CLASSIFICATION OF LINEAR MODULATIONS BY MEAN OF A FOURTH-ORDER CUMULANT

Denys Boiteau and Christophe Le Martret***
* CESTA, 37 av. du général De Gaulle, 35170 Bruz, France
** Centre d'ÉLectronique de l'Armement, 35170, Bruz, France

ABSTRACT

In this paper, we present a new linear modulation classification method based on a fourth-order cumulant of the stationary signal. Under some hypothesis, this method can be applied to carrier-modulated or baseband signals and doesn't need the knowledge of the signal to noise ratio. An example of classification is given for 4 PSK *vs.* 16 QAM modulations. Theoretical performance are approximated and compared to simulation results. The system achieves more than 90 % of correct classification for only 500 transmitted symbols and a signal to noise ratio of 0 dB.

1 INTRODUCTION

Automatic classification of modulations is a challenging problem that has been investigated for several years. We find two different approaches applied to linear modulations. In a pattern recognition approach, classification is achieved by mean of extracted features like phase, frequency or amplitude of the signals whereas in a log likelihood approach, the log likelihood function of the signal (or some of its parameters) is processed and compared to an appropriated threshold. In both cases, it can be noted that many proposed systems use explicitly or not some fourth order statistics as discriminating parameters [1-4].

In this paper, we are interested in a N PSK, $N=2^n$, $n>1$ (N state Phase Keying) and M QAM, $M=4^m$, $m>1$ (M states Quadrature Amplitude) modulation recognition task. It is well known that these modulations have identical stationary or cyclostationary statistics of order two so they cannot be discriminated. We derived here some relations which show that classification of N PSK *vs.* M QAM or M_1 QAM *vs.* M_2 QAM ($M_1 \neq M_2$) can be theoretically achieved using a fourth-order cumulant of the baseband stationary signal. For these modulations, the discriminating feature are not proportional so that a matched filter (maximum likelihood) approach can be used and, unlike in [3,4], no threshold adjusting will be needed. An application is presented for the classification of 4 PSK *vs.* 16 QAM modulations.

The paper is composed as follows. In Part 2, we derive the fourth-order cumulant expressions of the carrier-modulated and baseband signals. In Part 3, we give the principle of the classification method and provide a solution to obtain a time efficient system. The theoretical performance of the developed 4 PSK *vs.* 16 QAM classification system are approximated in Part 4 and compared to simulation results in Part 5.

2 SIGNALS AND FOURTH-ORDER CUMULANT

2.1 Carrier-modulated and baseband signals

Although the numerical modulated signals are cyclostationary, we consider here their stationary description. The N PSK or M QAM modulated signal $x_c(t)$ transmitted on a carrier frequency f_0 can be expressed (we omit in all the paper to write the random variable needed for the stationarity of the process) as

$$x_c(t) = \mathrm{Re}\left[x(t)\exp(2i\pi f_0 t + \phi_0)\right] \tag{1}$$

where ϕ_0 is a fixed phase and $x(t)$ is the complex-valued baseband signal given by

$$x(t) = \sum_{k=-\infty}^{\infty} c_k h(t - kT) \tag{2}$$

where T is the symbol period, $h(t) = \mathbb{1}_{[-T/2,T/2]}$ and $\{c_k = a_k + ib_k\}$ is an independant and identically distributed symbol sequence. The symbols are defined by $(a_k, b_k) = (\cos\theta_k, \sin\theta_k)$, $\theta_k = (2j+1)\pi/N$, j=0,N-1 for N PSK modulations and by $a_k, b_k = \pm 1, \pm 3, \cdots, \pm(2^m - 1)$ for M QAM modulations.

Let us define

$$C^{xmod}_{x,p+q,p}(\tau_1, \ldots, \tau_{p+q-1}) = \mathrm{Cum}\big(x(t), x(t+\tau_1), \ldots, x(t+\tau_{p-1}), x^*(t+\tau_p), \ldots, x^*(t+\tau_{p+q-1})\big) \tag{3}$$

the cumulant of order $(p+q)$ of a *xmod* modulated signal $x(t)$ (*xmod* = N PSK or M QAM). Then, the fourth-order cumulant $C_{x_c,4,0}^{xmod}(\tau_1,\tau_2,\tau_3)$ of the carrier-modulated signal can be expressed, according to (1), by:

$$
\begin{aligned}
&C_{x_c,4,4}^{xmod}(\tau_1,\tau_2,\tau_3) = \\
&\quad \tfrac{1}{8}\Big\{\mathrm{Re}[C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)]\exp(2i\pi f_0(\tau_1-\tau_2-\tau_3)) + \\
&\qquad \mathrm{Re}[C_{x,4,2}^{xmod}(\tau_2,\tau_1,\tau_3)]\exp(2i\pi f_0(\tau_2-\tau_3-\tau_3)) + \\
&\qquad \mathrm{Re}[C_{x,4,2}^{xmod}(\tau_3,\tau_2,\tau_1)]\exp(2i\pi f_0(\tau_3-\tau_2-\tau_1))\Big\} \quad (4)
\end{aligned}
$$

We see that the cumulant $C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ of the baseband signal appears in (4). Then, we can say that all signal processing systems based on $C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ may be extrapolated to carrier-modulated signals if the carrier frequency is known. We choose in the sequel to work on the baseband signal.

2.2 Cumulant of the complex-valued baseband signals

We can show that the cumulant $C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ of the baseband signal (2) can be written

$$
\begin{aligned}
C_{x,4,2}^{xmod}(\tau_2,\tau_1,\tau_3) = {} & \frac{2}{T}(\gamma_4+\psi_4-4\gamma_2^2).C_h(\tau_1,\tau_2,\tau_3) \\
& +\frac{\gamma_2^2}{T}\sum_{i\neq 0} C_h(\tau_1+iT,\tau_2+iT,\tau_3) \\
& +\frac{\gamma_2^2}{T}\sum_{i\neq 0} C_h(\tau_1+iT,\tau_2,\tau_3+iT) \\
& -\frac{\gamma_2^2}{T^2}\int_{-\infty}^{\infty} C_h(\tau_1+u_1,\tau_2+u_1,\tau_3)\,du_1 \\
& -\frac{\gamma_2^2}{T^2}\int_{-\infty}^{\infty} C_h(\tau_1+u_1,\tau_2,\tau_3+u_1)\,du_1 \qquad (5)
\end{aligned}
$$

where

$$C_h(\tau_1,\tau_2,\tau_3) = \int_{-\infty}^{\infty} h(u)h(u+\tau_1)h(u+\tau_2)h(u+\tau_3)\,du$$

and $\gamma_2 = \mathbb{E}[a_k^2]$, $\gamma_4 = \mathbb{E}[a_k^4]$ and $\psi_4 = \mathbb{E}[a_k^2 b_k^2]$. For signals of equal power P, $\forall\, M, N$ and $M_1 < M_2$, we have the following relations:

$$
\begin{aligned}
&\gamma_2^{N\,\mathrm{PSK}} = \gamma_2^{M\,\mathrm{QAM}} \\
&(\gamma_4+\psi_4)^{N\,\mathrm{PSK}} = \mathrm{c^{te}} \\
&(\gamma_4+\psi_4)^{N\,\mathrm{PSK}} < (\gamma_4+\psi_4)^{M\,\mathrm{QAM}} \\
&(\gamma_4+\psi_4)^{M_1\,\mathrm{QAM}} < (\gamma_4+\psi_4)^{M_2\,\mathrm{QAM}}.
\end{aligned}
$$

Cumulants (5) for N PSK, M_1 QAM and M_2 QAM modulations are therefore different. Moreover, we can show that, under some conditions on the pulse shape $h(t)$ (*cf.* Appendix), they are not proportional. The cumulant $C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ is then a discriminant pattern and modulation recognition using a matched filter framework will avoid the knowledge of the signal to noise ratio. We present in the following the system we set up for the classification of a 4 PSK *vs.* 16 QAM modulations with equal symbol period. For these modulations we have $\gamma_2^{16\mathrm{QAM}} = \gamma_2^{4\mathrm{PSK}} = P/2$, $(\gamma_4+\psi_4)^{4\mathrm{PSK}} = P^2/2$ and $(\gamma_4+\psi_4)^{16\mathrm{QAM}} = 66P^2/100$.

3 DESCRIPTION OF THE SYSTEM

3.1 The correlator

The proposed system can be described as in figure 1.

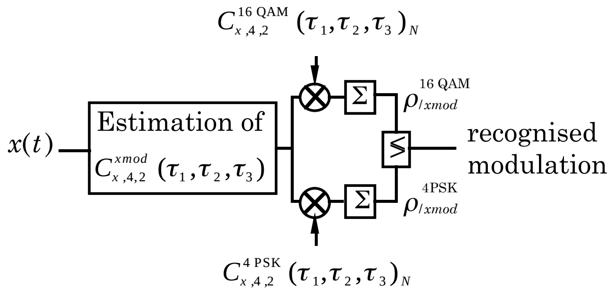

Fig.1: *The 4 PSK vs. 16 QAM classification system*

We first estimate the cumulant of the received *xmod* modulated signal. Then we compute the cross-correlations $\rho_{/xmod}^{tmod}$ between the estimated cumulant $\tilde{C}_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ and the two theoretical normalised cumulants $C_{x,4,2}^{tmod}(\tau_1,\tau_2,\tau_3)_N$ where $tmod \in \{4\,\mathrm{PSK}, 16\,\mathrm{QAM}\}$. The strongest correlation measure provides the recognised modulation.
The cross-correlation is expressed as:

$$\rho_{/xmod}^{tmod} = \sum_{\tau_1,\tau_2,\tau_3} \tilde{C}_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3).C_{x,4,2}^{tmod}(\tau_1,\tau_2,\tau_3)_N \qquad (6)$$

and the normalised cumulants are defined by:

$$C_{x,4,2}^{tmod}(\tau_1,\tau_2,\tau_3)_N = \frac{C_{x,4,2}^{tmod}(\tau_1,\tau_2,\tau_3)}{\sqrt{\sum_{\tau_1,\tau_2,\tau_3}\left(C_{x,4,2}^{tmod}(\tau_1,\tau_2,\tau_3)\right)^2}} \qquad (7)$$

This normalisation is necessary not to bias the cross-correlation measure.

3.2 A lower complexity classifier using only a slice of the cumulant

The fourth order cumulant $C_{x,4,2}^{xmod}(\tau_1,\tau_2,\tau_3)$ is a 3 dimension pattern which complete estimation is very time consuming. In order to obtain a lower complexity system (and so time efficient) , we propose to estimate only a slice $C_s^{xmod}(\tau)$ of the cumulant (5), under the condition this slice still contains discriminant information. Among several possible candidates, the slice we have retained is the one which minimises, according to the classification procedure, the correlation coefficient ρ between the slices obtained for the two modulations

$$\rho = \sum_{\tau} C_s^{16\text{QAM}}(\tau)_N . C_s^{4\text{PSK}}(\tau)_N \tag{8}$$

The selected slice is found to be

$$\begin{aligned} C_s^{xmod}(\tau) &= C_{x,4,2}^{xmod}(0,\tau,\tau) \\ &= \mathbb{E}[x^2(0)x^{2*}(\tau)] - 2\mathbb{E}^2[x(0)x^*(\tau)] \end{aligned} \tag{9}$$

or, by developing (5)

$$C_s^{xmod}(\tau) = 2(\gamma_4 + \psi_4).\Delta_T(\tau) - 8\gamma_2^2.\Delta_T{}^2(\tau) \tag{10}$$

with $\Delta_T(\tau) = (T - |\tau|)/T \cdot \mathbb{1}_{[-T,T]}$.

An asymptotically unbiased and gaussian estimation of (9) is classically obtained by time averaging over the available data, using the approximation

$$\mathbb{E}[y(t)] \approx \frac{1}{N_d}\sum_{i=1}^{N_d} y(i) \tag{11}$$

where $y(t)$ (as $x(t)$) is an ergodic and stationary process.

4 THEORETICAL PERFORMANCE

The probability of correct classification $\mathrm{P_{cc}}$ is given by:

$$\begin{aligned} P_{cc} = 1 &- \tfrac{1}{2}\Pr\left\{\rho_{/4\text{PSK}}^{4\text{PSK}} < \rho_{/4\text{PSK}}^{16\text{QAM}}\right\} \\ &- \tfrac{1}{2}\Pr\left\{\rho_{/16\text{QAM}}^{4\text{PSK}} < \rho_{/16\text{QAM}}^{16\text{QAM}}\right\} \end{aligned} \tag{12}$$

where $\rho_{/mod2}^{mod1}$ is defined by (6). We note

$$\rho_{/xmod}^{diff} = \rho_{/xmod}^{4\text{PSK}} - \rho_{/xmod}^{16\text{QAM}} \tag{13.a}$$

$$\mathbb{E}[\rho_{/xmod}^{diff}] = m_{/xmod}^{diff} \tag{13.b}$$

$$\mathbb{E}[(\rho_{/xmod}^{diff} - m_{/xmod}^{diff})^2] = (\sigma_{/xmod}^{diff})^2. \tag{13.c}$$

Under the hypothesis that the coefficients $\rho_{/xmod}^{diff}$ are gaussian[1], the $\mathrm{P_{cc}}$ can be written, using (13)

$$\mathrm{P}_{cc} = 1 - \frac{1}{2}\left[\operatorname{erfc}\left(-\frac{m_{/4\text{PSK}}^{diff}}{\sqrt{2}\sigma_{/4\text{PSK}}^{diff}}\right) + \operatorname{erfc}\left(\frac{m_{/16\text{QAM}}^{diff}}{\sqrt{2}\sigma_{/16\text{QAM}}^{diff}}\right)\right] \tag{14}$$

with $\operatorname{erfc}(\mathrm{t}) = \int_t^{\infty} \exp(-u^2)\,du$.

The parameters $m_{/xmod}^{diff}$ and $\sigma_{/xmod}^{diff}$ are given as functions of the means and variances of the cumulant estimates. If we note $\alpha_\tau = C_s^{4\text{PSK}}(\tau)_N$ and $\beta_\tau = C_s^{16\text{QAM}}(\tau)_N$, then it can be easily shown that

$$m_{/xmod}^{diff} = \sum_{\tau}(\alpha_\tau - \beta_\tau).C_s^{xmod}(\tau) \tag{15.a}$$

and

$$\begin{aligned} \left(\sigma_{/xmod}^{diff}\right)^2 = \sum_{\tau_1,\tau_2}\Big\{&(\alpha_{\tau_1} - \beta_{\tau_1})(\alpha_{\tau_2} - \beta_{\tau_2}) \cdot \\ &\operatorname{cov}\left[\tilde{C}_s^{xmod}(\tau_1), \tilde{C}_s^{xmod}(\tau_2)\right]\Big\}. \end{aligned} \tag{15.b}$$

Unlike $m_{/xmod}^{diff}$, the theoretical value of the covariance $\operatorname{cov}\left[\tilde{C}_s^{xmod}(\tau_1), \tilde{C}_s^{xmod}(\tau_2)\right]$ can not be reasonably calculated. To obtain an approximation of the theoretical error probability of the system, we use an approximation of the covariance by applying the formula given in [5] with $M_T = T$ and processing only one realisation of the process.

5 SIMULATION AND DISCUSSION

Simulation have been performed on simulated baseband signals in white gaussian noise for different signal to noise ratio S/N (S/N = -5, 0, 5 dB). The number of transmitted symbols Ns (symbol period T=10) varies from 50 (500 sampling data) to 5000 (50000 sampling data). For each couple ($S/N,Ns$), 1000 noisy and different signals (different symbol sequences, noise samples and timing phases) are generated for each modulation. The figures (2 a-c) give the theoretical (equation (14)) and experimental performance obtained for different Ns and at a given S/N.

We can see that simulation results are quite close to the theoretical performance. It seems to be reasonable according to the approximations made: 1) gaussian approximation of the cumulant estimates (asymptotically verified), 2) gaussian approximation of the correlation measure $\rho_{/xmod}^{diff}$ (experimentally verified), 3) approximation of the estimated covariance, and 4) approximation to 0 of the bias of the cumulant estimates (asymptotically verified).

For $S/N \geq 0$, the system offers good performance. We obtain a false classification lower than 10 % for only 500 transmitted symbols. Confusion matrices have shown that the errors are principally due to a misclassification of the 16 QAM signals which can be explained by the greater variability of the symbol values for a 16 QAM compared to a 4 PSK modulation inducing a worth estimate of the statistics.

6 CONCLUSION

We have presented a classification system for 4 PSK *vs.* 16 QAM modulations based on the recognition of the estimated slice $C_{x,4,2}^{xmod}(0,\tau,\tau)$ of the received signal. We have shown that this method can be applied to complex (baseband) or real-valued (carrier-modulated) signals. Under the hypothesis

---

1 This hypothesis stands only if the estimated cumulants for different offsets τ are independants. Although this condition is not satisfied, we note however in practice that the probability density function of the coefficient $\rho_{/xmod}^{diff}$ is correctly approximated by a gaussian function.

that we *a priori* know the carrier parameters and the symbol period, very good performance can be obtained for low S/N with white gaussian noise. This hypothesis can be however avoided by processing the module of the cumulant of the equivalent baseband signal and by searching the symbol period which maximises the cross-correlation. The approximate theoretical performance are close to simulation results and the system gives good performance for S/N ≥ 0 dB and $Ns \geq 500$.

APPENDIX

The frequency dual of (6) is the polyspectrum of the signal $x(t)$ which can be written (application of the 3D Fourier Transform (FT))

$$S_{x,4,2}^{xmod} = \begin{bmatrix} \frac{2}{T}(\gamma_4 + \psi_4 - 4\gamma_2^2) \\ +\frac{\gamma_2^2}{T^2}\sum_{i\neq 0}\delta(f_1+f_2+\frac{i}{T}) \\ +\frac{\gamma_2^2}{T^2}\sum_{i\neq 0}\delta(f_1+f_3+\frac{i}{T}) \end{bmatrix} .C_H(f_1,f_2,f_3) \quad \text{(A1)}$$

where $H(f) = \mathrm{FT}(h(t))$ is real-valued and even, $\delta(.)$ is the Dirac distribution and

$$C_H(f_1,f_2,f_3) = H(f_1)H(f_2)H(f_3)H(f_1+f_2+f_3).$$

According to the values of $(\gamma_4 + \psi_4)$ and γ_2 for the N PSK and the M QAM modulations, we see that the polyspectrum (and therefore the cumulants) for the two modulations are different and not proportional if

$$\frac{\gamma_2^2}{T^2}\sum_{i\neq 0}\delta\left(f_1+f_2+\frac{i}{T}\right).C_H(f_1,f_2,f_3) \neq 0 \quad \text{(A2}$$

or equivalently

$$\sum_{i\neq 0} H(f_1)H(-f_1+\frac{i}{T})H(f_3)H(f_3-\frac{i}{T})\delta(f_1+f_2+\frac{i}{T}) \neq 0 \,\text{(A3)}$$

which is equivalent to the condition

$$\left(\text{bandwith of } H(f)\right) > \frac{1}{T}. \quad \text{(A4)}$$

For $h(t) = \mathbb{1}_{[-T/2,T/2]}$, we have $H(f) = \sin(\pi f T)/\pi f$ and (A4) is verified.

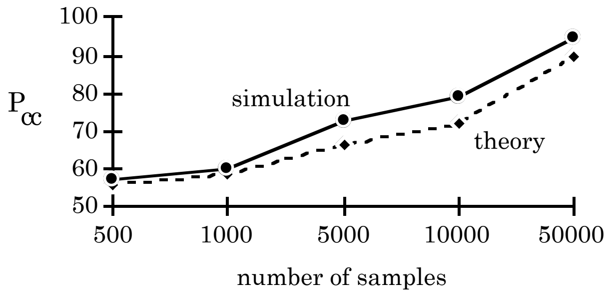

Fig.2a: *P_{cc} for S/N=-5 dB*

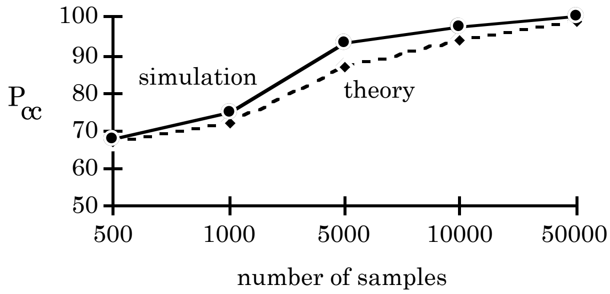

Fig.2b: *P_{cc} for S/N=0 dB*

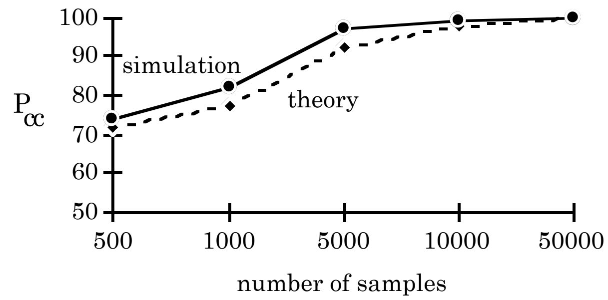

Fig.2c: *P_{cc} for S/N=5 dB*

BIBLIOGRAPHY

[1] C. Louis & P. Sehier, "Automatic modulation recognition with a hierarchical neural network", IEEE Milcom conf., 1994.

[2] S.S. Soliman & S.-Z. Hsur, "Signal classification using statistical moments", IEEE Tr. on Communications, vol. 40, n° 5, 1992.

[3] C.S. Long & al, "Further results in likelihood classification of QAM signals", IEEE Milcom conf., 1994.

[4] C.Y. Huang & A. Polydoros, "Likelihood methods for MPSK modulation classification", IEEE Tr. on Communications, vol. 43, n° 2/3/4, 1995.

[5] G.B. Giannakis & M.K. Tsatsanis, "Time domain test for gaussianity and time-reversibility", IEEE Tr. on Signal Processing, vol. 42, n° 12, Dec. 1994.