EVOLUTIONARY ARMA MODELLING FOR AERONAUTICAL COMMUNICATIONS

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ABSTRACT

This contribution deals with modelling and equalization for the aeronautical channel. This channel is subject to multipath and is characterized by a time-variant impulse response. An evolutionary ARMA model is proposed for such a non stationary channel . Evolutionary ARMA models are presented, they are used to derive a parameter estimator which is based upon an eigenformulation for a minimization criteria.
A technique is presented for an equalizer design, based on the above model.

1 INTRODUCTION

Multipath between aircraft and ground terminal is a fundamental problem encountered in aeronautical communications. This channel can be modelled as a randomly time-variant linear filter [1], described by a time-varying impulse response , $h(t,u)$, which transforms the input x into the output y as:

$$y(t) = \int_R h(t,u)x(u)du \quad (1)$$

The impulse response $h(t,u)$ represents the response at time t to a unit impulse at time u, i.e. for $x(t) = \delta(t-u)$, $y(t) = h(t,u)$, where $\delta(.)$ is the Dirac delta function.
We propose a model for the aeronautical channel. It is based on ARMA evolutionary modelling of the time-varying impulse response describing the channel.
The paper is structured as follows: section 2 reviews evolutionary ARMA models, and defines the modelling error. Section 3, introduces a parameter estimator for the evolutionary ARMA model which is based upon an eigenformulation of the error minimization. Section 4 supports the validity of the model. Results obtained by fitting the model to the impulse response of an aeronautical channel, simulated by means of a realistic geometric model . In particular, the propagation between aircraft and the ground terminal is studied during the final approach phase. The multipath delay occurring during this phase depends on the aircraft speed, its location with respect to the receiving antenna and the changing physical characteristics of the propagation medium or other physical phenomena.
Section 5 designs an equalizer using the proposed model and . Section 6 concludes the paper.

2 EVOLUTIONARY ARMA MODELLING

2.1 Evolutionary ARMA models

Let the received signal be modelled by the output of an evolutionary ARMA model. The input signal $x(n)$ and the modelled output signal $\widehat{y}(n)$ are related via the difference equation [3]:

$$\sum_{k=0}^{p} a_k(n-k)\,\widehat{y}(n-k) - \sum_{k=0}^{q} b_k(n-k)\,x(n-k) = 0 \quad (2)$$

The possibly time-varying parameters $a_k(n)$ and $b_k(n)$ are assumed to be real. Under a fairly general regularity condition, $a_k(n)$ and $b_k(n)$ can be expanded over a set of basis functions $\{f_m(t)\}$. To give the problem a finite complexity, it is usually assumed [3] that these time-varying coefficients can be represented, with a sufficient accuracy by a finite expansion:

$$a_k(n) = \sum_{m=1}^{d_a} a_{km} f_m(n) \quad \text{and} \quad b_k(n) = \sum_{m=1}^{d_b} b_{km} f_m(n)$$

Equation (2) then yields:

$$\sum_{k=0}^{p}\left(\sum_{m=1}^{d_a} a_{km} f_m(n-k)\right)\widehat{y}(n-k) = \sum_{k=0}^{q}\left(\sum_{m=1}^{d_b} b_{km} f_m(n-k)\right)x(n-k) \quad (3)$$

Consequently, the time-varying parameters are replaced by a set of $(p+1)d_a$ constant parameters $\{a_{km}\}$ and a set of $(q+1)d_b$ constant parameters $\{b_{km}\}$. The non stationary problem thus becomes a time-invariant problem [3] with respect to conveniently defined vectors.

2.2 Problem formulation

An time-varying multipath channel is characterized by the time spread introduced in the transmitted signal.

The equivalent received low-pass signal is [4]:

$$y(t) = \sum_{k=1}^{K} \alpha_k(t) \cos(2\pi f_c \tau_k(t))\, x(t - \tau_k(t)) \qquad (4)$$

the propagation delay τ_k and the attenuation α_k associated with each path are time-variant. K is the number of paths, f_c is the carrier frequency. Consequently the impulse response, $h(t,u)$, of the physical linear dispersive fading channel is expressed as the sum of K delayed pulses:

$$h(t,u) = \sum_{k=1}^{K} \alpha_k(t) \times \cos(2\pi f_c \tau_k(t))\, \delta(t - u - \tau_k(t)) \qquad (5)$$

The example we are interested in is that of multipath propagation between aircraft and ground terminal. When taking into account the geometry of the problem (aircraft speed, path lengths, etc...) the time delay τ_k between the direct path and the reflected paths are shown [2] to have slow variations while faster changes are due to the term $2\pi f_c \tau_k(t)$.
In order to model the faster changes in the impulse response, the expression (5) of $h(t,u)$ reduces to:

$$h(t,u) = \sum_{k=1}^{K} \alpha_k \cos(2\pi f_c \tau_k(t))\, \delta(t - u - \tau_k(t)) \qquad (6)$$

Let $\tau = t - u$, then:

$$h(t,u) = g(t,\tau) = \sum_{k=1}^{K} \alpha_k \cos(2\pi f_c \tau_k(t))\, \delta(\tau - \tau_k(t))$$

with $\tau_k(t) \simeq p_k T_E$, where T_E is the sampling period. Therefore for $t = nT$, the received signal (4) is:

$$y(n) = \sum_{p_{k=0}}^{q_{\max}} \alpha_k \cos(2\pi f_c \tau_k(n))\, x(n - p_k) \qquad (7)$$

This received signal form can be expressed as:

$$y(n) = \sum_{j=o}^{q} b_j(n)\, x(n-j)$$

or [3]

$$y(n) = \sum_{j=o}^{q} b_j(n-j)\, x(n-j) \qquad (8)$$

(8) justifies an MA or an ARMA evolutionary modelling of the time-varying impulse response [2].

3 PARAMETER ESTIMATION

The key step in characterizing the time-varying impulse response is to find a suitable basis function [2] and a set of fitting parameters $\{a_{km}\}$ and $\{b_{km}\}$ so that the model reproduce the channel behavior. The ARMA error definition given in [5]:

$$e(n) = \sum_{k=0}^{p} a_k(n-k)\, y(n-k) - \sum_{k=0}^{q} b_k(n-k) x(n-k)$$

was extended to the evolutionary case.
The criterion to be minimized with respect to the parameters is:

$$J = \sum_{n} e^2(n) \quad n = 0.....N$$

A method has been developed [2], for estimating the constant parameters vector θ:

$$\theta = [a_{01}...a_{0d_a}.....a_{p1}...a_{pd_a}\ b_{01}...b_{0d_b}.....b_{q1}...b_{qd_b}]^T$$

The expression of the criterion J has the following form:

$$J = \theta^T R^T R\, \theta$$

Using Rayleigh's theorem[6], the minimum of J with the constraint $\|\theta\| \neq 0$, is the minimum eigenvalue of $R^T R$. The corresponding eigenvector is the optimum parameter vector θ.

4 MODEL VALIDATION

The evolutionary ARMA model presented is now applied to a simulated geometrical model of an aeronautical channel. Most of the impulse response variations are caused by the evolution $\cos(2\pi f_c \tau_k(t))$ in (5). Figure 1 shows an example of an impulse response for the simulated channel. Figure 2 displays the response of the corresponding estimated MA(q) model. Figure 3 shows both the response of the simulated channel and of the estimated channel fed with the emitted signal x, which is a gaussian white noise. The algorithm performance for the channel estimate is given in term of the mean-squared-error (MSE).

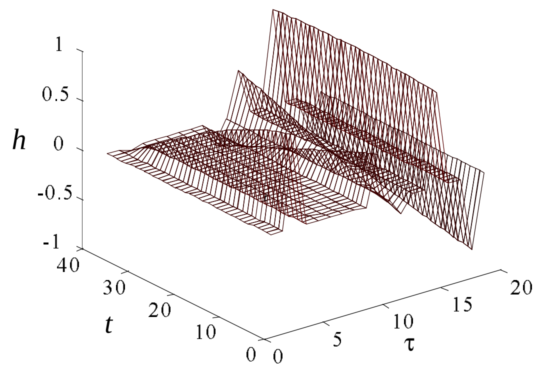

figure1 : time-varying impulse response of the simulated channel

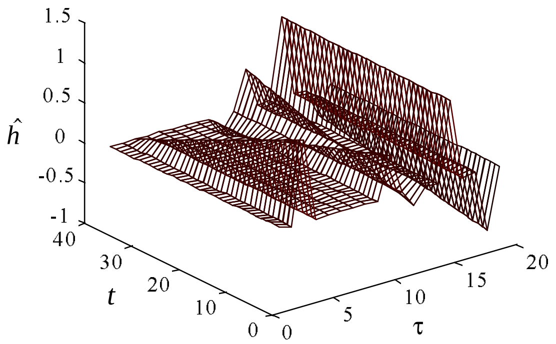

figure 2 : imulpse response of the estimated model

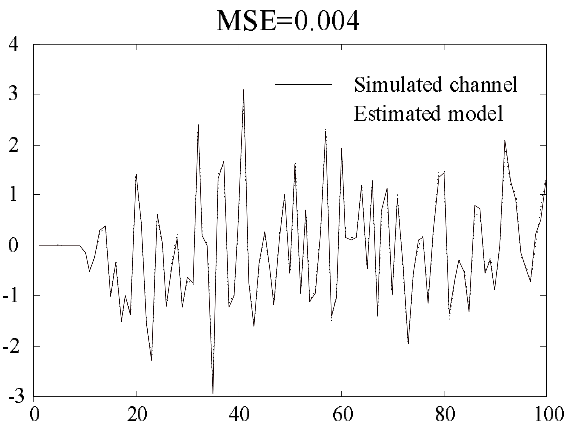

figure 3 : response of the simulated channel and the corresponding estimated model

5 EQUALIZATION

An important digital communication problem is message recovery in the presence of noise and intersymbol interference. This is the task of the equalizer filter. We have studied many methods based on the evolutionary ARMA modelling of the channel. These methods may be classified in three approaches. The first approach determines the equalizer coefficients using the model estimate of the channel. The second approach uses the fact that the channel has an evolutionary ARMA model and is directed at finding an equivalent evolutionary ARMA model for the equalizer.The last approach and the most robust is presented here. It consists at determining a bloc equalizer in a least square sense.
The input-output relationship

$$y(n) = \sum_{u<=n} h(n,u) \cdot x(u)$$

where x is a binary signal, may be expressed in matrix form as:

$$\underline{y} = H.\underline{x}$$

where

$$\underline{y} = [y(0)\ \ y(1)\ .....y(N)]\ ^T \text{ and } \underline{x} = [x(0)\ \ x(1)\ .....x(N)]$$

T standing for the transpose, N is the observation window length and the nth row of H is:

$$[h(n,0)\ \ h(n,1)\ ....h(n,n)\,0\,...0]$$

An emitted signal estimate is provided by solving the following problem:

$$min_{\underline{x}_e \in R^{1xN}}\ \left\|H.\underline{x}_e - \underline{y}\right\|$$

This is a standard least square problem whose solution is given by $\underline{x}_e = H^+.\underline{y}$ where H^+ denotes the pseudo-inverse of H [7]. This method uses a training data sequence to estimate the channel model. This model is valid as long as the impulse response exhibits the same variation law (6)
Eye diagram of figure 4 shows, at $SNR = 40\,dB$, the great amount of interference at the received signal for the channel of figure 1. Figure 5 illustrates the correct eye opening performed by the equalizer. The algorithm performance results are given in terms of the error probability at the equalizer output (pdf) and the bit-error-rate (BER) in detected data sequence. Figure 6 gives the pdf at the channel output and Figure 7 gives the pdf at the equalizer output . Figure 8 displays the BER versus SNR without and with equalization, it shows the good performances of the proposed equalizer. The main advantage of this equalizer is its validity whenever the channel variation has the same law, whereas a stationary equalizer is valid only when the channel can be assumed stationary.

6 CONCLUSION

This paper has proposed an ARMA evolutionary model for the aeronautical channel.
The ARMA evolutionary model was presented which was used to derive a parameter estimator based upon an eigenformulation of the optimization condition.
This model was then applied to simulated data derived from a geometrical model of the channel. An equalizer which was based on the impulse response model was introduced, The great interest of this equalizer is its validity whenever the channel variation exhibits the same law, whereas a stationary equalizer is valid only when the stationary assumption of the channel can be assumed. Some illustrative results were given.

References

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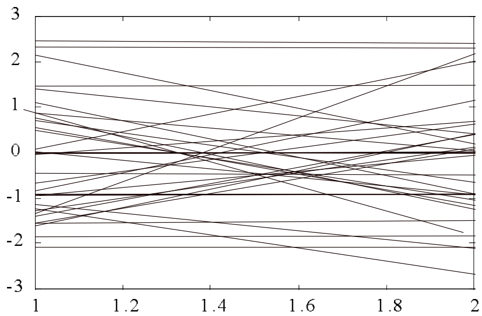

figure 4 : eye diagram at the channel output (SNR=40dB)

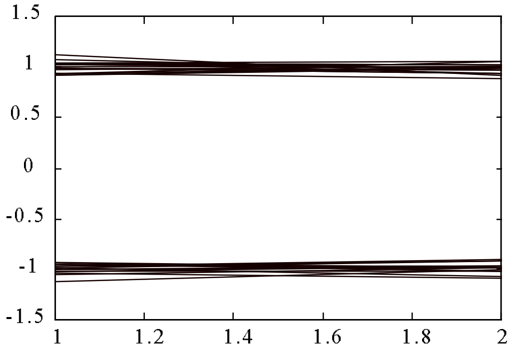

figure 5 : eye diagram at the equalizer output

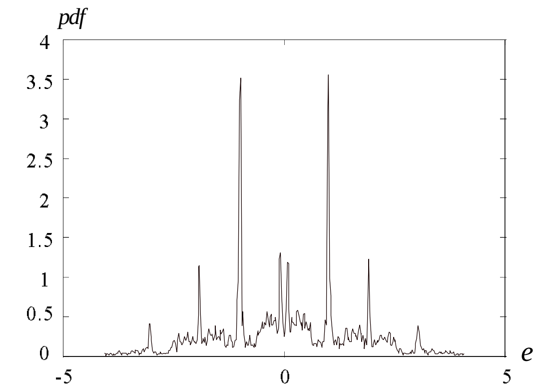

figure 6 : error probability density function without equalization

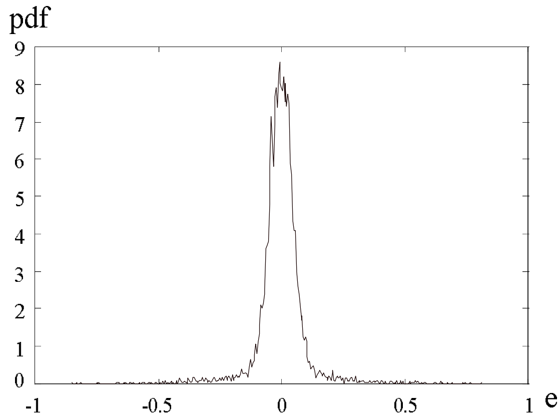

figure 7: error probability density function at the equalizer output

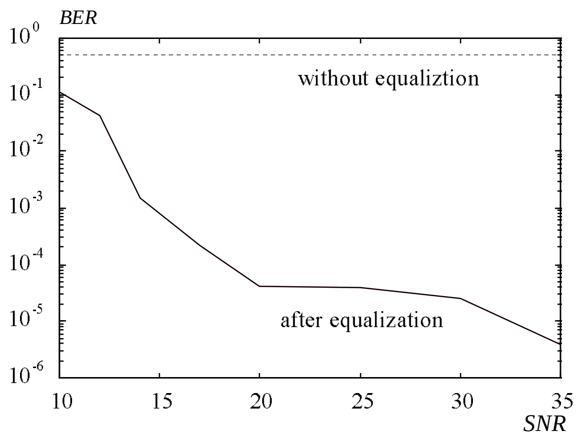

figure 8: bit-error-rate versus SNR