A NEW DESIGN METHOD OF LINEAR-PHASE PARAUNITARY FILTER BANKS WITH AN ODD NUMBER OF CHANNELS

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ABSTRACT

In this work, a new design method of M-channel linear-phase paraunitary filter banks (LPPUFB) is proposed for odd M with a cascade structure. The conventional cascade structure has a problem that one of the filters is restricted to be of length M. In the proposed method, all filters are permitted to be of the same length as each other and longer than M. The significance of our proposed method is verified by showing some design examples.

1 INTRODUCTION

The linear-phase (LP) and paraunitary (PU) properties of filter banks are particularly significant for subband coding of images [1, 2]. Thus, several linear-phase paraunitary filter banks (LPPUFB) have been studied so far [3–7]. In the article [3], a special case of such systems, which is known as the lapped orthogonal transforms (LOT), was shown. Then, the more general systems were established [4, 5]. For even M, such systems have been well developed, especially with the cascade structure [6, 7]. Those structures enable us to design LPPUFB in systematic ways. Furthermore, by using the symmetric extension methods [8–11], efficient structures of them for finite-duration sequences were constructed [3, 12]. Note that, however, LPPUFB for even M can not be applied to construct M-band wavelets which have more than one vanishing moment [13, Theorem 2.1]. On the other hand, for odd M, there still remains the possiblity to improve the limitations.

Soman *et al.* showed the existence of LPPUFB for odd M, and provided the cascade structure in [5, Section V]. However, a problem exists that one of the analysis filters and one of the synthesis filters are restricted to be of length M, while other filters are permitted to be longer than M. In other words, all of the filters can not to be of the same length as each other, except in a special case. This limitation affects the achievable performance such as coding gain [14] and stop-band attenuation.

Therefore, in order to solve this problem, we propose a new product form of LPPUFB for odd M. The proposed filter banks can be regarded as the odd M version of the generalized lapped orthogonal transforms (GenLOT) [6, 7] from the structure. In Sec.2, we review LPPUFB. In Sec.3, we provide overlap-save method (OLS) based on LP orthonormal matrices, and propose a new cascade structure of LPPUFB for odd M. In Sec.4, we also propose the design procedure and, to verify the significance of our proposed method, show some design examples.

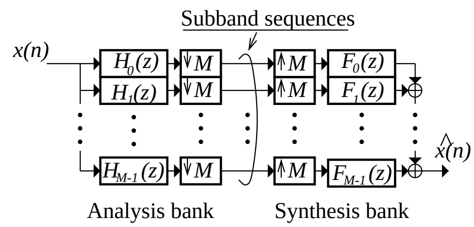


Figure 1: M-channel maximally decimated filter banks The box including $\downarrow M$ and $\uparrow M$ denote the down- and up-sampler with the factor M, respectively.

2 REVIEW OF LPPUFB

As a preliminary, we review M-channel LPPUFB. All through this work, the notations $\mathbf{\Gamma}_M$, $\mathbf{I}_M$, and $\mathbf{J}_M$ denote the $M \times M$ diagonal matrix which has $+1$ and -1 elements alternatively on the diagonal, identity matrix and reversal matrix [1], Besides, $\mathbf{O}$ and $\mathbf{o}$ are the null matrix and vector, respectively, and the superscript 'T' on a matrix or a vector represents the transposition.

Figure 1 shows a parallel structure of M-channel maximally decimated filter banks [1], where $H_k(z)$ and $F_k(z)$ are the analysis and synthesis filters, respectively. When the reconstructed output sequence $\hat{x}(n)$ is identical to the input $x(n)$, except for the delay and scaling, the analysis-synthesis system is called perfect reconstruction (PR) filter banks. Let $\mathbf{E}(z)$ and $\mathbf{R}(z)$ denote the $M \times M$ polyphase matrices of analysis and synthesis banks, respectively [1]. If $\mathbf{E}(z)$ and $\mathbf{R}(z)$ satisfy the following condition:

$$\mathbf{R}(z)\mathbf{E}(z) = cz^{-N}\mathbf{I}_M \tag{1}$$

for some integer N, then the system has PR property [1]. In addition, if $\mathbf{E}(z)$ satisfies the following condition:

$$\tilde{\mathbf{E}}(z)\mathbf{E}(z) = \mathbf{I}_M, \tag{2}$$

then it is said to be paraunitary (PU), where $\tilde{\mathbf{E}}(z)$ is the paraconjugation of $\mathbf{E}(z)$ [1]. The condition as in Eq.(2) is sufficient to construct PR filter banks, since the PR property as in Eq.(1) is guaranteed by choosing the synthesis polyphase matrix as $\mathbf{R}(z) = cz^{-N}\tilde{\mathbf{E}}(z)$.

Then, let us consider LP property of filter banks. We assume that the elements of the polyphase matrix $\mathbf{E}(z)$ is real, causal and FIR of order N. On this assumption, the corresponding analysis filters $H_k(z)$ are also real, causal and FIR, and the order results in $K = (N+1)M-1$. If $\mathbf{E}(z)$ further satisfies the following property:

$$z^{-N}\mathbf{\Gamma}_M\mathbf{E}(z^{-1})\mathbf{J}_M = \mathbf{E}(z), \tag{3}$$

then each analysis filter $H_k(z)$ for even k is symmetric and one for odd k is antisymmetric with the center of symmetry $K/2$ [4, 5].

In this work, we consider constructing valid LP-PUFB's for odd M, which satisfy both Eqs.(2) and (3).

3 PROPOSED LPPUFB FOR ODD M

In this section, we propose a new product form of polyphase matrices satisfying both Eqs.(2) and (3) for odd M. Our proposed product form provides a new cascade structure of LPPUFB.

3.1 OLS with LP Orthonormal Matrices

For the latter discussion, we provide an FIR filtering technique based on odd-size LP orthonormal matrices, as done on the basis of even-size ones in the article [7]. The technique can be regarded as a modification of the generalized overlap-save method [2], and has an important role for constructing LPPUFB for odd M.

Let $H(z)$ be an FIR filter and $\mathbf{e}(z)$ be the $M\times 1$ vector defined by $\mathbf{e}(z) = [E_0(z), E_1(z), \cdots, E_{M-1}(z)]^T$, where $E_\ell(z)$ is the ℓ-th type-I polyphase component of $H(z)$ with the decomposition factor M. In the followings, we assume that the factor M is odd.

Firstly, we decompose $\mathbf{e}(z)$ into the symmetric vector $\mathbf{s}(z)$ and antisymmetric vector $\mathbf{a}(z)$, as follows:

$$\mathbf{s}(z) = \frac{\mathbf{e}(z) + \mathbf{J}_M\mathbf{e}(z)}{2}, \tag{4}$$

$$\mathbf{a}(z) = \frac{\mathbf{e}(z) - \mathbf{J}_M\mathbf{e}(z)}{2}. \tag{5}$$

There is a relation $\mathbf{e}(z) = \mathbf{s}(z) + \mathbf{a}(z)$. Note that $\mathbf{s}(z)$ and $\mathbf{a}(z)$ are uniquely determined from their own $(M+1)/2\times 1$ and $(M-1)/2\times 1$ bottom vectors, respectively.

Let $\mathbf{s}^{\mathrm{r}}(z)$ and $\mathbf{a}^{\mathrm{r}}(z)$ be those bottom vectors of $\mathbf{s}(z)$ and $\mathbf{a}(z)$, respectively, and define transform coefficient vectors $\mathbf{g}_{\mathrm{E}}(z)$ and $\mathbf{g}_{\mathrm{O}}(z)$ of $\mathbf{s}^{\mathrm{r}}(z)$ and $\mathbf{a}^{\mathrm{r}}(z)$ by

$$\mathbf{g}_{\mathrm{E}}(z) = \mathbf{S}\mathbf{J}_{\frac{M+1}{2}}\mathbf{s}^{\mathrm{r}}(z), \tag{6}$$

$$\mathbf{g}_{\mathrm{O}}(z) = \mathbf{A}\mathbf{J}_{\frac{M-1}{2}}\mathbf{a}^{\mathrm{r}}(z), \tag{7}$$

where $\mathbf{S}$ and $\mathbf{A}$ denote arbitrary $(M+1)/2\times(M+1)/2$ and $(M-1)/2\times(M-1)/2$ orthonormal matrices, respectively. In terms of $\mathbf{g}_{\mathrm{E}}(z)$ and $\mathbf{g}_{\mathrm{O}}(z)$, the vector $\mathbf{e}(z)$ can be rewritten as follows:

$$\mathbf{e}^T(z) = \sqrt{2}\left[\mathbf{g}_{\mathrm{E}}^T(z)\ \ \mathbf{g}_{\mathrm{O}}^T(z)\right]\mathbf{C}\mathbf{J}_M, \tag{8}$$

where $\mathbf{C}$ is the $M\times M$ LP orthonormal matrix provided as follows:

$$\mathbf{C} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{S} & \mathbf{O} \\ \mathbf{O} & \mathbf{A}\end{bmatrix}\begin{bmatrix} \mathbf{I}_{\frac{M-1}{2}} & \mathbf{o} & \mathbf{J}_{\frac{M-1}{2}} \\ \mathbf{o}^T & \sqrt{2} & \mathbf{o}^T \\ \mathbf{J}_{\frac{M-1}{2}} & \mathbf{o} & -\mathbf{I}_{\frac{M-1}{2}}\end{bmatrix}. \tag{9}$$

Eq.(8) can be regarded as a special case of the generalized OLS in transform-domain filtering technique [2].

When the order of the polyphase component vector $\mathbf{e}(z)$ is N, the order of $H(z)$ results in $K = (N+1)M-1$. Note that if and only if $H(z)$ is symmetric with the center of symmetry $K/2$, that is, the case that $z^{-N}\mathbf{e}^T(z^{-1})\mathbf{J}_M = \mathbf{e}^T(z)$, then the following properties are satisfied with $\gamma_{\mathrm{E}} = 1$ and $\gamma_{\mathrm{O}} = -1$:

$$\mathbf{g}_{\mathrm{E}}(z) = \gamma_{\mathrm{E}}z^{-N}\mathbf{g}_{\mathrm{E}}(z^{-1}), \tag{10}$$

$$\mathbf{g}_{\mathrm{O}}(z) = \gamma_{\mathrm{O}}z^{-N}\mathbf{g}_{\mathrm{O}}(z^{-1}). \tag{11}$$

In addition, if and only if $H(z)$ is antisymmetric with the center of symmetry $K/2$, that is, the case that $-z^{-N}\mathbf{e}^T(z^{-1})\mathbf{J}_M = \mathbf{e}^T(z)$, the above properties are satisfied with $\gamma_{\mathrm{E}} = -1$ and $\gamma_{\mathrm{O}} = 1$.

3.2 New Product Form

By using OLS developed in the previous subsection, we derive a new product form of LPPUFB for odd M. Let $\mathbf{e}_k(z)$ be the type-I polyphase component vector of $H_k(z)$, that is, the transposition of the k-th row vector of $\mathbf{E}(z)$. Since $\mathbf{e}_k(z)$ can be represented as in Eq.(8), $\mathbf{E}(z)$ has the following form:

$$\mathbf{E}(z) = \mathbf{P}^T\mathbf{G}(z)\mathbf{C}\mathbf{J}_M, \tag{12}$$

where $\mathbf{G}(z)$ is the $M\times M$ matrix which consists of the transform coefficient vectors obtained from $\mathbf{e}_k(z)$ as in Eqs.(6) and (7), and $\mathbf{P}$ denotes the $M\times M$ matrix which permutes the even rows into the $(M+1)/2$ top rows and the odd rows into the $(M-1)/2$ bottom rows.

Then, we consider constructing $\mathbf{G}(z)$ under the PU constraint as in Eq.(2) and LP constraint as in Eq.(3). Note that if and only if $\mathbf{E}(z)$ is PU, $\mathbf{G}(z)$ is PU since all of $\mathbf{P}$, $\mathbf{J}_M$ and $\mathbf{C}$ are PU. For convenience of the further discussion, we define the $M\times M$ matrix $\mathbf{F}(z)$ by $\mathbf{F}(z) = \mathbf{T}\mathbf{B}\mathbf{G}(z)$ where

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{\frac{M+1}{2}} & \mathbf{O} \\ \mathbf{O} & \mathbf{J}_{\frac{M-1}{2}}\end{bmatrix}, \tag{13}$$

$$\mathbf{B} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_{\frac{M-1}{2}} & \mathbf{o} & \mathbf{I}_{\frac{M-1}{2}} \\ \mathbf{o}^T & \sqrt{2} & \mathbf{o}^T \\ \mathbf{I}_{\frac{M-1}{2}} & \mathbf{o} & -\mathbf{I}_{\frac{M-1}{2}}\end{bmatrix}. \tag{14}$$

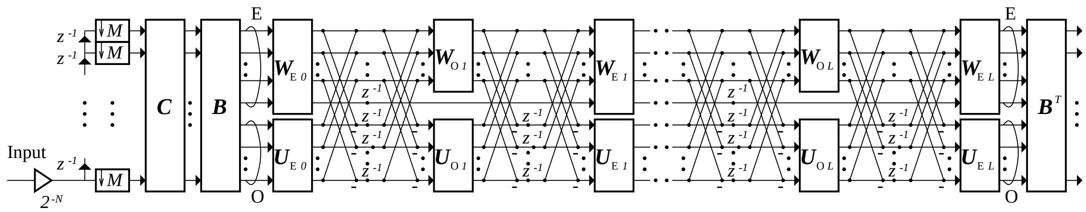


Figure 2: The proposed cascade structure of M-channel LPPU analysis filter bank for odd M.

It can be verified that if and only if $\mathbf{F}(z)$ is PU, then so is $\mathbf{G}(z)$. As a result, the PU property of $\mathbf{F}(z)$ implies that of $\mathbf{E}(z)$. In addition, the LP property of $\mathbf{E}(z)$ as in Eq.(3) can be represented in terms of $\mathbf{F}(z)$ as follows:

$$z^{-N}\mathbf{J}_M\mathbf{F}(z^{-1})\begin{bmatrix}\mathbf{I}_{\frac{M+1}{2}} & \mathbf{O}\\ \mathbf{O} & -\mathbf{I}_{\frac{M-1}{2}}\end{bmatrix} = \mathbf{F}(z). \tag{15}$$

The condition as in Eq.(15) is proofed from the fact that the transform coefficient vectors included in $\mathbf{G}(z)$ satisfy Eqs.(10) and Eq.(11) with $\gamma_{\mathrm{E}} = 1$ and $\gamma_{\mathrm{O}} = -1$ for top $(M+1)/2$ row vectors, and with $\gamma_{\mathrm{E}} = -1$ and $\gamma_{\mathrm{O}} = 1$ for bottom $(M-1)/2$ row vectors.

Let $\mathbf{F}_m(z)$ be the matrix of order m which satisfies both of PU property as in Eq.(2) and the condition as in Eq.(15), and let

$$\mathbf{R}_{\mathrm{E}\ell} = \begin{bmatrix}\mathbf{W}_{\mathrm{E}\ell} & \mathbf{O}\\ \mathbf{O} & \mathbf{U}_{\mathrm{E}\ell}\end{bmatrix} \tag{16}$$

$$\mathbf{R}_{\mathrm{O}\ell} = \begin{bmatrix}\mathbf{W}_{\mathrm{O}\ell} & \mathbf{o} & \mathbf{O}\\ \mathbf{o}^T & 1 & \mathbf{o}^T\\ \mathbf{O} & \mathbf{o} & \mathbf{U}_{\mathrm{O}\ell}\end{bmatrix} \tag{17}$$

where $\mathbf{W}_{\mathrm{E}\ell}$ is an $(M+1)/2 \times (M+1)/2$ orthonormal matrix, and all of $\mathbf{W}_{\mathrm{O}\ell}$, $\mathbf{U}_{\mathrm{E}\ell}$ and $\mathbf{U}_{\mathrm{O}\ell}$ are $(M-1)/2 \times (M-1)/2$ orthonormal matrices.

Then, we can construct $\mathbf{F}_{m+2}(z)$, which also satisfies Eqs.(2) and (15), from $\mathbf{F}_m$ as follows:

$$\mathbf{F}_{m+2}(z) = \mathbf{K}_{\mathrm{E},m+2}\mathbf{\Lambda}_{\mathrm{E}}(z)\mathbf{K}_{\mathrm{O},m+2}\mathbf{\Lambda}_{\mathrm{O}}(z)\mathbf{F}_m(z), \tag{18}$$

where $\mathbf{K}_{\mathrm{E},\ell} = \mathbf{TBR}_{\mathrm{E},\ell}\mathbf{BT}$, $\mathbf{K}_{\mathrm{O},\ell} = \mathbf{TBR}_{\mathrm{O},\ell}\mathbf{BT}$, and

$$\mathbf{\Lambda}_{\mathrm{E}}(z) = \begin{bmatrix}\mathbf{I}_{\frac{M+1}{2}} & \mathbf{O}\\ \mathbf{O} & z^{-1}\mathbf{I}_{\frac{M-1}{2}}\end{bmatrix}, \tag{19}$$

$$\mathbf{\Lambda}_{\mathrm{O}}(z) = \begin{bmatrix}\mathbf{I}_{\frac{M-1}{2}} & \mathbf{O}\\ \mathbf{O} & z^{-1}\mathbf{I}_{\frac{M+1}{2}}\end{bmatrix}. \tag{20}$$

As a result, by constructing $\mathbf{E}(z)$ with the following product form, we can obtain LPPUFB described by Eqs.(2) and (3) for odd M and even N, where N is the order of $\mathbf{E}(z)$.

$$\begin{aligned}\mathbf{E}(z) &= \mathbf{P}^T\mathbf{R}_{\mathrm{E}L}\mathbf{Q}_{\mathrm{E}}(z)\mathbf{R}_{\mathrm{O}L}\mathbf{Q}_{\mathrm{O}}(z)\cdots\\ &\quad\cdots\mathbf{R}_{\mathrm{E}1}\mathbf{Q}_{\mathrm{E}}(z)\mathbf{R}_{\mathrm{O}1}\mathbf{Q}_{\mathrm{O}}(z)\mathbf{R}_{\mathrm{E}0}\mathbf{CJ}_M,\end{aligned} \tag{21}$$

where $\mathbf{Q}_{\mathrm{E}}(z) = \mathbf{B\Lambda}_{\mathrm{E}}(z)\mathbf{B}$, $\mathbf{Q}_{\mathrm{O}}(z) = \mathbf{B\Lambda}_{\mathrm{O}}(z)\mathbf{B}$, and $L = N/2$. When $N = 0$, $\mathbf{E}(z) = \mathbf{P}^T\mathbf{R}_{\mathrm{E}0}\mathbf{CJ}_M$.

Eq.(21) provides us the cascade structure of LPPUFB for odd M and even N as shown in Fig.2. This system consists of $(M+1)/2$ symmetric and $(M-1)/2$ anti-symmetric filters of odd length. Note that the counterpart synthesis bank holding perfect reconstruction is simply obtained because of the PU property [1].

It can be verified that the product form as in Eq.(21) covers larger class of LPPUFB than that provided in the article [5]. The conventional product form can be viewed as the special case that the $(M+1)/2, (M+2)/2$-th elements of the matrices $\mathbf{W}_{\mathrm{E}\ell}$ for $\ell = 0, 1, \cdots, L-1$ are imposed to be 1. From the orthonormality, it implies that $(M+1)/2$-th row and column of each $\mathbf{W}_{\mathrm{E}\ell}$ consists of zeros except for the $(M+1)/2, (M+1)/2$-th element.

4 Design Procedure

By controlling orthonormal matrices $\mathbf{W}_{\mathrm{E}\ell}$, $\mathbf{W}_{\mathrm{O}\ell}$, $\mathbf{U}_{\mathrm{E}\ell}$ and $\mathbf{U}_{\mathrm{O}\ell}$ in the structure as shown in Fig.2, we can design LPPUFB for odd M, where N is even. Since $\mathbf{W}_{\mathrm{E}\ell}$ can be characterized in terms of $(M+1)(M-1)/8$ plane rotations and others can be done in terms of $M(M-2)/8$ ones [1], it is allowed to design such systems by an unconstrained optimization process to minimize (or maximize) some object function. Both of the PU and LP properties are guaranteed with this approach since these constraints are structurally imposed.

4.1 The Recursive Initialization Approach

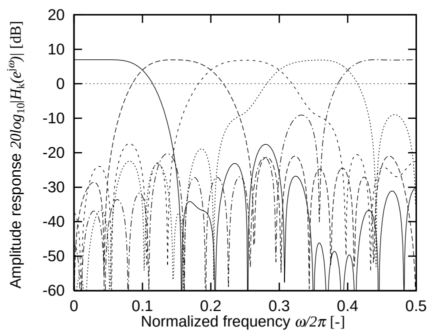
For the optimization process, we consider the recursive initialization approach to avoid insignificant local-minimum solutions. Suppose that $\mathbf{E}_N(z)$ be a matrix of order N provided as in Eq.(21). It can be verified that there exists the following relation:

$$\mathbf{E}_N(z) = z^{-1}\mathbf{E}_{N-2}(z), \tag{22}$$

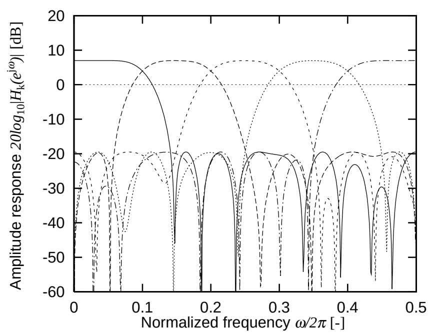
when

$$\mathbf{R}_{\mathrm{E}L} = \mathbf{R}_{\mathrm{O}L} = \begin{bmatrix}\mathbf{I}_{\frac{M+1}{2}} & \mathbf{O}\\ \mathbf{O} & -\mathbf{I}_{\frac{M-1}{2}}\end{bmatrix}, \tag{23}$$

where $L = N/2$. Eq.(22) implies that $\mathbf{E}_N(z)$ is identical to $\mathbf{E}_{N-2}(z)$ but with the delay. Thus, when $\mathbf{E}_{N-2}(z)$ has good performance, for example high coding gain, $\mathbf{E}_N(z)$ also does. From this fact, we can design a significant odd channel LPPUFB by the following procedure:



(a) Filters designed for maximum coding gain $G_{\rm TC}$ for an AR(1) signal with $\rho = 0.95$. $G_{\rm TC} = 8.95$dB.



(b) Filters designed for maximum stop-band attenuation $A_{\rm S}$, where the transition width of each filter is assumed to be half of the pass-band one. $A_{\rm S} = 26.5$dB.

Figure 3: Design examples: amplitude responses of 5 analysis filters, where $M = 5$, $N = 6$ and the length of each filter is 35.

Step 1: Start with proper LPPUFB $\mathbf{E}_0(z)$, for example the M-point type-I DCT [15], and optimize it.
Step 2: Initialize the two higher order system by adding the sections according to Eqs.(21) and (23).
Step 3: Optimize the system, and go to Step 2 until the order reaches to N.

4.2 Design Examples

To verify the significance of our proposed method, we provide two design examples in Fig.3, where $M = 5$, $N = 6$ and the matrix $\mathbf{C}$ in Eq.(21) is chosen to be the M-point type-I DCT, that is, $\mathbf{S}$ and $\mathbf{A}$ are chosen to be $(M+1)/2$-point type-I DCT and $(M-1)/2$-point type-III DCT, respectively [15]. In Fig.3, (a) and (b) show the amplitude responses of analysis filters designed for maximum coding gain $G_{\rm TC}$ [14], and those for maximum stop-band attenuation $A_{\rm S}$, respectively. Each analysis filter has $M(N+1) = 35$ tap length. These examples are obtained using the routines 'fminu' for (a) and 'minimax' for (b) provided by MATLAB optimization toolbox [16]. When one of the filters is restricted to be of length M as shown in [5], the results show worse performance than the examples presented here.

5 CONCLUSION

In this work, we proposed a new cascade structure of M-channel LPPUFB for odd M, which covers larger class than the conventional one, and provided the recursive initialization design method. We verified the significance of our proposed method by showing two design examples. As future works, it remains to construct M-band LP orthonormal wavelets which have more than one vanishing moment using our proposed filter banks.

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