COMPUTATIONALLY EFFICIENT REALIZATION OF MDFT FILTER BANKS

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ABSTRACT

A realization of the Modified DFT (MDFT) filter bank introduced in [1, 2, 3] was proposed in [4]. The analysis and synthesis filter bank consist each of two DFT polyphase filter banks, one without delay and one delayed by $M/2$ samples where M represents the number of channels of the MDFT filter bank.

In this paper, we will show that the two DFTs can be reduced to a single one for prototypes of the lengths $N = r \cdot M + 1$ and $N = r \cdot M + M/2 + 1$, respectively, by doing some simple combinations with the input signals. Hereby the modulation cost is nearly halved.

1 INTRODUCTION

As shown in Fig. 1, the MDFT filter bank can be derived from a complex modulated filter bank by decimating the sampling rate with and without a delay of $M/2$ samples and using either the real or the imaginary part, respectively, in the subbands. With these modifications, all odd alias spectra including adjacent channel aliasing are cancelled. Perfect reconstruction is achieved with prototypes which are also used for cosine modulated filter banks with half the number of channels [3].

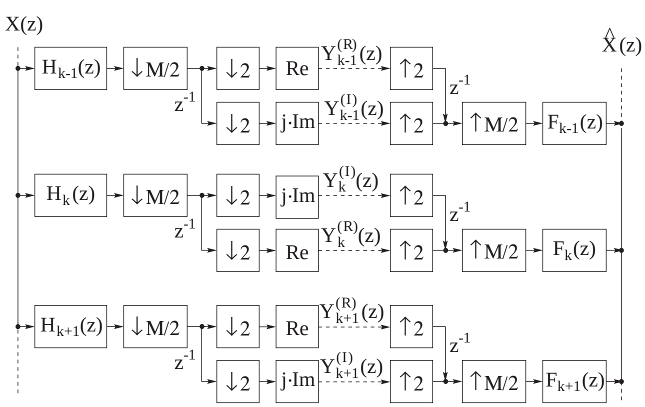

Figure 1: Modified complex modulated filter bank

The causal analysis and synthesis filters $H_k(z)$ and $F_k(z)$, respectively, are complex modulated and time shifted versions of a symmetrical, real valued, zero-phase, lowpass prototype $P(z)$ with bandwidth $2\pi/M$ (including positive and negative frequencies).

$$h_k(n) = f_k(n) = p(n - \frac{N-1}{2}) \cdot W^{-k(n-\frac{N-1}{2})},$$
$$n = 0, \ldots, N-1, \quad k = 0, \ldots, M-1,$$
$$W^k = \exp\left(-j\frac{2\pi k}{M}\right).$$

As was shown in [4], the MDFT filter bank can be realized by two DFT filter banks where one is delayed by $M/2$ samples (see Fig. 2).

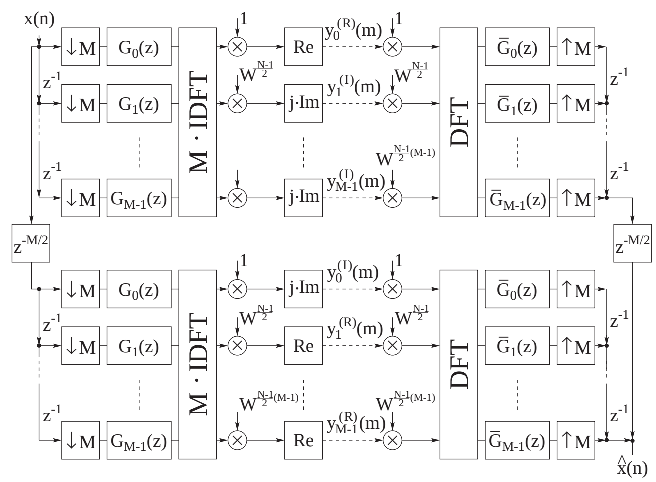

Figure 2: MDFT filter bank realized by two DFT polyphase filter banks

The input signal $x(n)$ is complex valued. $G_k(z)$ and $\bar{G}_k(z)$ represent different types of the k-th polyphase filter of the lowpass $H_0(z)$.

$$\begin{aligned} g_k(m) &= h_0(m \cdot M + k) \\ \bar{g}_k(m) &= h_0(m \cdot M - k) \end{aligned}$$

The multiplications of the subband signals with $W^{\frac{N-1}{2}k}$ are due to the assumption of causal analysis and synthesis filters.

2 COMPUTATIONALLY EFFICIENT REALIZATION

For certain filter orders, the operations between the analysis and synthesis filter banks' DFTs can be simplified leading to a significant reduction of the modulation cost:

[0]This work is supported by the "Deutsche Forschungsgemeinschaft" (contract # Fl 116-10). This paper as well as further publications are available via anonymous ftp to 'ftp.et2.tu-harburg.de'.

- $N = r \cdot M + 1$: here, $W^{\frac{N-1}{2}k} = (-1)^{kr}$.
  This leads to the structure given in Fig. 3a.

- $N = r \cdot M + M/2 + 1$: here, $W^{\frac{N-1}{2}k} = (-1)^{kr} \cdot (-j)^k$.
  We obtain, after minor modifications, Fig. 3b, where the subband signals' signs have been adjusted appropriately. From Fig. 3b, the filter banks proposed by Princen and Bradley [5] where r is limited to one and by Lin and Vaidyanathan [6] can be derived. Apart from the efficient implementation of the modulation (which will be shown in the sequel) the realization in Fig. 3b offers the advantage that complex valued signal processing is possible.

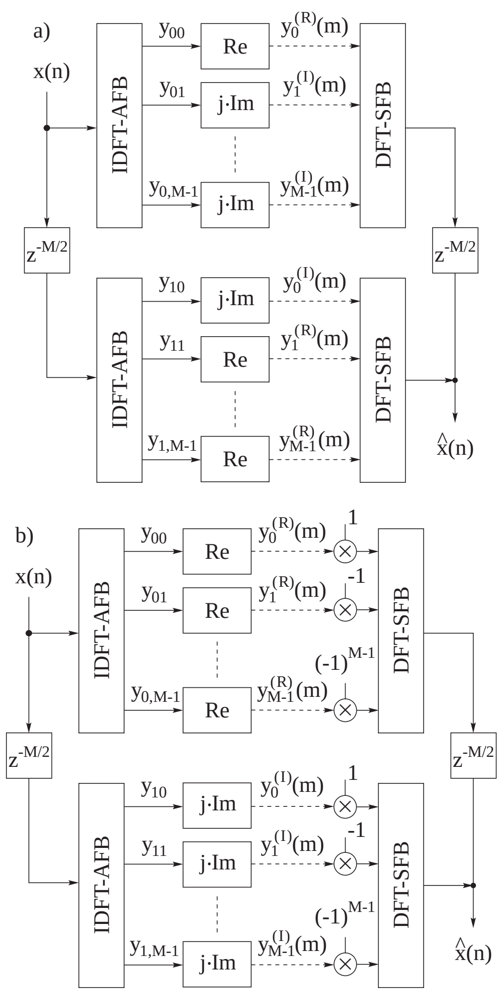

Figure 3: MDFT filter bank for prototypes with lengths a) $N = r \cdot M + 1$ and b) $N = r \cdot M + M/2 + 1$

Although there are M complex values at the input of each M point IDFT of the analysis filter bank, just the real or imaginary part is required in each subband. Instead of calculating the complex subband signals by two M point IDFTs and dismissing one of the real and imaginary parts, we will calculate the required M real parts and the M imaginary parts by one M point IDFT, only. For this purpose, we will exploit some properties of the discrete Fourier transform. The necessary modifications will be shown with the analysis filter bank in the case $N = m \cdot M + 1$ and $M = 4$.

1. In a first step, we exploit the DFT's property concerning a time shift of $M/4$ samples of the input signals:

$$\begin{aligned} \mathrm{IDFT}\left\{\tilde{x}(k)\right\} &= \mathrm{IDFT}\left\{x((M/4+k))_M\right\} \\ &= \frac{1}{M}\exp\left(-j\frac{2\pi k M}{4M}\right)\cdot y(k) \\ &= \frac{(-j)^k}{M}\cdot y(k) = \frac{\tilde{y}(k)}{M} \end{aligned}$$

   where $((M/4+k))_M$ stands for "$(M/4+k)$ modulo M", $\tilde{x}(k)$ for the k-th input signal of the M·IDFT operation and $\tilde{y}(k)$ for its k-th output signal, see Fig. 4.

   Taking the real part of the transformed sequence $\tilde{y}_{0k}$, $k = 0, \cdots, M-1$ with the upper, undelayed branch of the analysis filter bank and the imaginary part with the bottom, delayed branch, $\tilde{y}_{1k}$ we obtain the original subband signals apart from the sign and a multiplication with j. Fig. 4 clarifies this step.

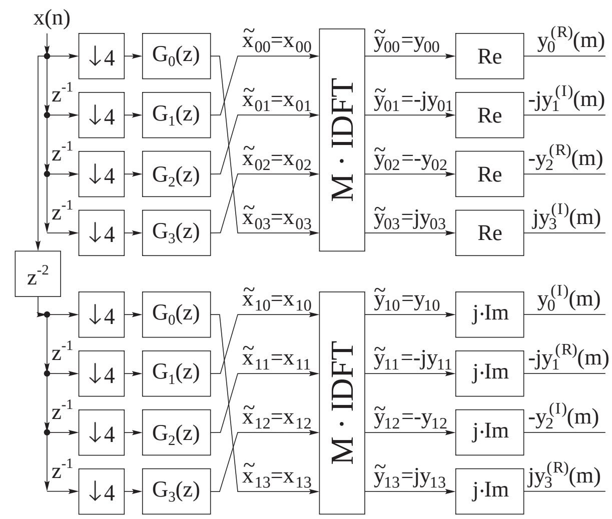

Figure 4: 4-channel MDFT analysis filter bank after the modifications described in item 1

2. At the output of the IDFTs, either the real or the imaginary part of the transformed sequence is required. To save the calculation of the irrelevant part, we combine the upper and lower input sequences into conjugate even and odd sequences, respectively (see Fig. 5).

$$\begin{gathered} \mathrm{Re}\left\{\tilde{y}_{0k}\right\} = \tfrac{1}{2}[\tilde{y}_{0k} + \tilde{y}^*_{0,M-k}] \\ = M\cdot\mathrm{IDFT}\left\{\tfrac{1}{2}[\tilde{x}_{0k} + \tilde{x}^*_{0,M-k}]\right\} = M\cdot\mathrm{IDFT}\left\{\tilde{x}_{ek}\right\} \\ j\mathrm{Im}\left\{\tilde{y}_{1k}\right\} = \tfrac{1}{2}[\tilde{y}_{1k} - \tilde{y}^*_{1k}] \\ = M\cdot\mathrm{IDFT}\left\{\tfrac{1}{2}[\tilde{x}_{1k} - \tilde{x}^*_{1,M-k}]\right\} = M\cdot\mathrm{IDFT}\left\{\tilde{x}_{ok}\right\} \end{gathered}$$

3. As the upper output sequence is a purely real signal and the lower one is purely imaginary valued, both sequences can be jointly calculated by adding the input sequences. Accordingly, the complex output sequence corresponds to the sum of the output sequences that have been calculated separately so far.

4. In order to obtain the original subband signals we have to undo the time shift introduced in step one after the addition of the upper and lower input sequence. Steps three and four are shown in Fig. 6.

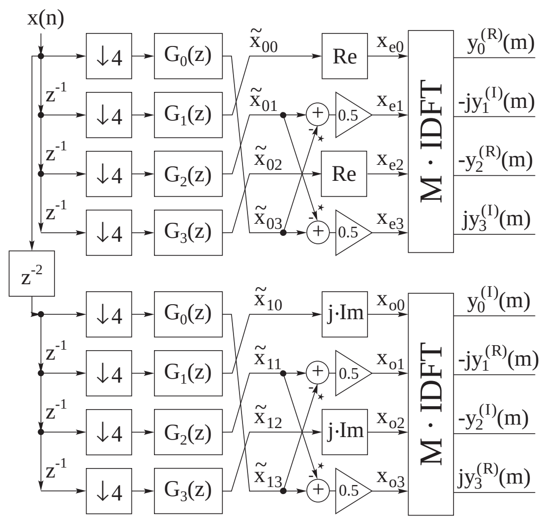

Figure 5: 4-channel MDFT analysis filter bank after the modifications described in items 1 and 2

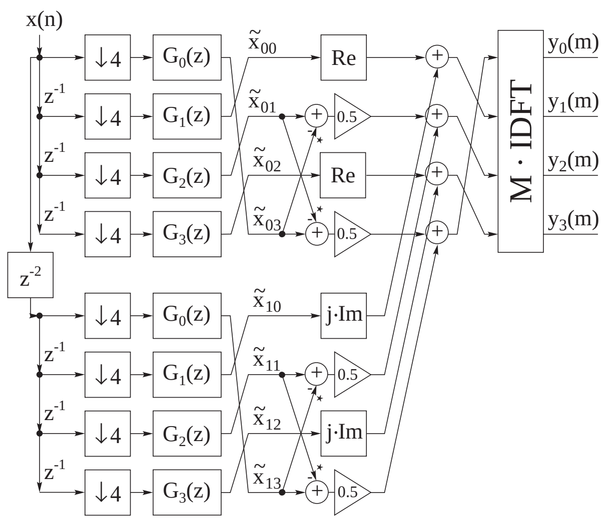

Figure 6: 4 channel MDFT analysis filter bank after the modifications described in items 1 to 4

For $N = m \cdot M + M/2 + 1$, we have to implement the steps 2 and 3, only.

All even order prototypes can be adapted to the above cases by appropriately padding their impulse responses with zeros so that the main impulse of the convolution of the lowpass filter $h_0(n)$ with itself is placed at time lags $r \cdot M$ and $r \cdot M + M/2$, respectively.

3 COMPARISON OF THE MODULATION COST

With the original polyphase realization of the MDFT filter bank given in Fig. 2, the modulation cost in the analysis filter bank is given by the number of operations within the two IDFTs. According to [7] the number of real additions $\alpha_{MDFT}(M)$ and multiplications $\mu_{MDFT}(M)$ is

$$\begin{aligned} \alpha_{MDFT}(M) &= 2 \cdot (3 \cdot \log_2 M - 3 + 4/M) f_s, \\ \mu_{MDFT}(M) &= 2 \cdot (\log_2 M - 3 + 4/M) f_s \end{aligned}$$

where f_s denotes the sampling rate. The input signal is supposed to be complex valued.

For the improved realization in Fig. 6 the modulation cost consists of the operations necessary for building the conjugate odd and even parts of the input signals, adding the two input sequences and performing one IDFT. Here, we obtain the following number of the required real additions and multiplications:

$$\begin{aligned} \alpha_{MDFT_1}(M) &= (3 \cdot \log_2 M + 3) f_s, \\ \mu_{MDFT_1}(M) &= (\log_2 M - 3 + 4/M) f_s. \end{aligned}$$

In the cosine modulated filter bank the modulation is done via an M point DCT-IV. Here, in case of a complex valued input signal, the number of real additions and multiplications equals [7]

$$\begin{aligned} \alpha_{DCT}(M) &= (3 \log_2 M) f_s, \\ \mu_{DCT}(M) &= (\log_2 M + 2) f_s. \end{aligned}$$

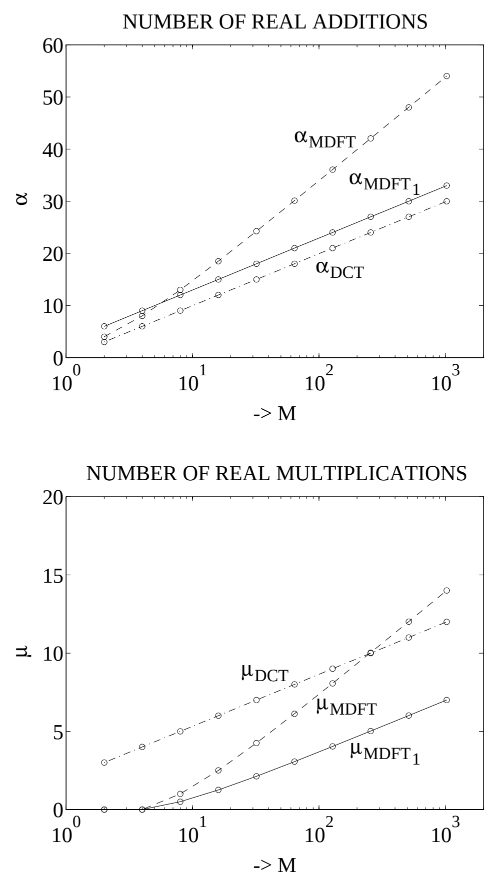

Figure 7: Comparison of the modulation costs

Fig. 7 compares the modulation costs necessary with the three realizations: MDFT (according Fig. 2), MDFT_1 (Fig. 6) and DCT-IV. In all cases the modulation cost grows with $O(\log M)$ but the improved realization of the MDFT filter bank has the lowest number of multiplications required. It is thus favorable for architectures where multiplications are more expensive than additions.

4 IMPLEMENTATION OF THE POLYPHASE FILTERS

So far, only the modulation complexity has been regarded. We have shown how to fit the two IDFTs into a single one. However, each polyphase filter still has to be realized twice. Having a closer look at the input sequences after subsampling, we see that the two polyphase filters $G_k(z)$ and $G_{((k+M/2))_M}(z)$ are fed with the same input signals apart from a possible delay. Therefore, we can use the same delay chain for both FIR filters, see Fig. 8.

For MDFT filter banks with perfect reconstruction these two polyphase filters are paraunitary and can be realized by the lattice structure proposed in [8].

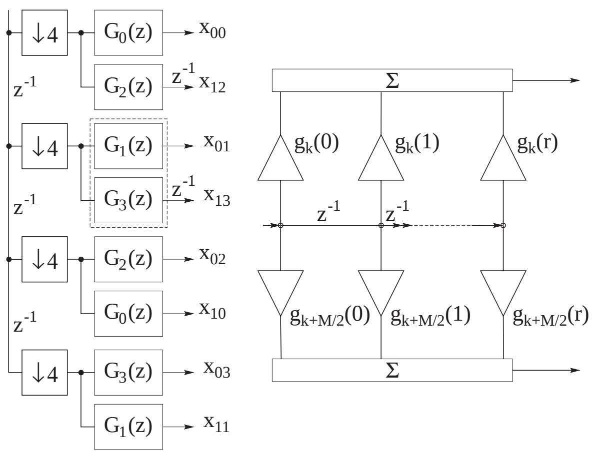

Figure 8: Joint realization of two polyphase filters

5 REAL SIGNAL PROCESSING

Up to now, we have assumed complex valued input sequences and have shown that the MDFT filter bank offers a very efficient realization in this case. However, in practical applications the input signals are often real valued. If there is more than one signal to be processed, we can build a complex input signal from two independent real signals by mapping them into the real and imaginary parts, respectively. In order to encode the signals separately we wish to have the same mapping for the subband signals. Unfortunately, the MDFT filter bank maps the real part of the input signal into both the real and the imaginary part of the subband signals [1, 2]. Therefore, we have to perform the operations described in Fig. 9 in order to get real (imaginary) subband signals for the real (imaginary) part of the input sequence.

6 CONCLUSION

In this paper we have shown that the modulation cost can be reduced significantly for analysis and synthesis filters of the lengths $N = r \cdot M + 1$ and $N = r \cdot M + M/2 + 1$, respectively. Each odd length prototype fits into these cases if its impulse response is properly filled up with zeros. Compared with [4], the modulation overhead is nearly halved. The proposed realization is efficient especially for complex valued input signals but can be adapted to real valued signals, too.

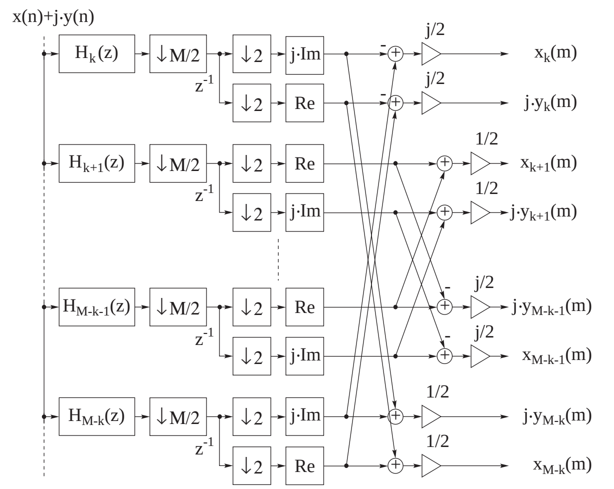

Figure 9: MDFT filter bank with normal subband signal mapping

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