SINGLE-LAYER PERCEPTRON BASED COMMUNICATION CHANNEL EQUALISATION WITH LEAST-MEAN-ABSOLUTE-ERROR ADAPTIVE ALGORITHM

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ABSTRACT

This paper* presents a novel approach to weight adaptation of single-layer perceptron (SLP) based communication channel equalisers, by developing the Least-Mean-Absolute-Error adaptive algorithm using the absolute-error cost function. Theoretical and experimental results are provided and comparisons made between the present algorithm and the traditional back-propagation, Rosenblatt and linear LMS algorithms. This work shows that the proposed algorithm is faster in adapting the weights of the SLP-based equalisers and leads to better estimation performance.

1. INTRODUCTION

Adaptive filtering algorithms have been widely used for adjusting the weights of channel equalisers in digital communication systems [1, 2]. In particular, the least-mean-squares (LMS) and back-propagation (BP) algorithms have been extensively exploited for the development of linear and non-linear equalisers, respectively [1, 5, 6]. The application of these adaptive algorithms is rooted in the clarity, in terms of the statistical concepts, and computational simplicity of the quadratic cost function that they employ. However, their performance may well not be universally the optimal. Other adaptive algorithms based upon non-quadratic cost function can also be defined to improve the adaptation performance. For example, the absolute-error cost function has been successfully applied to developing an algorithm for linear systems [3]. The present work investigates the use of such a cost function in the development of an adaptive algorithm for single-layer perceptron [4, 7] based non-linear filters for communication channel equalisation.

To be self-contained, a brief overview of the channel equalisers implemented using a SLP is provided next. The description of the Least-Mean-Absolute-Error adaptive algorithm is then presented. A substantial part of this paper is dedicated to the performance analysis of the proposed algorithm in terms of the mean-squared-error, convergence rate and bit-error-rate.

2. CHANNEL EQUALISATION USING SINGLE-LAYER PERCEPTRON

The digital data communication task considered herein requires transmitting a binary signal

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(1 or -1) sequence, $S(t) = \{s(t), s(t-1), s(t-2), \ldots, s(t-\Delta T), \ldots, s(t-N+1-M)\}$, passing through a dispersive channel modelled as a finite impulse response filter [2]. The task of a channel equaliser is to reconstruct the signal $s(t-\Delta T)$ using the observations $X(t) = \{x(t), x(t-1), \ldots, x(t-N+1)\}$, obtained at the channel output that are corrupted with noise, where M, N and ΔT are known as the order of the channel model, and the order and delay of the equaliser. In this paper, a channel equaliser is implemented with a single-layer perceptron (SLP) unless otherwise stated. Such an equaliser consists of a set of N input nodes and a single output node which performs the bipolar sigmoid non-linear mapping:

$$y_f(t) = \frac{1-e^{-y(t)}}{1+e^{-y(t)}}$$

where $y(t) = W^T(t)X(t)$ is a weighted sum of the past inputs, with $W(t) = \{ w_0(t), w_1(t), \ldots, w_{N-1}(t) \}$ representing the tap weights modified by an adaptive algorithm.

3. LEAST-MEAN-ABSOLUTE-ERROR ADAPTIVE ALGORITHM

The utilisation of a specific cost function plays an important role in the development of the corresponding adaptive algorithm while using the method of gradient decent. Different cost functions generally result in different algorithms. Here, the algorithm for weight adaptation is deduced using the following absolute-error cost function, where $|e(t)|$ is the absolute error between the transmitted signal $s(t-\Delta T)$ and the estimated signal $y_f(t)$:

$$J_1(t) = 2E[|e(t)|] = 2E[|s(t-\Delta T) - y_f(t)|]$$

Minimising this cost function, by following the method of gradient descent, leads to the formula for weight adaptation:

$$W(t+1) = W(t) + 2\mu[-\frac{\partial|e(t)|}{\partial w(t)}]$$

$$= W(t) + 2\mu[-sign(e(t))\frac{\partial e(t)}{\partial y_f(t)}\frac{\partial y_f(t)}{\partial y(t)}\frac{\partial y(t)}{\partial w(t)}]$$

$$= W(t) + 2\mu[sign(e(t))\frac{(1-y_f^2(t))}{2}X^T(t)]$$

$$= W(t) + \mu\, sign(e(t))(1-y_f^2(t))\, X^T(t)$$

where the parameter μ denotes the adaptation step size which controls the stability and the rate of convergence of this algorithm, and $sign(e(t)) = 1$ if $e(t) >= 0$, and -1 if $e(t) < 0$. To acknowledge the use of the absolute-error cost function, this method for weight modification is termed the Least-Mean-Absolute-Error (LMAE) adaptive algorithm.

In order to analyse the performance of this algorithm, it is interesting to compare it with two representative perceptron learning algorithms, i.e. the BP and Rosenblatt algorithms [4, 5, 7] which are based on the following two cost functions, respectively:

$$J_2(t) = E[e^2(t)]$$

$$J_3(t) = E[|y(t)| - s(t-\Delta T)y(t)]$$

For the SLP-based equalisers, these two traditional adaptive algorithms can be represented such that

$$W_B(t+1) = W_B(t) + \mu e(t)(1-y_f^2(t))X^T(t)$$

$$W_R(t+1) = W_R(t) + \mu e_r(t)X^T(t)$$

Obviously, the LMAE algorithm replaces the term $e(t)$ within the standard BP algorithm with $sign(e(t))$. This results in a number of important advantages in utilising the LMAE algorithm for developing SLP-based equalisers. Firstly, one floating point multiplication is saved for each weight adaptation, which leads to simpler computation and hence increases the efficiency of training the equaliser. Secondly, if the actual output $y_f(t)$, $y_f(t) \in [-1, 1]$, of the equaliser has the same sign as the transmitted signal $s(t-\Delta T)$, then $|e(t)| < 1$, therefore, $|sign(e(t))| > |e(t)|$ which allows more rapid convergence along the correct direction of weight adaptation. Thirdly, if $|e(t)| > 1$ or $y_f(t)$ has the opposite sign of $s(t-\Delta T)$, then $|sign(e(t))| < |e(t)|$. This allows the gradient value, $sign(e(t))$, within the LMAE algorithm to become lower than that within the BP algorithm, implying that the LMAE algorithm is less sensitive to noise.

Compared with the Rosenblatt algorithm, the weight adaptation process using the present approach is much more stable, even with a larger learning step size. This is due to the use of the bipolar sigmoid function in the SLP-based equalisers, instead of merely the hard-limiter (or sign operator). In fact, for the Rosenblatt algorithm, the output error is defined by $e_r(t) = s(t-\Delta T) - sign(y(t))$ which differs from that used in the LMAE and BP algorithms. In so doing, if $e_r(t) = 0$, no adaptation is carried out; whereas if $e_r(t) \neq 0$, the weights are changed by adding or subtracting a term of $2\mu x(t)$. Therefore, the weight modification process using the Rosenblatt algorithm is rather abrupt and is restricted to the employment of a much smaller adaptation step size, though it offers simpler computation than the other two.

4. SIMULATION RESULTS

The advantages of utilising the LMAE algorithm are confirmed with a variety of experimental results. Within this paper, a communication channel is simulated by a first-order non-minimum phase model: $H(z) = 0.5 + 1.0z^{-1}$. The channel output is corrupted by a white Gaussian noise with zero mean, and the order and delay of the equaliser are assumed to be 5 (with the SLP employed having 5 input nodes) and 1. For comparison purposes, simulations have also been carried out, under the same conditions, on another two SLP-based equalisers (one with the bipolar sigmoid mapping and the other with a hard-limiter) and on a linear equaliser with their weights being adapted using the BP, Rosenblatt and LMS algorithms, respectively.

The learning curves of the simulated systems are shown in Figure 1, where the signal-to-noise ratio (SNR) at the input of each equalisation system is 20dB. The average of the resulting estimation errors of 200 independent simulations is presented to demonstrate the performance of the LMAE algorithm. With respect to the same number of weight adaptations, the remaining mean-square-error (MSE) of the equaliser adapted with the LMAE algorithm is much less than that with the BP algorithm, although both equalisers share a common structure. Also, both LMAE and BP algorithms offer a better MSE performance compared with the Rosenblatt algorithm, demonstrating that the use of the bipolar sigmoid mapping provides better results than using a hard-limiter. Additionally, this figure shows a sharp contrast between the performances of the linear and non-linear equalisers. Indeed, no matter whether the Rosenblatt, BP or LMAE algorithm is utilised for adapting the SLP-based systems, the remaining estimation error is considerably smaller than that of the linear equaliser. This is owing to the non-linearity of the mapping function inherently embedded within such systems. Moreover, the convergence rate of the equaliser adapted with the LMAE algorithm is much faster than that of the equaliser of the same structure but trained with the BP algorithm.

In terms of bit error rate (BER), Figure 2 illustrates the results using the LMAE, BP, Rosenblatt and linear LMS algorithms for the same channel equalisation task with a varying signal-to-noise ratio (SNR). These results indicate that the use of the LMAE algorithm offers the best BER performance, particularly when the SNR is decreased. Also, the LMAE and BP algorithms both provide a better BER performance than that of using either the Rosenblatt algorithm or the linear LMS algorithm.

5. CONCLUSION

This paper presents an investigation of using the absolute-error cost function and the resulting Least-Mean-Absolute-Error (LMAE) adaptive algorithm for the development of single-layer perceptron based communication channel equalisation systems. This work demonstrates that the LMAE algorithm offers a good overall performance in developing channel equalisers with increased convergence rate, improved estimation accuracy and reduced bit error rate. Further, the use of this algorithm allows for the reduction of computation effort as compared with using the back-propagation algorithm.

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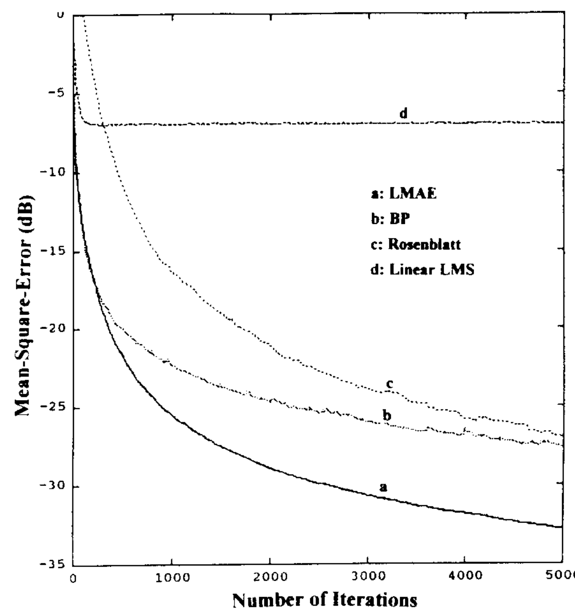

Fig. 1. Learning curves

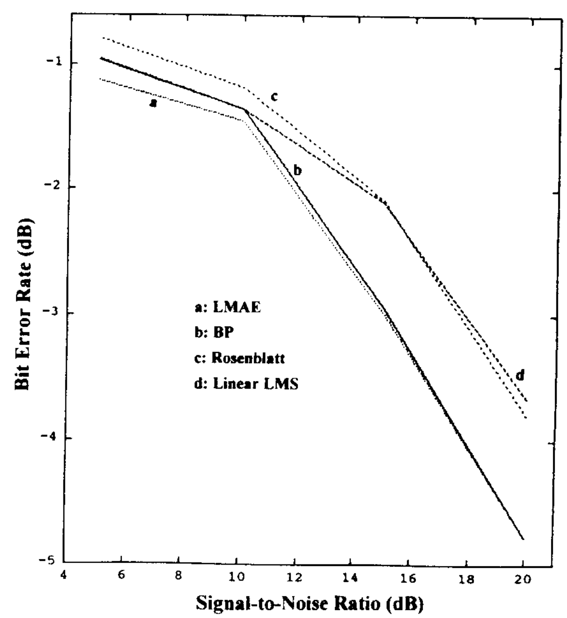

Fig. 2. BER vs. SNR