S(\omega) - \hat{S}(\omega)|^2]d\omega \tag{5}$$

is minimized, where ω indicates $(\omega_x, \omega_y, \omega_t)$, $S(\omega)$ and $\hat{S}(\omega)$ are the Fourier spectrum of $s(x,y,t)$ and $\hat{s}(x,y,t)$ respectively. $\boldsymbol{E}[\cdot]$ denotes the expectation on random variables of direction and velocity, and initial position.

As we mentioned, LTS discussed in this paper is specified uniquely by the parameters of initial position (x_0, y_0), velocity V and direction θ. We can treat signal $s(x,y,t)$ as a random field for given probability distribution of θ and V. That is, the random signal of LTS $s(x,y,t)$ is a set of LTS whose sample is a deterministic signal $s(x,y,t;x_0,y_0,\theta,V)$.

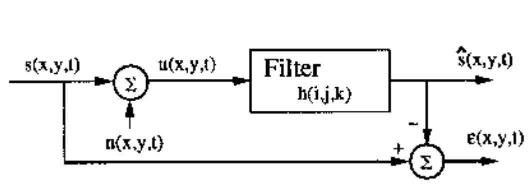


Figure 2: Wiener filter processing

The power spectrum of stochastic signal $s(x,y,t)$ is also the random process

$$|S(\omega)|^2 = \left| \iiint_{-\infty}^{\infty} s(x,y,t)e^{-j(\omega_x x + \omega_y y + \omega_t t)}dxdydt \right|^2. \tag{6}$$

If the probability density functions $p_V(V)$, $p_\theta(\theta)$ of random variables V, θ are given, we can obtain the power spectrum of random input stochastic process by

$$P_{ss}(\omega) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} |S(\omega;\theta,V)|^2 p_\theta(\theta)p_V(V)d\theta dV. \tag{7}$$

We can see that the spectrum of LTS is nonzero only in the particular plane $u\omega_x + v\omega_y + \omega_t = 0$. When we compute the values of power spectrum $P_{ss}(\omega)$ by the eq.(7), it is preferred to transform the random variables (θ, V) to the velocity random variables (u,v). Therefore, eq.(7) is rewritten by

$$P_{ss}(\omega) = \iint_{-\infty}^{\infty} |S(\omega;u,v)|^2 p(u,v)dudv \tag{8}$$

, where the probability density function $p(u,v)$ is given by the following transform

$$p(u,v) = \frac{p_V(V)p_\theta(\theta)}{\sqrt{u^2+v^2}}. \tag{9}$$

Furthermore,

$$P_{ss}(\omega) \tag{10}$$
$$= 4\pi^2 |W(\omega_x,\omega_y)|^2 \iint_{-\infty}^{\infty} \delta(u\omega_x + v\omega_y + \omega_t)p(u,v)dudv$$

where $W(\omega_x, \omega_y)$ is the Fourier spectrum of $w(x,y)$.

Now let us decompose noise signal $n(x,y,t)$ into noise moving object $n_1(x,y,t)$ with different velocity vector from the signal $s(x,y,t)$ and white noise $n_2(x,y,t)$. Then the Fourier spectrum of the input signal $u(x,y,t)$ is represented by the following equation from eq.(3),

$$
\begin{aligned}
&U(\omega) \qquad (11)\\
&= S(\omega; x_0, y_0, \theta, V) + N_1(\omega; x_0', y_0', \theta', V') + N_2(\omega)\\
&= S(\omega;\theta,V)\exp\{jq\} + N_1(\omega;\theta',V')\exp\{jr\} + N_2(\omega)
\end{aligned}
$$

where $q = \omega_x x_0 + \omega_y y_0$, $r = \omega_x x_0' + \omega_y y_0'$. In the frequency domain, the error $\epsilon(x,y,t)$ that results from the estimate $\hat{s}(x,y,t)$ of the signal $s(x,y,t)$ can be described by

$$
\begin{aligned}
E(\omega) &= S(\omega) - \hat{S}(\omega)\\
&= S(\omega;\theta,V)\exp\{jq\} - H(\omega)U(\omega)\\
&\triangleq M(\omega)\exp\{jq\} + L(\omega)\exp\{jr\} + K(\omega) \qquad (12)
\end{aligned}
$$

$$M(\omega)=S(\omega;\theta,V)\{1-H(\omega)\} \quad (13)$$

$$L(\omega)=-N_1(\omega;\theta',V')H(\omega) \quad (14)$$

$$K(\omega)=-N_2(\omega)H(\omega) \quad (15)$$

, where

$$H(\omega) \quad (16)$$

$$=\sum_{i=-M_1}^{M_1}\sum_{j=-M_2}^{M_2}\sum_{k=-M_3}^{M_3} h(i,j,k)\exp\{-(i\omega_x+j\omega_y+k\omega_t)\}$$

Additionally, the power spectrum of the estimation error $\epsilon(x,y,t)$ is written by

$$|E(\omega)|^2 = MM^*+LL^*+KK^*+2\mathrm{Re}[ML^*\exp\{j(q-r)\}] +2\mathrm{Re}[LK^*\exp\{jr\}]+2\mathrm{Re}[KM^*\exp\{-jq\}] \quad (17)$$

where the symbol $*$ means conjunguate operation. The error evaluation function e can be derived by substituting eq.(17) into eq.(5). It should be noticed that the function $\exp\{jq\}$,$\exp\{jr\}$ is periodic with respect to q,r. Since the aim of LTF is to extract the LTS with a particular velocity vector, initial position (x_0,y_0) of the object is not our concern. We then assume the (x_0,y_0) is distributed uniformly. That is, the signal $s(x,y,t)$ is statistically independent of noise $n_i(x,y,t)$, therefore we can derive the uniform distribution of q,r. The expectation of the periodic function $\exp\{jq\}$,$\exp\{jr\}$ on $x_0,y_0;x_0',y_0'$ is identical to that of $\exp\{jq\}$,$\exp\{jr\}$ over one period. Consequently, the terms of eq.(17) except $(MM^*+LL^*+KK^*)$ result to be zero. The expectation with respect to random variables θ, V, θ', V' should be considerated. As a result, the parameters (x_0,y_0) have no concern with our formulation. The expectation value of eq.(17) with respect to the radom variables θ and V is described by

$$\boldsymbol{E}[|E(\omega)|^2] \quad (18)$$

$$= \int_{-\pi}^{\pi}\int_{-\infty}^{\infty}|S(\omega;\theta,V)|^2\,|1-H(\omega)|^2\,p_\theta(\theta)p_V(V)d\theta dV + \int_{-\pi}^{\pi}\int_{-\infty}^{\infty}|N_1(\omega;\theta',V')|^2\,|H(\omega)|^2\,p_{\theta'}(\theta')p_{V'}(V')d\theta' dV' + |N_2(\omega)|^2\,|H(\omega)|^2\,.$$

The optimal solution of $h(i,j,k)$ which minimizes the e is obtained by the set of conditions $\partial e \,/\, \partial h(i,j,k)=0$ for all (i,j,k). To simplify the normal equation $\partial e \,/\, \partial h(i,j,k)=0$, we introduce the following matrices $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{R}$.

$$A \triangleq \begin{bmatrix} \int_\omega (P_{ss}+P_{nn})C_{-M_1,-M_2,-M_3}C_{-M_1,-M_2,-M_3}d\omega & \cdots & \int_\omega (P_{ss}+P_{nn})C_{M_1,M_2,M_3}C_{-M_1,-M_2,-M_3}d\omega \\ \vdots & \ddots & \vdots \\ \int_\omega (P_{ss}+P_{nn})C_{-M_1,-M_2,-M_3}C_{M_1,M_2,M_3}d\omega & \cdots & \int_\omega (P_{ss}+P_{nn})C_{M_1,M_2,M_3}C_{M_1,M_2,M_3}d\omega \end{bmatrix} \quad (19)$$

$$\boldsymbol{G}^T \triangleq [\; g(-M_1,-M_2,-M_3) \;\; g(-M_1,-M_2,-M_3+1) \;\; \cdots \;\; g(M_1,M_2,M_3) \;] \quad (20)$$

$$\boldsymbol{R}^T \triangleq [\; \int_\omega P_{ss}C_{-M_1,-M_2,-M_3}d\omega \;\; \int_\omega P_{ss}C_{-M_1,-M_2,-M_3+1}d\omega \;\; \cdots \;\; \int_\omega P_{ss}C_{M_1,M_2,M_3}d\omega \;] \quad (21)$$

$$C_{i,j,k}=\cos(\omega_x i+\omega_y j+\omega_t k) \quad (22)$$

, where the symbol ^{T} represents transposition. P_{ss} is given by eq.(7) and P_{nn} is the power spectrum of $n(x,y,t)$, written by

$$P_{nn}(\omega) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty}|N_1(\omega;\theta',V')|^2 \cdot p_{\theta'}(\theta')p_{V'}(V')d\theta' dV' + |N_2(\omega)|^2 \quad (23)$$

Then the impulse response $h(i,j,k)$ can be obtained as the solution of the next normal equation with this notation.

$$\boldsymbol{A}\cdot\boldsymbol{G}=\boldsymbol{R} \quad (24)$$

$$g(i,j,k)=\begin{cases} \frac{1}{4}h(i,j,k), & (i=j=k=0) \\ \frac{1}{2}h(i,j,k), & (either\ of\ \ i,j,k=0) \\ h(i,j,k), & (either\ of\ \ i,j,k\neq 0) \\ 2h(i,j,k), & (i,j,k\neq 0) \end{cases} \quad (25)$$

3 EXAMPLE

Let us consider a detection problem of a target LTS which has constant value on a rectangular-shaped region($2x_1\times 2y_1$[pixels]) contaminated by additive Gaussian white noise (variance Σ^2) and unwanted another target signal.

The target LTS is supposed to have 2-D object signal

$$w(x,y)=\begin{cases} 1 & (x,y);|x|\le x_1,|y|\le y_1 \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

The power spectrum $|S(\omega;u,v)|^2$ of input signal $s(x,y,t)$ can be obtained from eq.(6) as follows,

$$|S(\omega;u,v)|^2 \quad (27)$$

$$=|8\pi x_1y_1\mathrm{sinc}(\omega_x x_1)\mathrm{sinc}(\omega_y y_1)\delta(u\omega_x+v\omega_y+\omega_t)|^2$$

Eq.(8) is used to calculate $P_{ss}(\omega)$, that is,

$$P_{ss}(\omega)=\{8\pi x_1y_1\mathrm{sinc}(\omega_x x_1)\mathrm{sinc}(\omega_y y_1)\}^2 \cdot\iint_{-\infty}^{\infty}\delta(u\omega_x+v\omega_y+\omega_t)p(u,v)dudv \quad (28)$$

We set the probability density function $p_\theta(\theta)$ and $p_V(V)$ as follows:

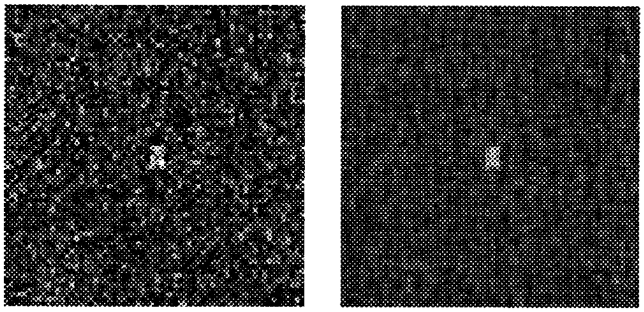
(i) θ: approximated Gaussian distribution on $[\theta_0-\pi,\theta_0+\pi]$

$$p_\theta(\theta)\cong\frac{1}{\sqrt{2\pi}\beta}exp\left\{-\frac{(\theta-\theta_0)^2}{2\beta^2}\right\} \qquad -\pi<\theta-\theta_0<\pi \quad (29)$$

(ii) V: Gaussian distribution,

$$p_V(V) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{-\frac{(V-V_0)^2}{2\sigma^2}\right\} \qquad (30)$$

where θ_0, V_0 and β^2, σ^2 are set to the mean and variance of θ, V, respectively. All elements in the matrices A and $\boldsymbol{R}^T$ are obtained by the numerical integration. Then we can obtain $h(i,j,k)$ by solving eq.(24).



(a) Input image (b) Output image

Figure 3: Example 1

In the first case by setting $\theta_0 = 0$ *or* $\pi(rad), V_0 = 1$ $(pixel/frame), \beta = 0.1\pi(rad), \sigma = 0.1(pixel/frame)$, $x_1 = 2(pixel), y_1 = 1(pixel), \Sigma^2 = 0.25$, a set of input and output frame images is shown in Fig.3 ($32nd$ $frame$). The obtained frequency response of optimal filter ($M_1 = M_2 = M_3 = 5, \omega_t = 0$) is shown in Fig.4. Fig.5(b) shows the output in the second case where a moving noise object is contained in the input. Finally, in the same condition of the first example, the shape of target object is changed to the 'L'-shaped as shown in Fig.5(c) . The 32nd output frame image is shown in Fig.5(d).

4 CONCLUSION

In this paper, we have proposed a design method of optimal filter for extracting a target LTS. The impulse response sequences of the optimal FIR filter is derived in a general form by using power spectrum of LTS with stochastically distributed direction and velocity. An optimal 3-D smoothing zero-phase FIR filter is obtained by using the information of a set of desired LTS and noise signals and their velocity vector distributions.

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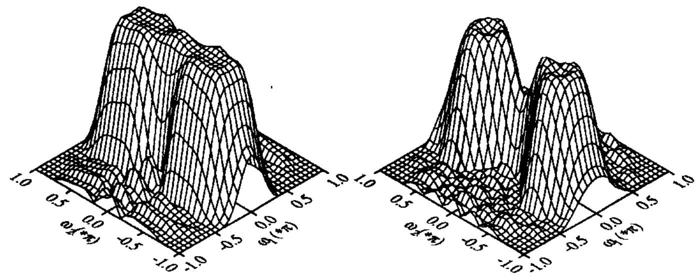

(a) CASE I (b) CASE II

Figure 4: Frequency response of FIR optimal filter

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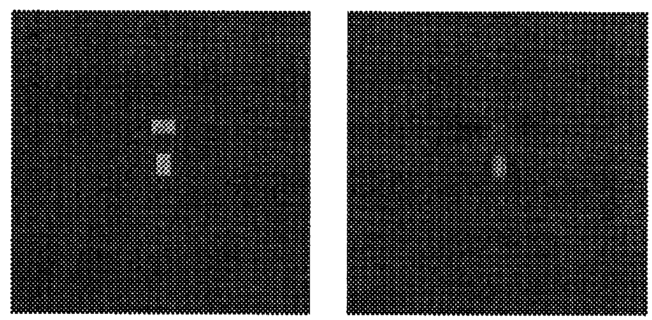
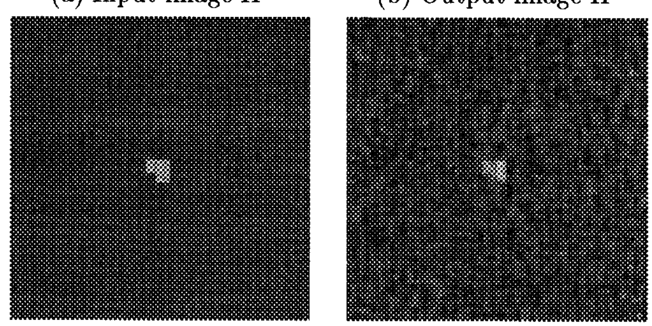


(a) Input image II (b) Output image II



(c) Target object II (d) Output image III

Figure 5: Example 2