NONLINEAR INTERFERENCE CANCELLATION USING A RADIAL BASIS FUNCTION NETWORK

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ABSTRACT

Conventional linear filtering techniques cannot suppress interference or noise in the same band as the signal without degrading the signal. However if the corrupting noise arises from a nonlinear low dimensional dynamical system, it is possible to model the noise as a deterministic process rather than a stochastic one. In this paper a combination of linear and nonlinear models are used to separate the linear signal from the nonlinear noise. The normalised gaussian radial basis function (RBF) network is used to model the nonlinear interference. Decimators have been implemented to reduce the computational cost of the RBF network and re-embed the filtered chaos.

1 INTRODUCTION

The problem considered in this paper is that of a narrowband signal corrupted by wide band interference or noise. With no a priori knowledge about this interference it is usually assumed to be a stochastic process. Conventional filtering techniques are then used which exploit the incoherence of the interference or spectral properties in the frequency domain. However, today it is known that many of these noise processes arise from nonlinear dynamical systems. Chaotic time series are a typical example of aperiodic time series which appear to be a stochastic process when analysed with second order statistics. Deterministic nonlinear behaviour can arise from all kinds off different physical systems. Electronic circuits (Chua's circuit)[1], mechanical systems (engine noise) [1] or fluids (sea clutter) [2] are known to exhibit chaotic behaviour. Nonlinear processes are widespread in nature. For this reason it is important to ask, whether the noise process should be modelled as a stochastic or a chaotic process. If the noise is chaotic then nonlinear methods which depend on the coherence are more suitable to cancel nonlinear interference.

2 BACKGROUND

Although chaotic behaviour has been encountered in a variety of nonlinear systems, few researchers have addressed the problem of cancelling nonlinear deterministic "noise" or interference from a signal of interest. A brief review of available methods can be found in [1]. Most methods are particularly restrictive with respect to the required properties of the signal and/or the noise (e.g. slow temporal variation of signal, discrete signals, low signal to noise ratio (SNR)). These restrictions severely limits their wider application. The method recently proposed by Broomhead et al. in [1], shown in Figure 1, replaces the restrictions on SNR with a requirement that the signal of interest should be extremely narrow band with respect to the noise. The technique presented here in Section 4 goes some way to removing the restriction on the bandwidth of the signal as well.

2.1 Delay Embedding

The following work in Section 2.2 and 4 is based on the delay embedding theorem from Takens [3]. The basic idea is to construct the state space of a dynamical system by time series observations. If the attractor of a dynamical system is contained within a finite dimensional manifold M, with $dim\ M = m < \infty$, then an embedding of M can be constructed from a time series using a delay register. The embedding described by Takens is a map $\Phi_{2m+1} : M \rightarrow R^{2m+1}$, defined by

$$\Phi_{2m+1}(x_0) = (v(x_0), v(x_{-1}), \ldots, v(x_{2m})) \quad (1)$$

where $v(x_0), v(x_{-1}), \ldots, v(x_{-2m})$ are a sequence of real-valued measurements $v : M \rightarrow R$ and $\ldots, x_{-1}, x_0, x_1, \ldots$ is a trajectory on the dynamical system on M.

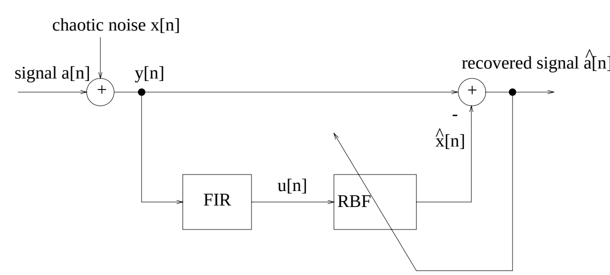

Figure 1: Nonlinear Interference Cancellation using an Orthogonal FIR Filter

2.2 Nonlinear Inverse Filtering

The FIR filter in Figure 1 is a bandstop $\sum_{k=0}^{N-1} h[k]a[n-k] = 0 \quad \forall n$. Where $h[k]$ are the coefficients of the FIR filter and $a[n-k]$ the signal of interest. Note that the FIR filter is *orthogonal* to the signal $a[n-k]$. Since FIR filters do not have a feedback path they preserve the structure of a dynamic process [4]. A delay map $\mathbf{F}_N \Phi_{L+N-1}$ can be constructed from the output of the FIR filter $v[n]$, where $\mathbf{F}_N$ is an $L \times (L+N-1)$ banded matrix [1] and Φ_{L+N-1} is the embedding of the manifold [2] K using $(L+N-1)$ delays of the unfiltered time series. $\mathbf{F}_N$ describes the effect that filtering has on the embedded attractor.

The aim of the nonlinear filter is to reconstruct the dynamic structure of the chaotic noise. The linear map $\mathbf{F}_N$ has cancelled the signal $a[n]$ and distorted the image of the chaotic attractor. An inverse $\mathbf{F}_N^{-1}$ has to be estimated $\widehat{\mathbf{F}}_N^{-1}$ by the nonlinear filter, in order to undo the distortion.

$$\mathbf{F}_N \mathbf{u} = \mathbf{F}_N \mathbf{x} \in \mathbf{F}_N \circ \Phi_{L+N-1} K \qquad (2)$$

$$\widehat{\mathbf{x}} = \widehat{\mathbf{F}}_N^{-1} \circ \mathbf{F}_N \mathbf{u} \qquad (3)$$

Once the estimated map $\widehat{\mathbf{F}}_N^{-1}$ has been found the signal $a[n]$ can be estimated as follows.

$$\widehat{a}[n] = u[n] - \left(\widehat{\mathbf{F}}_N^{-1} \circ \mathbf{F}_N \mathbf{u}\right)_1 \qquad (4)$$

This filter method has the drawback that the signal of interest has to be cancelled by a notch filter. Simulations show this in [1]. This requires complete knowledge of the signal. The scheme in Section 4 in this paper circumvents this problem by using filterbanks, multirate signal processing and an additional linear model.

3 DIMENSION AND SAMPLING

In order to exploit the determinism in the chaotic signal it is necessary to chose an approriate sampling rate and embedding dimension.

3.1 Singular System Analysis

To chose the right embedding dimension it is necessary to determine the degrees of freedom in the time series. If there is stochastic noise in the system as well as low-dimensional chaotic noise it would be plausible to assume that the data is high-dimensional and that it will fill more or less uniformly any low-dimensional space. A straightforward and easy to implement method is the singular system analysis approach [5] for estimating the dimension of a dynamical system.

[1] L is the embedding dimension of the attractor

[2] K is a finite-dimensional manifold containing the attractor of the dynamical system

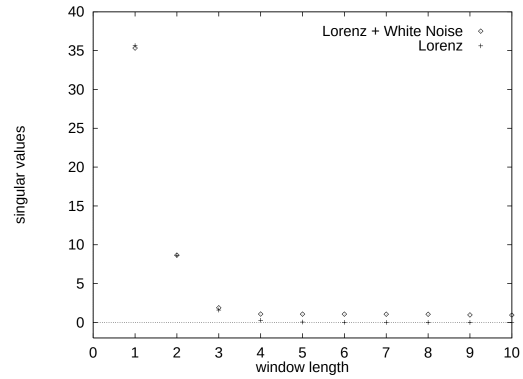

Figure 2: Singular System Analysis

The singular values are derived from the following covariance matrix

$$COV = \frac{1}{N} \sum_{n=1}^{N} [x(n) - \bar{x}][x(n) - \bar{x}]^T \qquad (5)$$

where $x[n]$ is the time series to be analysed. The chaotic noise in the simulations used is the Lorenz chaos. The results in Figure 2 show the Lorenz chaos with and without contamination. It is clear in both cases that the chaos is low-dimensional and the estimated dimension is three.

3.2 Average Mutual Information

To determine the sampling rate of the time series the average mutual information has been used. It measures the independence between two samples. The average mutual information is a kind of generalisation to the nonlinear world from the correlation function in the linear world. It has been suggested [6] that the optimum sampling rate is at the first minimum of the average mutual information $I(T)$.

The average amount of information about $s(n+T)$ when making the observation $s(n)$ is

$$I(T) = \sum_{n=1}^{N} P(s(n), s(n+T)) \log_2 \left[\frac{P(s(n), s(n+T))}{P(s(n))P(s(n+T))} \right] \qquad (6)$$

and $I(T) = 0$. In Figure 3 it can be seen that the minimum occurs around 0.02s. As the Lorenz data is sampled at 100Hz it is appropriate to downsample the data by factor $M = 2$. Note also the slight information (correlation) increase between two samples when passing the Lorenz data through a linear bandpass or lowpass filter.

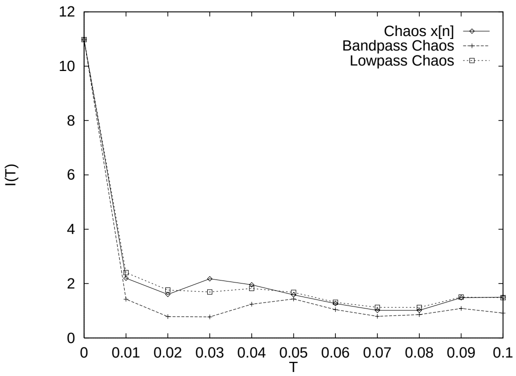

Figure 3: Average Mutual Information $I(T)$

4 NONLINEAR INTERFERENCE CANCELLATION

This cancellation method proposed exploits the deterministic nature of the noise process. The scheme is shown in Figure 4. The task of the linear FIR lowpass filter LP1 is to provide the RBF network with a reference signal $v[n]$ without the signal of interest. The bandpass filter BP1 provides the desired signal $d[n]$ for the linear adaptive FIR filter. As these two signals $d[n]$ and $v[n]$ are now bandlimited it is possible to decimate the signals by the integer factor M effectively re-embedding the filtered chaotic signal. Decimating the signals also makes the scheme computational more efficient because the RBF network has a smaller size, and is implemented at a lower speed [7].

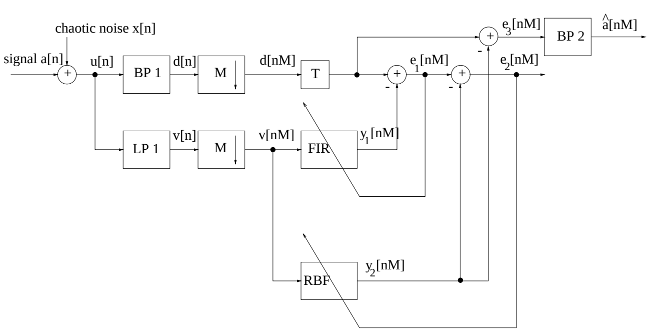

Figure 4: Nonlinear Interference Cancellation

In general the signal of interest, $a[n]$, will be attenuated by LP1 but not removed. Therefore it is necessary to remove the linear map between $v[nM]$ and $d[nM]$. The linear adaptive FIR filter is trained to model the linear map. After convergence in the least square sense the error signal $e_1[nM]$ will be orthogonal to the signal $v[nM]$. This means that there is no linear map between $v[nM]$ and $e_1[nM]$. The RBF network then, has to estimate a nonlinear mapping between the signals $v[nM]$ and $e_1[nM]$. The output of the RBF network $y_2[n]$ is the estimated nonlinear noise. Therefore an estimate for the signal of interest can be found by $e_3[nM] = d[nM] - y_2[nM]$.

The bandpass filter BP2 is necessary, because the RBF network introduces high frequency components outside the bandwidth of interest.

4.1 Nonlinear Model

The nonlinear static network structure is a normalised Gaussian RBF network. This network features either localised response like an ordinary RBF network or non-localised behaviour like that of a sigmoid, depending on the location of the centre. This behaviour leads to an enhanced approximation capability [8].

$$f(\mathbf{x}(k)) = \sum_{i=0}^{n} w_i \phi_i(k) \quad (7)$$

$$\phi_i(k) = \frac{\exp(- \| \mathbf{x}(k) - \mathbf{c}_i \|^2 / \sigma_i^2)}{\sum_{j=1}^{n} \exp(- \| \mathbf{x}(k) - \mathbf{c}_j \|^2 / \sigma_j^2)} \quad (8)$$

The positions of the centres $\mathbf{c}$ were determined by the adaptive k-means algorithm or by spreading the centres on the input data evenly depending on the dimension of the input space. As Figure 5 shows the mean square error (MSE) of the centres variance in a two dimensional input space (Logistic map) is a magnitude smaller than in the three dimensional input space of the Lorenz data. This means that the centres in the three dimensional case have a greater centre variation. The adaptive k-means algorithm tries to converge to equidistant spacing between the centres. For that reason each centre has the same bandwith σ_i^2. The input data in the three dimensional space is therefore not covered smoothly by the centres and hence the performance of the RBF network decreases. For higher dimensions than two in the input space an evenly spaced grid has been used.

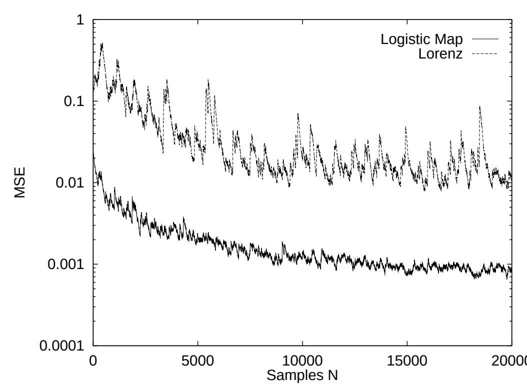

Figure 5: Mean Square Error of the Variance

5 SIMULATIONS AND RESULTS

To compare the normalised RBF network with another nonlinear model, a Volterra filter with quadratic and cubic terms was chosen. A linear filter, which is a combination of bandpass BP1, decimated by factor M and followed by bandpass BP2 is provided as a benchmark for comparison. The chaotic noise as mentioned earlier is derived from the three nonlinear differential equations from Lorenz. The signal of interest was a sine wave. The power spectral density of the input scenario $a[n] + x[n]$ is shown in Figure 6. Figures 7 and 8 show the achieved output SNR versus the input SNR across a range from -5dB to 5dB.

The results in Figure 7 were obtained by using an embedding dimension of four for the input space of the RBF network and the Volterra filter. From Takens embedding theorem in Section 2.1, it is clear that the embedding dimension is greater than four. Therefore, it is not surprising that the linear filters perform as well as the nonlinear filters. The situation changes in Figure 8 where an embedding dimension of five has been chosen. The chaos is embedded in the correct way and the nonlinear filters are able to perform better than the linear filters.

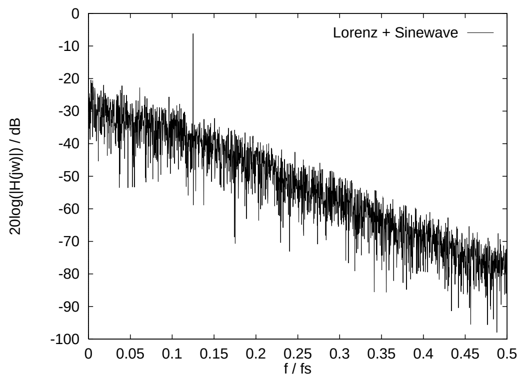

Figure 6: Chaotic Noise + Sine Wave of Interest

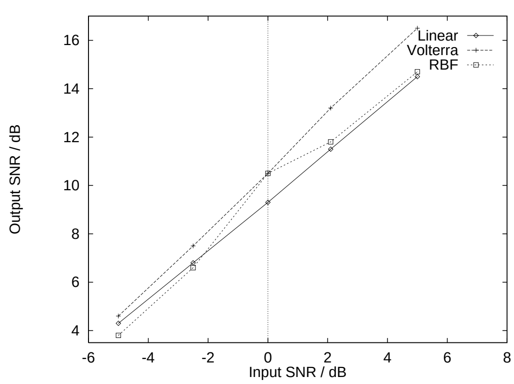

Figure 7: Signal to Noise Ratio in dB (Embedding Dimension = 4)

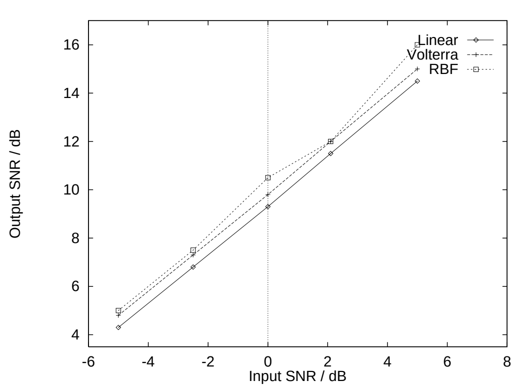

Figure 8: Signal to Noise Ratio in dB (Embedding Dimension = 5)

6 CONCLUSION

It has been shown that nonlinear deterministic noise can be modelled by a nonlinear model. If an appropriate sampling rate and embedding dimension is chosen, it is possible to enhance the SNR in comparison to conventional linear methods. Further investigations are underway out to determine the optimum lengths of the filterbanks and network configuration of the RBF model. Another aspect is to use a more complex signal of interest. Simulations with a narrowbanded AR process as a signal of interest are also under investigation.

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