WIDEBAND ARRAY PROCESSING USING A PARTITIONED SPECTRAL MATRIX

S. Bourennane and M. Frikel
C.M.C.S. URA 2053 CNRS
B.P. 52, Quartier Grossetti, 20250 Corte France
Tel: (+33) 95 450034; Fax: (+33) 95 610551
e-mail: bourenna@univ-corse.fr - frikel@univ-corse.fr

ABSTRACT

This paper presents a propagator method for high resolution estimation of the angles of arrival of multiple wideband plane waves without eigendecomposition. The technique is based on a partition of the array spectral matrix. The noisy situation is considered and an algorithm to eliminate the noise contribution is given. The results of simulations support the theoretical predictions are presented.

1 INTRODUCTION

High resolution techniques for estimating the arriving angles of radiation sources by sensor arrays have been discussed extensively in the past [1-4]. Many of the most commonly employed high resolution estimation procedures utilize the eigenvectors associated with the smallest eigenvalues (noise subspace) of the array's cross-spectral matrix. The main difficult of eigenstructure based methods is their rather expensive computational load which limits their implementation in real time. To reduce the computational load, several papers have been proposed. The basic idea is to estimate the noise subspace without eigendecomposition [5-7]. A partition of the cross-spectral matrix is generally used, which leads asymptotically to the signal subspace. The aim of the developed methods is to reduce the computational load used during the treatment and to alleviate the noise effect.

The propagator method [5] is one of these methods, an algebraic operator extracted from the spectral matrix of the received data. The spectral matrix is partitioned and using the linear dependency property existing between the rows of the transfer matrix of the source-sensors, an estimate of the noise subspace is obtained. In noise free environnement, it is shown that the partition of the spetcral matrix leads to estimate the propagator. In this study, an other partition is proposed for eliminating the noise and then the exact solution for the propagator is obtained in noisy situation. Then the estimation of the directions of the sources is improved.

2 PROBLEM FORMULATION

Consider a uniform linear array composed of N identical sensors separated from each other by a distance d. Let $P, (P < N)$ Wideband plane waves, impinge on the array from the directions $\{\theta_1, \theta_2, \ldots, \theta_P\}$. The signal received at the ith sensor can be expressed as :

$$\mathbf{r}_i(t) = \sum_{p=1}^{P} \mathbf{s}_p(t - \tau_{ip}) + \mathbf{n}_i(t) \quad (1)$$

Where $\mathbf{n}_i(t)$ is the additive white noise at the ith sensor. $\mathbf{s}(t - \tau_{ip})$ is the signal emitted by the P sources.
Rewritting (1) in matrix notation, in the frequency domain, we obtain :

$$\mathbf{r}(f_j) = \mathbf{A}(f_j)\mathbf{s}(f_j) + \mathbf{n}(f_j), j = 1, .., M$$

Where $\mathbf{r}(f_j)$ is the Fourier transform of the array ouput vector, $\mathbf{s}(f_j)$ is the $P \times 1$ vector of complex signals of P wavefronts :

$$\mathbf{s}(f_j) = [s_1(f_j), s_2(f_j), ..., s_P(f_j)]^T$$

$\mathbf{n}(f_j)$ is the $N \times 1$ vector of additive noise in sensors

$$\mathbf{n}(f_j) = [n_1(f_j), n_2(f_j), ..., n_N(f_j)]^T$$

and $\mathbf{A}(f_j)$ is the $N \times P$ transfer matrix of the source-sensor array systems with respect to some chosen reference point.

$$\mathbf{A}(f_j) = [\mathbf{a}(f_j, \theta_1), \mathbf{a}(f_j, \theta_2), ..., \mathbf{a}(f_j, \theta_P)] .$$

$\mathbf{a}(f_j, \theta_i)$ is the steering vector of the array toward the direction θ_i at the frequency f_j.
The steering vector of a linear uniform array with N sensors is given by :

$$\mathbf{a}(f_j, \theta_i) = [1 \quad e^{j\varphi_i} \quad e^{2j\varphi_i} ... \quad e^{(N-1)j\varphi_i}]^T$$

Where,
$\varphi_i = 2\pi f_j \frac{d}{c} sin(\theta_i)$.
d : sensor spacing.
θ_i : direction of arrival (DOA) of the ith source as measured from the array broadside.
c : velocity wave propagation.
f_j : analysis frequency.

Assume that the signals and the additive noises are stationary and ergodic zero mean complex valued random processes. In addition, the noises are assumed to be uncorrelated between sensors, and to have the same variance $\sigma^2(f_j)$ at each sensor. It follows from these assumptions that the spatial $N \times N$ cross-spectral matrix of the observation vector at frequency f_j is given by :

$$\mathbf{\Gamma}(f_j) = E[\mathbf{r}(f_j)\mathbf{r}^+(f_j)]$$

$$\mathbf{\Gamma}(f_j) = \mathbf{A}(f_j)\mathbf{\Gamma}_s(f_j)\mathbf{A}^+(f_j) + \sigma^2(f_j)\mathbf{I}$$

Where $E[.]$ denotes the expectation operator, the superscript $^+$ represents conjugate transpose,
and $\mathbf{\Gamma}_s(f_j) = E[\mathbf{s}(f_j)\mathbf{s}^+(f_j)]$ is the $P \times P$ sources cross-spectral matrix, and $\mathbf{\Gamma}_s(f_j)$ is diagonal when the signals are uncorrelated.
Assuming that the columns of $\mathbf{A}^+(f_j)$ are all different and linearly independent it follows that for nonsingular $\mathbf{\Gamma}_s(f_j)$, the rank of $\mathbf{A}(f_n)\mathbf{\Gamma}_s(f_n)\mathbf{A}^+(f_n)$ is P. If the eigenvalues and the corresponding eigenvectors of $\mathbf{\Gamma}(f_j)$ are denoted by $\{\lambda_i\}_{i=1..N}$ and $\{\mathbf{v}_i\}_{i=1..N}$ then this rank property implies that :
- The minimal eigenvalue of $\mathbf{\Gamma}(f_j)$ is equal to $\sigma^2(f_j)$ with multiplicity $N-P$: $\lambda_{P+1} = \ldots = \lambda_N = \sigma^2(f_j)$.
- The eigenvectors corresponding to the minimal eigenvalues are orthogonal to the columns of the matrix $\mathbf{A}(f_j)$, namely, the steering vectors of the signals
$\{\mathbf{v}_{P+1} \ldots \mathbf{v}_N\} \perp \{\mathbf{a}(\theta_1) \ldots \mathbf{a}(\theta_P)\}$.
The eigenstructure based techniques are based on the exploitation of these properties. Generally, these methods have better performance than the classical methods. In practice, their rather expensive computational load is an inconvenience for their implementation in real time. In the last five years, several papers have been published [5,6] to reduce the computational load needfull for the eigendecomposition of the spectral matrix.
In this study, the propagator method which is considered as an alternative of Music method is analysed. Initially, this method is introduced by J. Munier [5] for estimating the direction of arrival of narrow-band sources, in the no-noisy or high SNR situation. We propose an extension of this method to the wideband sources and we present a new algorithm for the noisy situation.

3 PROPAGATOR METHOD

The number P of the radiating sources is assumed to be known or can be estimated using the detection criteria [7]. This problem is not considered in this paper.
Using the assumption, that the matrix $\mathbf{A}(f_j)$ is of rank P, there exists a $(N-P) \times P$ matrix such that :

$$\mathbf{A}_2(f_j) = \mathbf{\Pi}^+(f_j)\mathbf{A}_1(f_j), for j = 1, \ldots, M$$

Where $\mathbf{A}_1(f_j)$ and $\mathbf{A}_2(f_j)$ are two block matrices of the matrix $\mathbf{A}(f_j)$, of dimension $(P \times P)$ and $(N-P) \times P$, respectively. $\mathbf{\Pi}(f_j)$ is so-called propagator operator [5]. The $N-P$ last rows of $\mathbf{A}(f_j)$ are linearly dependent of the P first rows. It follows :

3.1 No-noisy situation

The cross-spectral matrix of the received data is :

$$\mathbf{\Gamma}(f_j) = \begin{bmatrix} \mathbf{\Gamma}_{11}(f_j) & \mathbf{\Gamma}_{12}(f_j) \\ \mathbf{\Gamma}_{21}(f_j) & \mathbf{\Gamma}_{22}(f_j) \end{bmatrix}$$

where,

$$\mathbf{\Gamma}_{11}(f_j) = \mathbf{A}_1(f_j)\mathbf{\Gamma}_s(f_j)\mathbf{A}_1^+(f_j)$$

$$\mathbf{\Gamma}_{12}(f_j) = \mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}(f_j)$$

$$\mathbf{\Gamma}_{21}(f_j) = \mathbf{\Pi}^+(f_j)\mathbf{\Gamma}_{11}(f_j)$$

$$\mathbf{\Gamma}_{22}(f_j) = \mathbf{\Pi}^+(f_j)\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}(f_j)$$

For example, by using

$$\mathbf{\Gamma}_{12}(f_j) = \mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}(f_j), \; j = 1, \ldots, M$$

an estimation of the propagator is given by :

$$\mathbf{\Pi}(f_j) = \mathbf{\Gamma}_{11}^{-1}(f_j)\mathbf{\Gamma}_{12}(f_j) \qquad (2)$$

The expression (2) shows that, in practice, the propagator can be constructed directly from the received data.

3.2 Noisy situation

In [5], the least mean squares method is used to estimate the propagator. The optimal propagator, in the presence of the noise, is obtained by minimizing :
$\| \mathbf{H}(f_j) - \mathbf{G}(f_j)\mathbf{\Pi}(f_j) \|_F$, where $\mathbf{H}(f_j)$ and $\mathbf{G}(f_j)$ are two block matrices of dimension, $(N \times P)$ and $N \times (N-P)$, respectively, such that :

$$\mathbf{\Gamma}(f_j) = [\mathbf{G}(f_j) \mid \mathbf{H}(f_j)]$$

The optimal estimate of $\mathbf{\Pi}(f_j)$ is given by:

$$\mathbf{\Pi}(f_j) = \left[\mathbf{G}^+(f_j)\mathbf{G}(f_j)\right]^{-1}\mathbf{G}^+(f_j)\mathbf{H}(f_j)$$

It is clear that the quality of this estimation is dependent of the noise power. To improve this method, we propose another partition of the croos-spectral matrix such that the propagator components are estimated from the uncorrupted blocks of the cross-spectral matrix.
Let us the propagator operator is partitioned as follows

$$\mathbf{\Pi}^+(f_j) = \begin{bmatrix} \mathbf{\Pi}_1^+(f_j) \\ --- \\ \mathbf{\Pi}_2^+(f_j) \end{bmatrix}$$

where, $\mathbf{\Pi}_1(f_j)$ is a $(P \times 1)$ vector and,
$\mathbf{\Pi}_2(f_j)$ is a $P \times (N-P-1)$ matrix.
In this part we assume that $N > P+1$.
Using the former partition of the matrix $\mathbf{A}(f_j)$, the array spectral matrix is :

$$\widetilde{\mathbf{\Gamma}}(f_j) = \begin{bmatrix} \widetilde{\mathbf{\Gamma}}_{11}(f_j) & \widetilde{\mathbf{\Gamma}}_{12}(f_j) \\ \widetilde{\mathbf{\Gamma}}_{21}(f_j) & \widetilde{\mathbf{\Gamma}}_{22}(f_j) \end{bmatrix}$$

Where :

$$\widetilde{\mathbf{\Gamma}}_{11}(f_j) = \mathbf{\Gamma}_{11}(f_j) + \sigma^2(f_j)\mathbf{I}_P$$

$$\widetilde{\mathbf{\Gamma}}_{12}(f_j) = [\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_1(f_j) \mid \mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_2(f_j)]$$

$$\widetilde{\mathbf{\Gamma}}_{12}(f_j) = \left[\widetilde{\mathbf{\Gamma}}_{12}^1(f_j) \mid \widetilde{\mathbf{\Gamma}}_{12}^2(f_j)\right]$$

$$\widetilde{\mathbf{\Gamma}}_{21}(f_j) = \widetilde{\mathbf{\Gamma}}_{12}^+(f_j)$$

and,

$$\widetilde{\mathbf{\Gamma}}_{22}(f_j) = \begin{bmatrix} \widetilde{\mathbf{\Gamma}}_{22}^{11}(f_j) & \widetilde{\mathbf{\Gamma}}_{22}^{12}(f_j) \\ \widetilde{\mathbf{\Gamma}}_{21}^{21}(f_j) & \widetilde{\mathbf{\Gamma}}_{22}^{22}(f_j) \end{bmatrix}$$

with :

$$\widetilde{\mathbf{\Gamma}}_{22}^{11}(f_j) = \mathbf{\Pi}_1^+(f_j)\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_1(f_j)$$

$$\widetilde{\mathbf{\Gamma}}_{22}^{12}(f_j) = \mathbf{\Pi}_1^+(f_j)\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_2(f_j)$$

$$\widetilde{\mathbf{\Gamma}}_{22}^{21}(f_j) = \mathbf{\Pi}_2^+(f_j)\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_1(f_j)$$

$$\widetilde{\mathbf{\Gamma}}_{22}^{22}(f_j) = \mathbf{\Pi}_2^+(f_j)\mathbf{\Gamma}_{11}(f_j)\mathbf{\Pi}_2(f_j) + \sigma^2(f_j)\mathbf{I}_{N-P-1}$$

Using the above expressions, we obtain :

$$\mathbf{\Pi}_1^+(f_j) = \widetilde{\mathbf{\Gamma}}_{22}^{12}(f_j)\left[\widetilde{\mathbf{\Gamma}}_{12}^{2}(f_j)\right]^+\left[\widetilde{\mathbf{\Gamma}}_{12}^{2}(f_j)\left(\widetilde{\mathbf{\Gamma}}_{12}^{2}(f_j)\right)^+\right]^{-1}$$

$$\mathbf{\Pi}_2^+(f_j) = \widetilde{\mathbf{\Gamma}}_{22}^{21}(f_j)\left[\widetilde{\mathbf{\Gamma}}_{12}^{1}(f_j)\right]^+\left[\widetilde{\mathbf{\Gamma}}_{12}^{1}(f_j)\left(\widetilde{\mathbf{\Gamma}}_{12}^{1}(f_j)\right)^+\right]^{-1}$$

These expressions are used to estimate the propagator operator.

To estimate the direction of arrival of the sources, the propagator can be used at each frequency bin and then the final result is obtained by averaging the different results. In the following, the coherent propagator is presented.

4 COHERENT PROPAGATOR

The transformations matrices [1,10-13] are used at each frequency bin such that we obtain the coherent propagator which span the same subspace as the coherent noise subspace, we have :

$$\mathbf{D}(f_j)\mathbf{\Pi}(f_j) = \widehat{\mathbf{\Pi}}(f_o), j = 1, \ldots, M$$

where, f_o is a focusing frequency, $\mathbf{D}(f_j)$ is the focusing matrix, and $\widehat{\mathbf{\Pi}}(f_o)$ is given by :

$$\widehat{\mathbf{\Pi}}(f_o) = \left[\widehat{\mathbf{A}}_1(f_o)\widehat{\mathbf{\Gamma}}_s(f_o)\widehat{\mathbf{A}}_1^+(f_o)\right]^{-1}\mathbf{\Gamma}_{12}(f_o)$$

$$\widehat{\mathbf{\Gamma}}_s(f_o) = \frac{1}{M}\sum_{j=1}^{M}\widehat{\mathbf{A}}_1^{-1}(f_j)\mathbf{\Gamma}_{11}(f_j)\left(\widehat{\mathbf{A}}_1^+(f_j)\right)^{-1}$$

$\widehat{\mathbf{A}}_1(f_j)$ is obtained by using an initial estimates of the direction of arrival of the sources and $\mathbf{\Gamma}_{12}(f_o)$ is a block matrix of $\mathbf{\Gamma}(f_o)$.

The transformation matrix is given by :

$$\mathbf{D}(f_j) = \widehat{\mathbf{\Pi}}(f_o)\mathbf{\Gamma}_{12}^+(f_j)\left[\mathbf{\Gamma}_{12}(f_j)\mathbf{\Gamma}_{12}^+(f_j)\right]^{-1}\mathbf{\Gamma}_{11}^+(f_j)$$

The coherent propagator is then :

$$\widetilde{\mathbf{\Pi}}(f_o) = \frac{1}{M}\sum_{j=1}^{M}\mathbf{D}(f_j)\mathbf{\Pi}(f_j)$$

The obtained propagator is used to construct the coherent matrix, given by :

$$\widetilde{\mathbf{Q}}^+(f_o) = \left[\widetilde{\mathbf{\Pi}}^+(f_o) \mid -\mathbf{I}\right]$$

We have :

$$\widetilde{\mathbf{Q}}^+(f_o)\mathbf{A}(f_o) = \widetilde{\mathbf{\Pi}}^+(f_o)\mathbf{A}_1(f_o) - \mathbf{A}_2(f_o) = \mathbf{0}$$

$$Or, \widetilde{\mathbf{Q}}^+(f_o)\mathbf{a}_p(f_o) = 0, for\ p = 1, \ldots, P$$

This result shows that the columns of $\widetilde{\mathbf{Q}}^+(f_o)$ are orthogonal to the columns of $\mathbf{A}(f_o)$. This means that the subspace spanned by the columns of $\widetilde{\mathbf{Q}}^+(f_o)$ is the same spanned by the noise subspace.

The directions of the sources are the values of $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, for which the function $J(\theta)$ is maximized:

$$J(\theta) = \frac{1}{\mid \widetilde{\mathbf{Q}}^+(f_o)\mathbf{a}_p(f_o, \theta) \mid}$$

5 SIMULATIONS RESULTS

In the following simulations, a linear array of $N = 10$ equispaced sensors with equal interlement spacing $d = \frac{c}{2f_o}$ is used. The source signals are temporally stationary zero-mean bandpass white gaussian processes with the same bandwidth $B = [100, 131Hz]$.

Three source signals impinge on the array at $\theta_1 = 20°, \theta_2 = 22°$ and $\theta_3 = 24°$, respectively, in the presence of noise. The array noise is stationary zero-mean bandpass white gaussian vector process, independent of the signals and with the same variance. The SNR is equal to $3dB$. The analysis band is divided into $M = 32$ frequency bins via FFT.

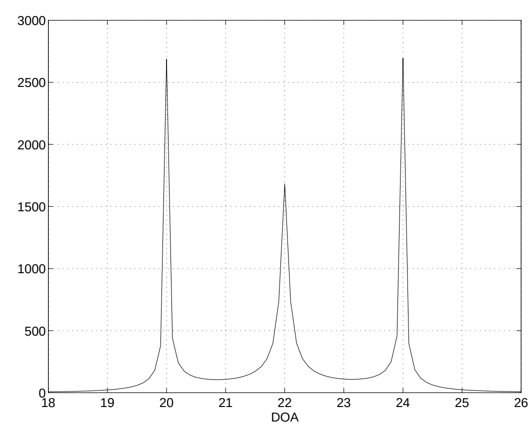

Figure 1: Estimation of the DOA with coherent propagator method in no-noise case

Figure 1 exhibits DOA of the sources using the coherent propagator in the no-noise case. The three sources are localized. Figures 2 and 3 exhibit the directions of the sources using the coherent propagator estimated using the no-noisy case algorithm and the least mean squares algorithm estimates respectively, Figure 4 shows the DOA of the sources with the proposed algorithm for estimating the coherent propagator, the sources are perfectly localized.

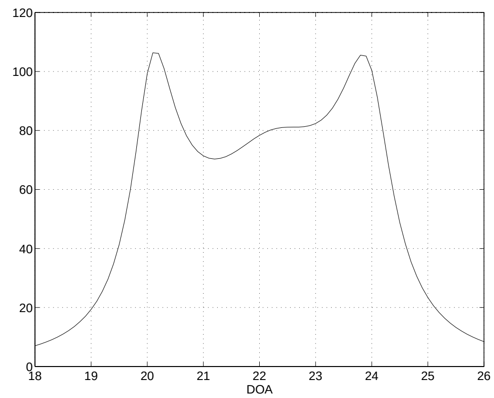

Figure 2: Estimation of the DOA with coherent propagator method in noise case

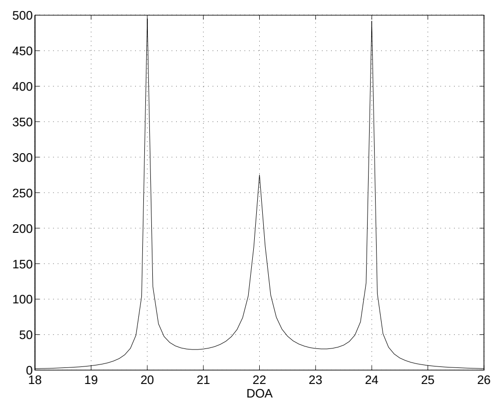

Figure 3: Estimation of the DOA with optimal (LMS) coherent propagator method in noise case

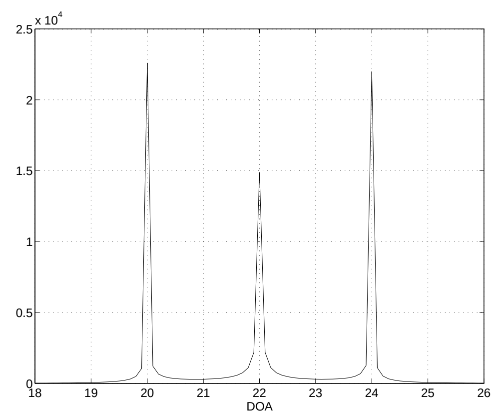

Figure 4: Estimation of the DOA with proposed coherent propagator method in noise case

6 CONCLUSION

A new algorithm for estimating the DOA of the wideband sources using a propagator method is presented. It appeared that the presence of noise degrads the performances of the existing method. The developed method improves the treatment and gives the good estimates of the DOA. Because the noise is eliminated.

References

[1] H. Wang and M. Kaveh, *Coherent Signal-Subspace Processing for the detection and estimation of angles of arrival of multiple wideband sources*, IEEE Trans. ASSP, Vol. 33, n° 4, Aug. 1985.

[2] G. Bienvenu, L. Kopp, *Optimality of high resolution array processing using the eigensystem approach*, IEEE Trans. ASSP, Vol. 33, n° 4, Aug. 1985.

[3] H. Hung and M. Kaveh, *Focusing matrices for coherent signal-subspace processing*, IEEE Trans. ASSP, Vol. 36, pp. 1272-1281, Aug. 1988.

[4] H. Hung and M. Kaveh, *Coherent wideband ESPRIT method for directions-of-arrival estimation of multiple wideband sources*, IEEE Trans. ASSP, Vol. 38, n° 2, Feb. 1990.

[5] J. Munier, *Identification de fronts d'ondes correlés et distordus*, Revue Traitement du signal, vol. 4, n° 4, 1987.

[6] C. E. Davila, M. Azmoodeh, *Efficient estimation of the signal subspace without eigendecomposition*, IEEE Trans. on Sig. Proc., 1994.

[7] M. Frikel and S.Bourennane, *Detection methods using eigenvalues*, Acta Acustica, vol. 82, Jan.-Feb. 1996.

[8] J. Krolik, M. Eizenmann, *Minimum variance spectral estimation for broad-band source location using steered covariance matrices*, ICASSP'88, NY, 1988.

[9] J. Krolik, D. N. Swingler, *High resolution bearing estimation of multiple broadband sources using steered covariance matrice, a linear predictive methods*, ICASSP'88, NY, 1988.

[10] M. A. Doron, J. Weiss, *On focusing for wideband array processing*, IEEE trans. on sign. Proc. Vol. 40, n° 6, Jun. 1992.

[11] S. Valaee and P. Kabal, *Wideband array processing using a two-sided correlation transformation*, IEEE trans. on sign. Proc. Vol. 43, n° 1, Jan. 1995.

[12] S. Bourennane, B. Faure and J. L. Lacoume, *Traitement d'antenne pour des sources large bande*, Ann. Telecom., 1990.

[13] S.Bourennane, M. Frikel, *Localization of the wideband sources with estimation of the antenna shape*, IEEE-Workshop on Statistical & Array Processing, 1996.