FAST ALGORITHM FOR THE WIDEBAND ARRAY PROCESSING USING A TWO-SIDED CORRELATION TRANSFORMATION

M. Frikel and S. Bourennane
C.M.C.S. URA 2053 CNRS
B.P. 52, Quartier Grossetti, 20250 Corte France
Tel: (+33) 95 450071; Fax: (+33) 95 610551
e-mail: frikel@univ-corse.fr - bourenna@univ-corse.fr

ABSTRACT

The purpose of this paper is the passive angular location of the wideband sources using an array of sensors. The improvement of the two-sided correlation transformation (TCT) is proposed, only the signal subspace estimated at each frequency is transformed by focusing matrices such that to obtain the coherent signal subspace for all the analysis band. The simulation results show that the proposed algorithm reduce the computational load compared to the original version TCT.

1 INTRODUCTION

Array processing is used in diverse areas such as radar, sonar, communications, and seismic exploration. Usually the parameters of interest are the direction of arrival of the radiating sources from the recorded data. Recent literature in array processing show a growing interest in the analysis of wideband signals [1-4]. Some methods sample the frequency spectrum to create narrowband signals. Then at each frequency bin a narrowband signal methods are used to estimate the direction of arrival of the wideband sources : that is the incoherent method [6,7]. In the coherent signal subspace method [1-4] the cross-spectral matrices at different frequency bins are combined to form an average cross-spectral matrix. Then, the high resolution algorithm, such as Music [8,9], is used to estimate the DOA. In the coherent signal subspace method, the combination of the narrowband samples is done through of the observation vector or the cross-spectral matrices. This is called focusing. The focusing operator is a matrix that transforms the location matrix of the array at a sampling frequency to the location matrix at the focusing frequency. An improved version of the coherent signal subsapce method is also reported in the [2] that uses unitary focusing matrices. A two-sided transformation is applied on the spectral matrices of the array.

In this paper, we introduce a modification of this algorithm to estimate the unitary focusing matrices. The simulation results show that the proposed leads to reduce the computer time, compared to the version proposed in [2].

2 PROBLEM FORMULATION

We consider an array of N sensors which received the wavefield generated by P wideband sources, with M frequency bins, in the presence of an additive noise. The received signal vector, in the frequency domain, is given by :

$$\mathbf{r}(f_n) = \mathbf{A}(f_n)\mathbf{s}(f_n) + \mathbf{n}(f_n), n = 1, .., M$$

Where $\mathbf{r}(f_n)$ is the Fourier transform of the array ouput vector, $\mathbf{s}(f_n)$ is the $P \times 1$ vecor of complex signals of P wavefronts :

$$\mathbf{s}(f_n) = [s_1(f_n), s_2(f_n), ..., s_P(f_n)]^T.$$

$\mathbf{n}(f_n)$ is the $N \times 1$ vector of additive noise in sensors

$$\mathbf{n}(f_n) = [n_1(f_n), n_2(f_n), ..., n_N(f_n)]^T$$

and $\mathbf{A}(f_n)$ is the $N \times P$ transfer matrix of the source-sensor array systems with respect to some chosen reference point.

$$\mathbf{A}(f_n) = [\mathbf{a}(f_n, \theta_1), \mathbf{a}(f_n, \theta_2), ..., \mathbf{a}(f_n, \theta_P)]$$

$\mathbf{a}(f_n, \theta_i)$ is the steering vector of the array toward the direction θ_i. For example, the steering vector of a linear uniform array with N sensors is given by :

$$\mathbf{a}(f_n, \theta_i) = \begin{bmatrix} 1 & e^{j\varphi_i} & e^{2j\varphi_i} ... & e^{(N-1)j\varphi_i} \end{bmatrix}^T$$

Where, $\varphi_i = 2\pi f_n \frac{d}{c} sin(\theta_i)$.
d : sensor spacing.
θ_i : direction of arrival (DOA) of the ith source as measured from the array broadside.
c : velocity wave propagation.
f_n : analysis frequency.

Assume that the signals and the additive noises are stationary and ergodic zero mean complex valued random processes. In addition, the noises are assumed to be uncorrelated between sensors, and to have the same variance $\sigma^2(f_n)$ at each sensor. It follows from these assumptions that the spatial $N \times N$ cross-spectral matrix of the observation vector at frequency f_n is given by :

$$\mathbf{\Gamma}(f_n) = E[\mathbf{r}(f_n)\mathbf{r}^+(f_n)]$$

$$\mathbf{\Gamma}(f_n) = \mathbf{A}(f_n)\mathbf{\Gamma}_s(f_n)\mathbf{A}^+(f_n) + \sigma^2(f_n)\mathbf{I}$$

Where $E[.]$ denotes the expectation operator, the superscript $^+$ represents conjugate transpose,
and $\mathbf{\Gamma}_s(f_n) = E[\mathbf{s}(f_n)\mathbf{s}^+(f_n)]$ is the $P \times P$ sources cross-spectral matrix.
The eigendecomposition of the cross-spectral matrix $\mathbf{\Gamma}(f_n)$ yields : $\mathbf{\Gamma}(f_n) = \sum_{i=1}^{N} \lambda_i(f_n)\mathbf{v}_i(f_n)\mathbf{v}_i^+(f_n)$
Where $\lambda_i(f_n)$, $i = 1..N$ $\left(\lambda_1 \geq \lambda_2 \geq .. \geq \lambda_N\right)$, and $\mathbf{v}_i(f_n)$ are the ith eigenvalue and ith corresponding eigenvector, repectively.

Our aim is to estimate the angles $\theta_i, i = 1, ..., P$, from the received data. In this paper, the detection of the sources is not treated. We assume that the number of the sources P is supposed known or can be estimated [5].

For locating the wideband sources several solutions have been proposed in the literature and are summarized as following :
- The incoherent subspace methods: the analysis bandwidth is divided into several frequency bins and then at each frequency the treatment is applied and the obtained results are combined to obtain the final result [6,7].
- The coherent subspace methods: the different subspaces are transformed in a predefined subsapce using the focusing matrices [1-4].

3 COHERENT SIGNAL SUBSPACE METHOD

The focusing matrices $\mathbf{H}(f_o, f_n)$'s are the solutions of the equations :

$$\mathbf{H}(f_o, f_n)\mathbf{A}(f_n) = \mathbf{A}(f_o), \forall f_n \in L, \tag{1}$$

Where f_o is the focusing frequency and $\mathbf{A}(f_o)$ is the focusing location matrix.
The matrices $\mathbf{A}(f_o)$ and $\mathbf{A}(f_n)$ are functions of the DOA's θ (Direction of arrival). An ordinary beamforming pre-process gives an estimate of the angles-of-arrival that can be used in (1). Using the focusing matrices $\mathbf{H}(f_o, f_n)$, the observation vectors at different frequency bins are transformed into the focusing subspace.
We present, in the following section, the TCT approach.

4 TCT METHOD

In [2], the TCT approach is based on transformation of the matrices :

$$\mathbf{P}(f_n) = \mathbf{A}(f_n)\mathbf{\Gamma}_S(f_n)\mathbf{A}^+(f_n)$$

$$Or, \mathbf{P}(f_n) = \mathbf{\Gamma}(f_n) - \sigma^2(f_n)\mathbf{I}$$

Where $\mathbf{P}(f_n)$ is the cross-spectral matrix of the received data at the nth frequency bin in a noise-free environnement. Let $\mathbf{P}(f_o)$ be the focusing noise-free cross-spectral matrix. The TCT focusing matrices are found by minimising :

$$\begin{cases} \min\limits_{\mathbf{H}(f_o,f_n)} \parallel \mathbf{P}(f_o) - \mathbf{H}(f_o, f_n)\mathbf{P}(f_n)\mathbf{H}^+(f_o, f_n) \parallel \\ \mathbf{H}^+(f_o, f_n)\mathbf{H}(f_o, f_n) = \mathbf{I} \end{cases} \tag{2}$$

It is shown [2] that the optimal solution of (2) is given by the eigenvectors of the spectral matrix at the frequencies f_o and f_n. The solution of the above equation system (2) is [2] :

$$\mathbf{H}(f_o, f_n) = \mathbf{X}(f_o)\mathbf{X}^+(f_n)$$

Where $\mathbf{X}(f_o)$ and $\mathbf{X}(f_n)$ are the eigenvector matrices of $\mathbf{P}(f_o)$ and $\mathbf{P}(f_n)$, respectively : $\mathbf{P}(.) = \mathbf{X}(.)\mathbf{\Pi}(.)\mathbf{X}^+(.)$, with $\mathbf{\Pi}(.)$ is the eigenvalue diagonal matrix

5 FAST TCT METHOD (FTCT)

In this section, we present the focusing operator based in the rotation of the source subspace only at the frequency f_n to the source subspace at the focusing frequency f_o. This limitation to the transformation of the signal subspace reduces the computational load, and has the same performance than the TCT method.
The partition of the eigenvector matrix $\mathbf{X}(.)$ on :

$$\mathbf{X}(.) = [\mathbf{X}_S(.) \mid \mathbf{X}_B(.)]$$

Where $\mathbf{X}_S(.)$ is $N \times P$ of P largest eigenvectors, and $\mathbf{X}_B(.)$ is $N \times (N - P)$ of $(N - P)$ smallest eigenvectors of the cross-spectral matrix $\mathbf{P}(.)$., the eigenvalues of the cross-spectral of the received data $\mathbf{P}(.)$:

$$\mathbf{\Pi}(.) = \begin{bmatrix} \mathbf{\Pi_S}(.) & \mathbf{0} \\ \mathbf{0} & \mathbf{\Pi_B}(.) \end{bmatrix}$$

Where $\mathbf{\Pi}_S(.)$ is $P \times P$ of P largest eigenvalues, and $\mathbf{\Pi}_B(.)$ is $(N - P) \times (N - P)$ of $(N - P)$ smallest eigenvalues of $\mathbf{P}(.)$.
The proposed focusing operator is :

$$\mathbf{H}_S(f_o, f_n) = \mathbf{X}_S(f_o)\mathbf{X}_S^+(f_n)$$

The average cross-spectral matrix is :

$$\widetilde{\mathbf{P}}(f_o) = \frac{1}{M}\sum_{n=1}^{M} \mathbf{H}_S(f_o, f_n)\mathbf{P}(f_n)\mathbf{H}_S^+(f_o, f_n)$$

It is shown [8,9] that the noise and signal subspaces are orthogonal, we have :

$$\mathbf{X}_S^+(f_n)\mathbf{X}_B(f_n) = \mathbf{X}_B^+(f_n)\mathbf{X}_S(f_n) = 0$$

The P eigenvectors corresponding to the P largest eigenvalues of the cross-spectral matrix of the observation are orthonormalized [8,9], so we have :

$$\mathbf{X}_S^+(f_n)\mathbf{X}_S(f_n) = \mathbf{I}$$

Using the above properties, we obtain :

$$\widetilde{\mathbf{P}}(f_o) = \frac{1}{M}\sum_{n=1}^{M} \mathbf{X}_S(f_o)\mathbf{\Pi}_S(f_n)\mathbf{X}_S^+(f_o)$$

$$Or, \widetilde{\mathbf{P}}(f_o) = \mathbf{X}_S(f_o)\left[\frac{1}{M}\sum_{n=1}^{M}\mathbf{\Pi}_S(f_n)\right]\mathbf{X}_S^+(f_o)$$

This formula shows that the proposed focusing operator focus the signal subspace into the focusing frequency f_o, all the power of the different signal subspaces of the analysis band.
The eigendecomposition of $\widetilde{\mathbf{P}}(f_o)$ is:

$$\widetilde{\mathbf{P}}(f_o) = \widetilde{\mathbf{X}}(f_o)\widetilde{\mathbf{\Pi}}(f_o)\widetilde{\mathbf{X}}^+(f_o)$$

The partition of the eigenvector matrix $\widetilde{\mathbf{X}}(f_o)$ is :

$$\widetilde{\mathbf{X}}(f_o) = \left[\widetilde{\mathbf{X}}_S(f_o) \mid \widetilde{\mathbf{X}}_B(f_o)\right]$$

We have : $\widetilde{\mathbf{X}}_S(f_o) \perp \widetilde{\mathbf{X}}_B(f_o)$, this property is used to estimate the DOA [8,9].

6 FTCT ALGORITHM

The FTCT algorithm is summarise as follows :
step 1 : Use an ordinary estimator to scan the space and find an initial estimate of the DOA.

step 2 : Form $\widehat{\mathbf{A}}(f_n)$ and estimate:

$$\widehat{\mathbf{\Gamma}}_S(f_n) = \left(\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n)\right)^{-1}\widehat{\mathbf{A}}^+(f_n)\left[\widehat{\mathbf{\Gamma}}(f_n) - \widehat{\sigma}^2(f_n)\mathbf{I}\right]\widehat{\mathbf{A}}(f_n)\left(\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n)\right)^{-1}$$

Where $\widehat{\mathbf{\Gamma}}(f_n)$ is the sample cross-spectral matrix of the observation, $\widehat{\Gamma}(f_n) = \frac{1}{K}\sum_{k=1}^{K}\mathbf{r}_k(f_n)\mathbf{r}_k^+(f_n)$ where K is the number of snapshots, the noise power is given by :

$$\widehat{\sigma}^2(f_n) = \frac{1}{(N-P)}\sum_{i=P+1}^{N}\lambda_i\left(\widehat{\mathbf{\Gamma}}(f_n)\right)$$

The number P of the sources is supposed known or can be estimated [5].

step 3 : Average the source cross-spectral matrices to obtain: $\widetilde{\mathbf{\Gamma}}_S(f_o) = \frac{1}{M}\sum_{n=1}^{M}\widehat{\mathbf{\Gamma}}_S(f_n)$
Estimate the cross-spectral matrix of the received data

$$\widetilde{\mathbf{\Gamma}}(f_o) = \widehat{\mathbf{A}}(f_o)\widetilde{\mathbf{\Gamma}}_S(f_o)\widehat{\mathbf{A}}(f_o) + \widehat{\sigma}^2(f_o)\mathbf{I}$$

step 4 : Find $\widetilde{\mathbf{P}}(f_o) = \widehat{\mathbf{A}}(f_o)\widetilde{\mathbf{\Gamma}}_S(f_o)\widehat{\mathbf{A}}^+(f_o)$

$$and, \widehat{\mathbf{P}}(f_n) = \widehat{\mathbf{\Gamma}}(f_n) - \widehat{\sigma}^2(f_n)\mathbf{I}, n = 1, ..., M$$

step 5 : Determine the focusing operator

$$\mathbf{H}_S(f_o, f_n) = \mathbf{X}_S(f_o)\mathbf{X}_S^+(f_n)$$

step 6 : Multiply these matrices by the sample cross-spectral matrices, and average the results:

$$\widetilde{\mathbf{P}}(f_o) = \mathbf{X}_S(f_o)\left[\frac{1}{M}\sum_{n=1}^{M}\mathbf{\Pi}_S(f_n)\right]\mathbf{X}_S^+(f_o)$$

step 7 : Apply a localization method e.g. MUSIC [8,9] to find the DOA of the sources.

In this method, the estimation of the noise power is necessary for improving the SNR, and then the localization of the sources is increasing in accuracy, in the following, we present others focusing matrices in the high SNR situations.

7 HIGH SNR SITUATION

The computational complexity of the TCT method appears at the calcul of the noise power of the sensor output at the nth frequency bin, and at the determination of the unitary transformation matrices. However, $\mathbf{P}(f_n)$ can be a negative matrix.
In this section, we show that, we can extract directly the focusing matrices [4] from the cross-spectral matrix of observation $\mathbf{\Gamma}(f_n)$.
The focusing matrix presented in [4] is :

$$\mathbf{H}_\Gamma(f_o, f_n) = \mathbf{V}(f_o)\mathbf{V}^+(f_n) \quad (3)$$

Where $\mathbf{V}(f_o)$ and $\mathbf{V}(f_n)$ are the eigenvector matrices of $\mathbf{\Gamma}(f_o)$ and $\mathbf{\Gamma}(f_n)$, respectively : $\mathbf{\Gamma}(.) = \mathbf{V}(.)\mathbf{\Sigma}(.)\mathbf{V}^+(.)$, with $\mathbf{\Sigma}(.)$ is the diagonal eigenvalue matrix of $\mathbf{\Gamma}(.)$.
The partition of the eigenvector matrix $\mathbf{V}(.)$ on :

$$\mathbf{V}(.) = [\mathbf{V}_S(.) \mid \mathbf{V}_B(.)]$$

Where $\mathbf{V}_S(.)$ is $N \times P$ of P first eigenvectors, and $\mathbf{V}_B(.)$ is $N \times (N-P)$ of $(N-P)$ last eigenvectors.
The proposed focusing operator is :

$$\mathbf{H}_S^\Gamma(f_o, f_n) = \mathbf{V}_S(f_o)\mathbf{V}_S^+(f_n) \quad (4)$$

In **step 5** of the **section 6**, we can use the focusing operators (3) or (4) to estimate the DOA of the wideband sources.

8 SIMULATION RESULTS

In the following simulations, a linear array of $N = 20$ omniderictional sensors with equal interlement spacing $d = \frac{c}{2f_o}$ is used, where f_o is the midband frequency and c is the velocity of propagation. The rayleigh angle resolution limit for this array is about $\frac{2}{N-1} \approx 0,3$ radius $\approx 7,4$ degres. The source signals are temporally stationary zero-mean bandpass white gaussian processes whith the same central frequency $f_o = 115Hz$, and the same bandwidth $BW = 32Hz$. The array noise is stationary zero-mean bandpass (the same passband as that of the signals) white gaussian vector process, independent of the signals, and statistically independent and identical. Three equipower uncorrelated sources impinging from the angles $10°, 12°$ and $14°$, with the $SNR = 3dB$.

Figure 1, and Figure 2 show the inverse null-spectrum of the described methods.

The norm of bias for two methods is shown (Figure 3), this result point out that the FTCT algorithm is more accurate than the TCT algorithm. To compare the computational complexity of TCT algorithm and the proposed fast algorithm , the number of multiplications is estimated when the number of sources is $P << N$, in **step 5**, ($\approx MN^3$) complex multiplications is needed with the TCT method, the proposed algorithm requires $\approx PMN^2$.

In the studied cas $N = 20$ sensors, $P = 2$ sources and $M = 32$ frequency number : The **step 5**, for the TCT algorithm requires 256.10^3 complex multiplications, and the fast-TCT algorithm requires 256.10^2 complex multiplications, the time of computational is divided by 10.

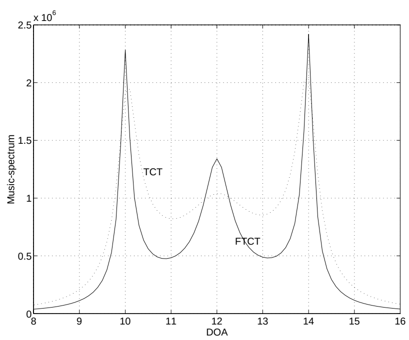

Figure 1: DOA estimation with Music method : _. : TCT algortihm, _ : FTCT algortihm

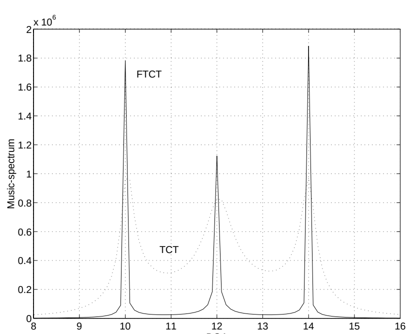

Figure 2: DOA estimation with Music method : _. : TCT algortihm without noise power estimation, _ : FTCT algortihm without noise power estimation

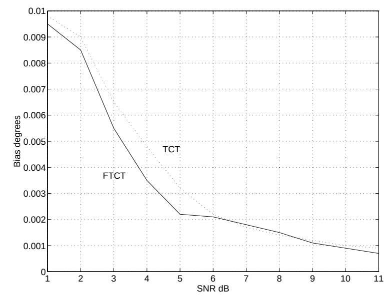

Figure 3: Norm of the averaged bias vector

9 CONCLUSION

In this paper, we have introduced a fast algorithm (FTCT) for the wideband signals, the objective is to reduce the time of computational without reducing the performances of the TCT method. In the high SNR situation, others focusing operators are presented, the estimation of the noise power at each frequency is not necessary.

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