A NEW ALGORITHM FOR DESIGNING PROTOTYPE FILTERS FOR M-BAND PSEUDO QMF BANKS

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ABSTRACT

This paper presents a simple and efficient method to design M-band Pseudo Quadrature Mirror Filter (QMF) banks. This method does not rely on a conventional nonlinear optimization method but rather on an Iterative Least Squares algorithm. The algorithm is rapidly converging, simple to implement and flexible. Its convergence does not depend on the starting point. Moreover iteratively calculated weighting functions can be used to shape the stopband of the prototype filter and the filter bank transfer function, and perform the minimax or the gain constrained least squares approximation. Design examples and a MATLAB program implementing the proposed algorithm are included.

1 INTRODUCTION

Digital filter banks are used in a number of applications such as subband coders for speech and image signals. The design of filter banks is generally performed by general purpose nonlinear optimization methods [1]. These optimization procedures are computationally very intensive, their convergences towards optimum filter banks are slow and uncertain since the cost functions have generally many local minima. As noted in [2] and [4], the starting points are critical and significant human intervention is necessary to obtain acceptable filters.

To simplify, improve and speed up the design of filter banks, we have investigated ways to use the Iterative Least Squares (ILS) method. In the past ten years, the ILS approach has been used in many filter design contexts: two dimensional minimax FIR filter design [5], log-IIR filter design [6], minimax and gain constrained least squares FIR filter design [7]-[8], allpass IIR equalizers design [9].

The use of the ILS approach to design filter banks has been proposed in [10]-[12]. Jain's and Chen's ILS methods [10]-[11] are however restricted to the design of 2-band QMF banks. Nayebi's method [12] is very general and can be used to design M-band low delay filter banks, however the ILS method is associated with a complicated gradient based optimization method.

In this paper we extend Jain's and Chen's ILS methods to the design of M-band Pseudo QMF banks. The algorithm is based on the iterative approximation of the objective function by a quadratic quantity. It is rapidly converging, simple to implement and flexible. Its convergence does not depend on the starting point. Moreover it can be coupled with an Iterative Reweighted Least Squares algorithm to perform the minimax or the gain constrained least squares approximation of the prototype stopband or/and the filter bank magnitude transfer function.

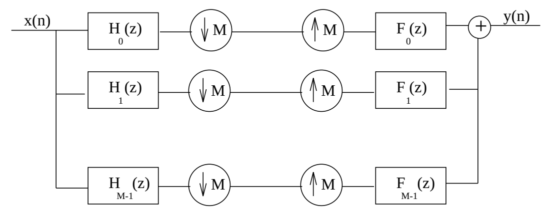

Figure 1: Uniform M-band filter bank

2 PROBLEM DESCRIPTION

A typical M-band filter bank is shown in Fig. 1 where $H_k(z)$ are the analysis filters and $F_k(z)$ are the synthesis filters. The perfect reconstruction property is satisfied when the output signal is a delayed version of the input signal, i.e. $y(n) = x(n-n_0)$ for some number n_0. Without any restriction on the analysis and synthesis filters, the output signal will be corrupted by three kinds of distortion [1]: phase, aliasing, and magnitude distortions.

In this paper we address the design of cosine modulated banks where the analysis and the synthesis filters are cosine modulated versions of a single linear phase lowpass FIR filter, $H(z)$. The cosine modulation has three main attractive properties [1]: the aliasing between adjacent bands and the phase distortion are entirely canceled; only one filter, i.e. the prototype filter, has to be designed; and there exists a fast implementation using the discrete cosine transform. As shown in [13], when the residual aliasing distortion is neglected, the filter bank transfer function can be expressed as

$$T(z) = \frac{z^{-N}}{M} \sum_{k=0}^{2M-1} H(zW_{2M}^{k+.5})H(z^{-1}W_{2M}^{-(k+.5)}) \qquad (1)$$

where N is the order of the prototype filter and $W_{2M} = e^{-j\frac{\pi}{M}}$. Alternatively T(z) can be expressed in terms of the prototype filter impulse response, $h(n)$ for $n = 0, 1, ..., N$, as follows

$$T(z) = 2z^{-N} \sum_{k=-E[\frac{N}{2M}]}^{E[\frac{N}{2M}]} (-1)^k \sum_{r=0}^{N-2M|k|} h(r)h(2M|k|+r)z^{-2Mk} \qquad (2)$$

where $E[x]$ is the integer part of x.

As proposed in [3], M-band Pseudo QMF banks can be designed by extending Johnston's 2-band QMF bank design method [2]. This method consists of minimizing the energy related to the magnitude reconstruction error of the filter bank, e_m, as well as the stopband ripple energy of the prototype filter, e_s, as follows

$$\min(e_s + \gamma e_m) \tag{3}$$

with

$$e_s = \frac{1}{\pi - \omega_s}\int_{\omega_s}^{\pi} |H(e^{j\omega})|^2 d\omega \tag{4}$$

$$e_m = \frac{1}{\pi}\int_0^{\pi} (|T(e^{j\omega})| - 1)^2 d\omega \tag{5}$$

where ω_s is the stopband frequency edge of the prototype filter and γ is a positive number corresponding to a relative weight factor associated with the minimization of e_m. The larger γ is, the smaller the magnitude distortion is. When $M > 2$, there is aliasing distortion between non adjacent bands. As pointed out in [13], the maximum level of aliasing is of the order of the stopband attenuation of the prototype filter. Hence it can be kept small when the attenuation is sufficiently large for $\omega \geq \frac{\pi}{M}$.

3 PROPOSED METHOD

3.1 ILS design

The design of M-band Pseudo QMF banks using the unconstrained optimization formulation (3) is not straightforward since the objective function is not quadratic. Complicated and computationally intensive nonlinear optimization methods are generally used. To simplify and speed up the design, we propose an ILS method where the objective function is iteratively approximated by a quadratic quantity.

At each iteration i, the filter bank transfer function, $T(z)$, is approximated by $T_i(z)$ which coefficients depend linearly on the coefficients $h_i(r)$ as follows

$$\begin{aligned} T_i(z) = 2z^{-N} \textstyle\sum_{k=-E[\frac{N}{2M}]}^{E[\frac{N}{2M}]} (-1)^k \sum_{r=0}^{N-2M|k|} h_i(r) \\ .h_{i-1}(2M|k| + r) z^{-2Mk} \end{aligned} \tag{6}$$

where $h_{i-1}(r)$ are the coefficients found in the $(i-1)^{th}$ iteration. The approximated filter bank transfer function, $T_i(z)$, along with the Parseval relation leads to the approximation of e_m by a quadratic term defined as follows

$$\begin{aligned} \hat{e}_m(i) &= \textstyle\sum_{k=-E[\frac{N}{2M}]}^{E[\frac{N}{2M}]} (2(-1)^k \sum_{r=0}^{N-2M|k|} h_i(r) \\ &\qquad .h_{i-1}(2M|k| + r) - \delta(k))^2 \\ &= (G_i p_i - d)^t (G_i p_i - d) \end{aligned} \tag{7}$$

where $\delta(k)$ is the Kronecker symbol, i.e. $\delta(0) = 1$ and $\delta(k) = 0$ for $k \neq 0$, $p_i = [h_i(\frac{N+1}{2}), ..., h_i(N)]^t$, $d = [1, 0, ..., 0]^t$ and G_i is a size $(E[\frac{N}{2M}] + 1)$x$\frac{N+1}{2}$ matrix defined as follows

$$\begin{cases} G_i(1, r) = 4h_{i-1}(\frac{N+1}{2} + r - 1) & 1 \leq r \leq \frac{N+1}{2} \\ G_i(k, r) = 0 & 1 \leq r \leq M(k-1) \\ G_i(k, r) = 4\sqrt{2}(-1)^{k-1} h_{i-1}(\frac{N+1}{2} + r - 1 - 2M(k-1)) & \\ \quad \text{with} \quad M(k-1) + 1 \leq r \leq \frac{N+1}{2} & \end{cases} \tag{8}$$

for $2 \leq k \leq E[\frac{N}{2M}] + 1$.

Let us express the stopband energy of the prototype filter at the iteration i, $e_s(i)$, in terms of the vector p_i. For the sake of simplicity, Let us assume that the order of the prototype filter, N, is odd and that its impulse response shows an even symmetry. The prototype filter frequency response can be expressed as

$$H_i(e^{j\omega}) = e^{-j\omega\frac{N}{2}} \sum_{n=0}^{\frac{N-1}{2}} h_i(\frac{N+1}{2} + n) 2cos(\omega(n + .5)) \tag{9}$$

The evaluation of the frequency response amplitude of the prototype filter, i.e. $A_i(e^{j\omega}) = e^{j\omega\frac{N}{2}} H_i(e^{j\omega})$, on $\Omega = \{\omega_0, ..., \omega_{L-1}\}$, a dense and uniform frequency grid of $[\omega_s, \pi]$, can be written in a matrix form as

$$[A_i(e^{j\omega_0}), ..., A_i(e^{j\omega_{L-1}})]^t = Cp_i \tag{10}$$

where C is a size Lx$\frac{N+1}{2}$ matrix containing the cosine coefficients $C(r, n) = 2cos(\omega_r(n + .5))$ for $0 \leq n \leq L-1$ and $0 \leq r \leq \frac{N-1}{2}$. The stopband energy of the prototype filter, $e_s(i)$, can be calculated on Ω, leading to the following matrix expression

$$e_s(i) \approx \frac{1}{L}\sum_n |H_i(e^{j\omega_n})|^2 = \frac{1}{L} p_i^t C^t C p_i \tag{11}$$

The solution to the complicated optimization problem (3) is found by iteratively minimizing a simple quadratic function defined as follows

$$\frac{1}{L} p_i C^t C p_i + \gamma (G_i p_i - d)^t (G_i p_i - d) \tag{12}$$

The vector p_i is obtained by solving the following overdetermined system of linear equations in the least squares sense

$$\begin{bmatrix} G_i \\ \frac{1}{\sqrt{L\gamma}} C \end{bmatrix} p_i = \begin{bmatrix} d \\ 0 \end{bmatrix} \tag{13}$$

The pseudo inverse method leads to the following solution

$$p_i = [G_i^t G_i + \frac{1}{L\gamma} C^t C]^{-1} G_i{}^t d \tag{14}$$

To guarantee the mean of $|T(e^{j\omega})|$ to be equal to 1, we normalize the coefficients $h_i(n)$, obtained from (14) as follows

$$h_i(n) \longleftarrow \frac{h_i(n)}{\sqrt{2\sum_{r=0}^{N} h_i(r)^2}} \tag{15}$$

As for most of ILS methods used in filter design applications, the convergence of this algorithm is not theoretically proven. However, from the experimentation we have found that it does converge very fast, and that the designed filters do not depend on the initial coefficients p_0. The termination criterion can simply consist of a tolerance on the maximum variation of the coefficients of p_i. The algorithm can be summarized into the following steps:

1. choose M, N, ω_s, γ and ϵ
2. $i \longleftarrow 0$, initialize p_0 with random coefficients
3. $i \longleftarrow i + 1$, build the matrix G_i with p_{i-1} using (8)
4. find p_i using (14)
5. normalize p_i using (15)
6. go to step 3 unless $\max|p_i - p_{i-1}| \leq \epsilon$.

3.2 Weighted design

The ILS design of Pseudo QMF banks can be easily be coupled with an Iterative Reweighted Least Squares (IRLS) algorithm [7]-[8] to perform the minimax or the gain constrained least squares approximation of the prototype stopband or/and the filter bank transfer function. The minimax approximation is of interest since it avoids the large error peaks of the least squares approximation. The gain constrained least squares approximation offers a trade-off between the least squares and the minimax criteria. As pointed out in [14], it is possible to reduce the Chebyshev error of the least squares approximation with a slight increase in the squared error.

When the IRLS method is applied to the stopband of the prototype filter, the squared measure of the stopband is weighted at each iteration i as follows

$$\tilde{e}_s(i) = \frac{1}{L}\sum_{n=0}^{L-1} |w_i(\omega_n) H_i(e^{j\omega_n})|^2 = p_i^t C^t W_i^t W_i C p_i \qquad (16)$$

where $w_i(\omega_n)$ is the weighting function defined on Ω and W_i is the diagonal matrix diag$([w_i(\omega_0), ..., w_i(\omega_{L-1})])$. At each iteration i, the weighted least squares solution is obtained by simply replacing C by $W_i C$ in (14). If the least squares design subject to the constraint $|H(e^{j\omega_n})| \le g_{max}$ is desired, the weights are initialized with the value 1 and are updated as follows after the least squares design has converged

$$\begin{cases} w_{i+1}(\omega_n) = w_i(\omega_n)(env(|H_i(e^{j\omega_n})|)/g_{max})^{\frac{\theta}{2}} & \text{for } \omega_n \in \hat{\Omega}_i \\ w_{i+1}(\omega_n) = w_i(\omega_n) & \text{for } \omega_n \in \Omega - \hat{\Omega}_i \end{cases} \qquad (17)$$

where $env(|H_i(e^{j\omega_n})|)$ is the envelope of $|H_i(e^{j\omega_n})|$, i.e. the function which connects its maxima with straight line, $\hat{\Omega}_i$ is a subset of Ω where $env(|H_i(e^{j\omega_n})|) \ge g_{max}$ and θ is a positive number typically between .1 and 2 and. To keep the same γ at each iteration, the weights are normalized after each update (17) as follows

$$w_{i+1}(\omega_n) \longleftarrow \frac{L w_{i+1}(\omega_n)}{\sqrt{\sum_{n=0}^{L-1} w_{i+1}(\omega_n)^2}} \qquad (18)$$

When g_{max} is small, i.e. less than the Chebyshev norm of the corresponding minimax approximation, the minimax approximation is performed whereas when g_{max} is large enough the weights remain unchanged and a least squares approximation is performed.

In a similar way, the IRLS algorithm can be applied to the approximation of the filter bank transfer function, $T(e^{j\omega})$. At each iteration i, the energy of the magnitude reconstruction error is computed on a dense and uniform frequency grid $\{\varphi_0, ..., \varphi_{\tilde{L}-1}\}$ of $[0,\ \pi/M]$, and weighted as follows

$$\begin{aligned} \tilde{e}_m(i) &= \tfrac{1}{\tilde{L}}\textstyle\sum_{n=0}^{\tilde{L}} |e^{jN\omega} T_i(e^{j\varphi_n}) - 1|^2 \\ &= \tfrac{1}{\tilde{L}}(C_2 G_i p_i - u)^t \tilde{W}_i^t \tilde{W}_i (C_2 G_i p_i - u) \end{aligned} \qquad (19)$$

where $u = [1, ..., 1]^t$, $\tilde{w}_i(\varphi_n)$ is the weighting function, $\tilde{W}_i =$ diag$([\tilde{w}_i(\varphi_0), ..., \tilde{w}_i(\varphi_{\tilde{L}-1})])$ and C_2 is a size $(E[\frac{N}{2M}]+1)$x$\tilde{L}$ matrix which coefficients are $c_2(n,1) = 1$ and $c_2(n,l) = \sqrt{2}\cos(2M(l-1)\varphi_{n+1})$ for $l \ge 2$. The weighted least squares solution is obtained by simply replacing d by $\tilde{W}_i u/\sqrt{\tilde{L}}$ and G_i by $\tilde{W}_i C_2 G_i/\sqrt{\tilde{L}}$ in (14). The weights are computed according to the approximation error $|e^{jN\omega}T(e^{j\omega}) - 1|$ similarly as in (17)-(18).

4 DESIGN EXAMPLES

These examples have been performed using MATLAB on a SPARC-10 workstation.

Example 1: Comparison between the ILS and the non-linear optimization methods

Johnston's 2-band QMF banks [2] are not cosine modulated. They are designed by minimizing the objective function (3) with $M = 1$. The filters tabulated in [2] can be designed with the ILS method in very few iterations. For instance the design of Johnston's filter 32D required no more than 5 iterations, i.e. 0.2 cpu seconds. The nonlinear optimization using a quasi-Newton method ($fminu$ of MATLAB optimization toolbox) converged in 17 cpu seconds.

Example 2: 8-band Pseudo QMF bank

Let $M = 8$, $N + 1 = 140$, $\omega_s = \frac{\pi}{M}$ and $\gamma = 10^4$. The algorithm converged in 15 iterations, i.e. 4.2 cpu seconds. The normalized analysis filter bank, $H_k(e^{j\omega})/\sqrt{M}$ for $k = 0, ..., 7$, is plotted in Fig. 2. The filter bank magnitude transfer function, $|T(e^{j\omega})|$, is shown in Fig. 3. The maximum aliasing level is $-103dB$ which is of the same order as the prototype stopband attenuation. The gain of $H(e^{j\omega})/\sqrt{M}$ is $-83dB$ at ω_s. Using the IRLS method, we have re-designed the prototype filter with the constraint $|H(e^{j\omega})|/\sqrt{M} \le -95dB$ in the stopband. The IRLS algorithm converged in 40 iterations with $\theta = 1$. The normalized prototype filter, $H(e^{j\omega})/\sqrt{M}$, is shown in fig. 4. The magnitude transfer function of the filter bank obtained is roughly the same as with the least squares design, with peak values at $6E^{-7}dB$. The maximum level of aliasing is $-96dB$.

Example 3: 4-band Pseudo QMF banks with least squares and equiripple magnitude transfer function

Let $M = 4$, $N + 1 = 32$, $\omega_s = \frac{\pi}{M}$ and $\gamma = 1.5$. The ILS and IRLS algorithms have been used to design two filter banks with a least squares and a minimax magnitude transfer function respectively. The IRLS algorithm converged in 20 iterations with $\theta = 1$. The prototype filters obtained have roughly the same stopband frequency response: same attenuation, $45dB$, at ω_s and same maximum level of aliasing, $-55dB$. The overall magnitude transfer functions are shown on Fig. 5. We can see that the minimax approximation results in an equiripple magnitude transfer function.

5 MATLAB PROGRAM

```
function h=PQMF(M,Lh,f,gamma)
% Pseudo QMF BANK ILS design
% h:prototype, Lh:length(h)(even), M:nb of bands
% f:stopband freq., gamma: weight of e_m
% example: h=PQMF(8,140,1/8,10^4);
% Author: Michel Rossi, University of Ottawa, 1996
e=1E-8;t=cputime;R=Lh/2;p=rand(R,1);disp('es em');
d=-sin((2*[1:R]-1)*pi*f)/pi/(1-f)/4./([1:R]-.5)+.5;
for k1=1:R for k2=k1+1:R %Compute S=C'C/L
 S(k1,k2)=-[sin((k1+k2-1)*f*pi)/(k1+k2-1)+...
 sin((k2-k1)*f*pi)/(k2-k1)]/2/pi/(1-f);
end;end;S=[S;zeros(1,R)];S=4*(S'+S+diag(d));
for i=1:40  pold=p;G=[];
 % Build the matrix G
 for k=1:M:R
  G=[G;4*sqrt(2)*[zeros(1,k-1),...
 p(k-1:-1:max(1,1-(R-2*(k-1))))',p(1:R-2*(k-1))']];
 end;G(1,:)=G(1,:)/sqrt(2);
 %  Find the L2 solution
 p=4*(S/gamma+G'*G)\p;p=p/sqrt(4*sum(p.^2));
 es(i)=p'*S*p;mdeltap=max(abs(pold-p));
 em(i)=sum((G(2:floor(Lh/(2*M)),:)*p).^2);
 disp([num2str(es(i)),' ',num2str(em(i))]);
 if mdeltap<e break;end;
end;h=[flipud(p);p];disp([num2str(cputime-t),'s']);
```

6 CONCLUSIONS

In this paper we have presented an ILS algorithm to design M-band Pseudo QMF banks. The algorithm is fast converging and simple to implement. The convergence does not depend on the starting point. The flexibility of the ILS approach allows the use of an iteratively calculated weighting function to perform the minimax or the gain constrained least squares approximation of the prototype stopband or/and the filter bank magnitude transfer function. Interested readers can refer to [15]-[16] for the ILS design of Near Perfect Reconstruction Pseudo QMF banks and Perfect Reconstruction filter banks.

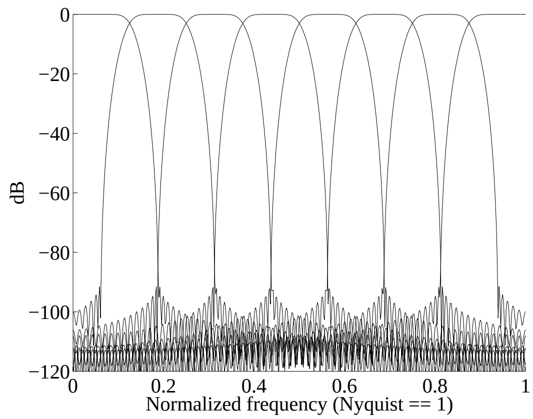

Figure 2: Example 2, 8-band Pseudo QMF analysis filter bank

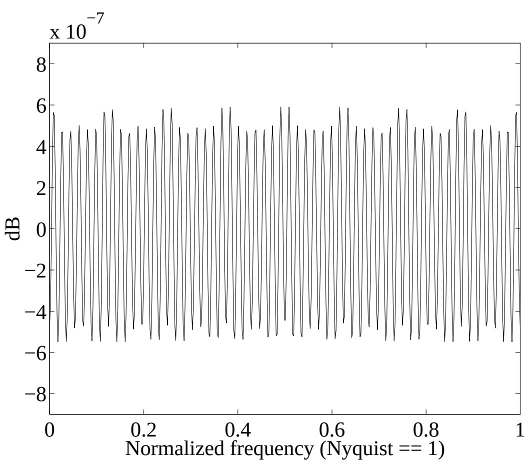

Figure 3: Example 2, filter bank magnitude transfer function, $|T(e^{j\omega})|$

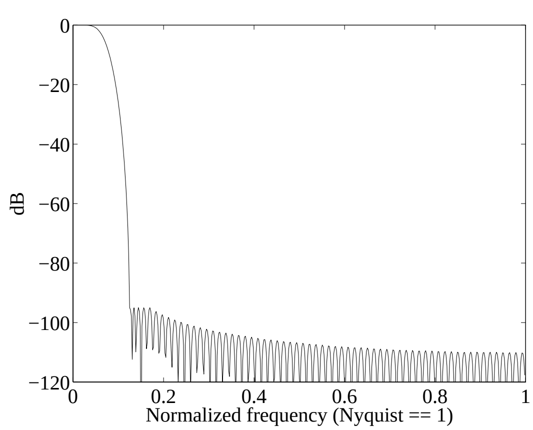

Figure 4: Example 2, 8-band Pseudo QMF prototype filter with a gain constrained least squares stopband

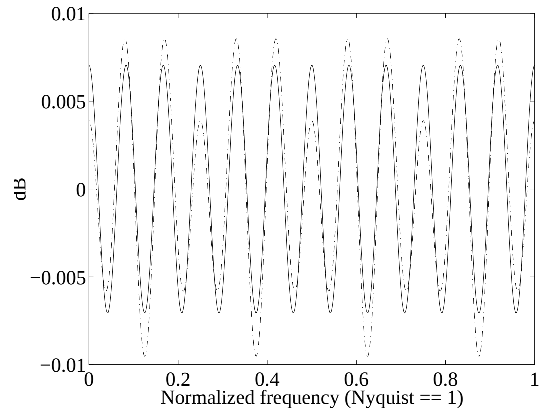

Figure 5: Example 3, 4-band Pseudo QMF bank magnitude transfer function, $|T(e^{j\omega})|$, $- \cdot - \cdot$ least squares design, —— equiripple design

7 ACKNOLEDGEMENTS

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