PERFORMANCE OF AN ADAPTIVE KALMAN EQUALISER ON TIME VARIANT MULTIPATH CHANNELS

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ABSTRACT

This paper develops an adaptive equaliser which utilises the Kalman filtering to reconstruct the transmitted sequence in time variant environments. The adaptive Kalman equaliser(AKE) addressed by Mulgrew and Cowan is modified by adopting a channel estimator, coefficients of which are updated by a gradient algorithm with fading memory prediction. By computer simulations, the performance of the AKE is investigated, and shown to be superior to that of the decision feedback equaliser(DFE) involving the adaptation of recursive least squares(RLS) algorithm in the case of a second order Markov communication channel model.

1 INTRODUCTION

There have been a number of attempts to tackle the problem of equalisation of time variant communications channels. The least mean square(LMS) and recursive least squares(RLS) algorithms are commonly used as the adaptation procedure for the equalisers.

Multipath channels containing the characteristics of time variation often causes channel output spectral nulls, leading to severe intersymbol interference which means ill-conditioning to any adaptive equaliser. This is typical on high-frequency(HF) channels and mobile radio channels. To handle such spectral nulls, a decision feedback equaliser(DFE) is preferred to a linear equaliser, because the DFE operates on a noise-free output from the decision circuit. In [1]-[3] it has been proposed that the RLS algorithm based on the structure of the DFE should be used to equalise time variant channels. However, the DFE has an inherent problem associated with the error propagation.

On the other hand, it has been shown that the adaptive Kalman equaliser(AKE) addressed by Mulgrew and Cowan[7] provides good performance in the HF channel[6]. In the AKE, the separation of the state and channel estimation processes is attempted. The structure of the AKE is basically that of a linear infinite impulse response filter, and thus involves a feedback path like the DFE. However, the AKE, unlike the DFE, does not suffer from error propagation, because it does not utilise previous decisions to obtain the equaliser output.

This paper modifies the AKE by Mulgrew and Cowan[7] to suit time variant environments. To deal with rapid time variation, a gradient algorithm with fading memory prediction is adopted as the adaptation procedure of the channel estimator. We demonstrate the performance of the AKE by computer simulations, in comparison with that of the conventional RLS DFE. The results show how the AKE is robust against channel fade rate and against additive noise.

2 CHANNEL MODEL

We assume that the channel is modeled as a discrete-time finite impulse response filter, the output of which is corrupted by additive noise. Thus if u_k is the transmitted sequence, assumed to have zero mean and unit variance, the output of the channel is a noise-corrupted sequence x_k given by

$$x_k = \sum_{i=0}^{L-1} h_i(k) u_{k-i} + n_k \tag{1}$$

where $h_o(k), h_1(k), , , , h_{L-1}(k)$ is the channel impulse response and n_k is a stationary sequence of Gaussian noise with zero mean and variance σ^2, which is assumed uncorrelated with u_k.

3 STRUCTURE AND ADAPTATION

The AKE basically consists of double adaptation processes; one is the channel estimation and the other is the Kalman filtering. The channel estimation process requires the estimate of the variance of the additive noise as well as the estimate of the channel coefficients. The original AKE involves a channel estimator, coefficients of which are updated by the LMS algorithm. However, here, the LMS algorithm is replaced by the gradient algorithm with degree-1 least square fading memory prediction[4].

3.1 Channel Estimation

Figure 1 shows a configuration of the modified AKE in the training mode where it is assumed that exact

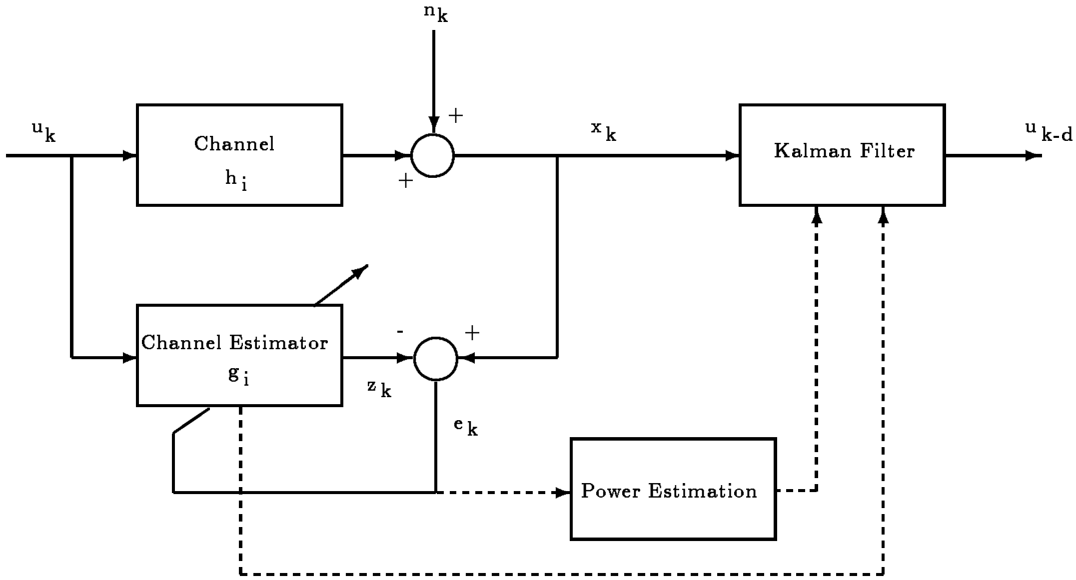

Figure 1: Configuration of the adaptive Kalman equaliser in the training mode.

knowledge of the training sequence is obtained at the receiver side. Suppose that the channel estimator has N adjustable tap coefficients represented by the vector $\mathbf{g}(k) = (g_0(k), g_1(k),,,, g_{N-1}(k))^T$, and the output of the channel estimator is the sequence z_k given by

$$z_k = \sum_{i=0}^{N-1} g_i(k) u_{k-i}. \tag{2}$$

Then the gradient algorithm is given by the following equations:

$$e_k = x_k - z_k \tag{3}$$

$$\mathbf{f}(k+1) = \mathbf{g}(k) + \mu \mathbf{u}(k) e_k \tag{4}$$

$$\mathbf{E}(k) = \mathbf{f}(k+1) - \mathbf{g}(k) \tag{5}$$

$$\alpha(k+1) = \alpha(k) + (1-\theta)^2 \mathbf{E}(k) \tag{6}$$

$$\mathbf{g}(k+1) = \mathbf{g}(k) + \alpha(k+1) + (1-\theta^2)\mathbf{E}(k) \tag{7}$$

where μ is the step-size parameter and $\mathbf{u}(k)$ is the input vector given by $\mathbf{u}(k) = (u_k, u_{k-1},,,, u_{k-N+1})^T$. $\mathbf{f}(k)$, $\mathbf{E}(k)$ and $\alpha(k)$ are N component row vectors, and θ is a real-valued constant in the range 0 to 1. This channel estimator, at each iteration, gives an estimate of the impulse response of the channel. The gradient algorithm (2)-(7) is known to be more cost-effective than the corresponding RLS-type algorithm[5].

For the purpose of estimating the variance of the additive noise, we use the following operation:

$$\hat{\sigma^2}(k) = (1 - 1/N)\hat{\sigma^2}(k-1) + (e_{k-i})^2/N, \tag{8}$$

because this operation compensates the Kalman filter for the uncertainty in the channel impulse response vector[7].

3.2 Kalman Filtering

Based on the results from the estimates of the impulse response of the channel and the variance of the additive noise, the following Kalman filter is implemented to obtain the equaliser output.

$$\hat{\mathbf{s}}(k/k-1) = \Phi \hat{\mathbf{s}}(k-1/k-1) \tag{9}$$

$$\hat{\mathbf{s}}(k/k) = \hat{\mathbf{s}}(k/k-1) + \mathbf{K}(k)[x_k - \mathbf{H}(k)\hat{\mathbf{s}}(k/k-1)] \tag{10}$$

$$\mathbf{V}(k/k-1) = \Phi \mathbf{V}(k-1/k-1)\Phi^T + \xi\xi^T \tag{11}$$

$$\mathbf{K}(k) = \mathbf{V}(k/k-1)\mathbf{H}^T(k)[\mathbf{H}(k)\mathbf{V}(k/k-1)\mathbf{H}^T(k) + \sigma^2]^{-1} \tag{12}$$

$$\mathbf{V}(k/k) = [\mathbf{I} - \mathbf{K}(k)\mathbf{H}(k)]\mathbf{V}(k/k-1) \tag{13}$$

where $\mathbf{s}(k)$ represents the state vector given by

$$\mathbf{s}^T(k) = [u_k u_{k-1} \ldots u_{k-N+1} \ldots u_{k-d}], \tag{14}$$

and $\hat{\mathbf{s}}(k/l)$ means the estimate of $\mathbf{s}(k)$ given data from 0 to sample l. Φ is a $(d+1)$ by $(d+1)$ shift matrix whose elements ϕ_{ij} are equal to unity if $i-j=1$ and are zero otherwise, and ξ is a vector with $(d+1)$ elements

$$\xi^T = [100\ldots0]. \tag{15}$$

Also, $\mathbf{K}(k)$ is the $(d+1)$ elements Kalman gain vector, $\mathbf{H}(k)$ is the 1 by $(d+1)$ observation vector given by

$$\mathbf{H}(k) = [h_0(k), h_1(k), \ldots, h_{N-1}(k), 0, 0, \ldots, 0], \tag{16}$$

and $\mathbf{V}(k/k)$ is the $(d+1)$ by $(d+1)$ error covariance matrix. Replacing $h_i(k)$ in (16) by $g_i(k)$ in (4) and substituting (8) in (12), we can obtain at each iteration the equaliser output, u_{k-d}, from the state vector $\mathbf{s}(k)$.

Equations (2)-(13) give the whole algorithm of the AKE for the training mode.

In the tracking mode, the equaliser output u_{k-d} is used as the input to the channel estimator, and, to produce correct estimates, x_k delayed by d, x_{k-d}, is used as the channel output instead of x_k.

4 SIMULATION RESULTS

Computer simulations were carried out to verify the performance of the AKE. A comparison was also made with the RLS DFE which is a DFE involving the adaptation of the Godard RLS algorithm. The channel used in our simulations consists of 3 taps and is given by

$$H(z) = h_0(k) + h_1(k)z^{-1} + h_2(k)z^{-2}. \qquad (17)$$

The time variant coefficients, $h_0(k)$, $h_1(k)$ and $h_2(k)$, were generated by passing a Gaussian white noise through a second order Butterworth filter which was designed with sampling rate of 2400 sample/s. The input of the channel was a pseudo-random sequence with values of $+1$ or -1. In these simulations the channel fade rate can be quoted as the 3dB bandwidth for the Markov process. Figure 2 shows an example of coefficient trajectory of this channel model with a fade rate of 2 Hz.

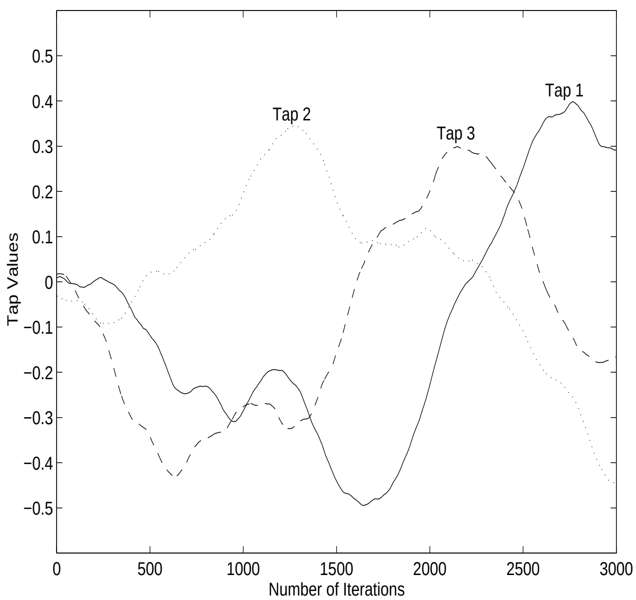

Figure 2: An example of coefficient trajectory. The Taps 1-3 correspond to $h_0(k)$, $h_1(k)$ and $h_2(k)$, respectively.

Figures 3 is the convergence for channel fade rates of 2 Hz and 10 Hz where the RLS DFE and the AKE are compared in the case of -50 dB additive noise. The equalisers have the same filter order $M_f = 14$ and $M_b = 2$ for the RLS DFE and $d = 16$ for the AKE, where M_f and M_b denote the feedforward and feedback filter

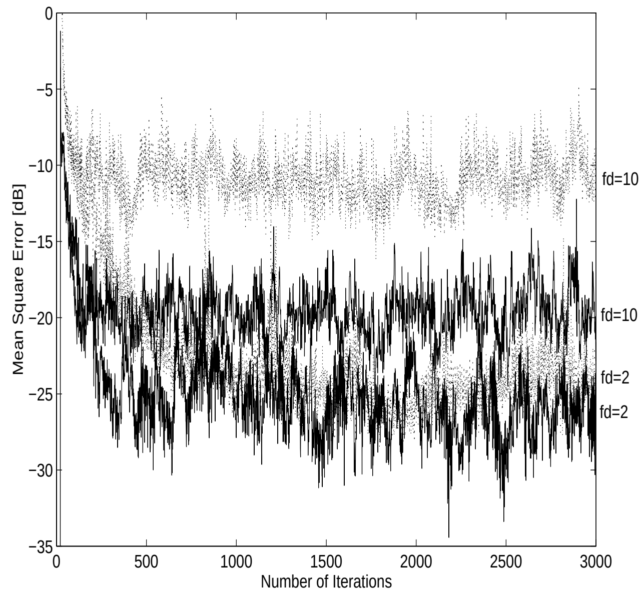

Figure 3: Convergence of the AKE and the RLS DFE. Solid and dotted lines correspond to the former and the latter, respectively. The fd denotes fade rate in Hz.

orders, respectively. (This means that both equalisers have about the same computational complexity which is proportional to the squares of the number of coefficients to be updated at each iteration.) The parameters for both equalisers have been optimised to give the best performance. For the purpose of investigating the equalisation performance against various time variant characteristics, Figure 3 has been evaluated by averaging 100 individual trials. From Figure 3 we see that as the fade rate increases, the steady state properties of the RLS DFE drastically degrade, while those of the AKE are comparatively robust. Looking carefully at Figure 3, we also see that the convergence curve of the RLS DFE has some spikes in the steady state, especially for the fade rate of 2 Hz. This may be a visualisation of the phenomenon of the error propagation invoked by the structure of the DFE. Figure 3 also shows that the AKE provides competitive or faster convergence.

Figure 4 is an illustration of the probability of error of both equalisers on the channel with fade rates ranging 0.5 to 5 Hz. The signal-to-noise(SN) ratio is 30 dB. Figure 4 shows that the AKE provides better performance than the RLS DFE for a wide range of fade rates. The AKE behaves robustly at high fade rates which are more than 2 Hz. This result is coincident with that of Figure 3.

Figure 5 illustrates the probability of error on the channel where the additive noise predominates. The fate rate is 2 Hz. This figure demonstrates the tolerance of the equalisers to the additive noise. Figure 5 shows that the AKE provides an improvement related

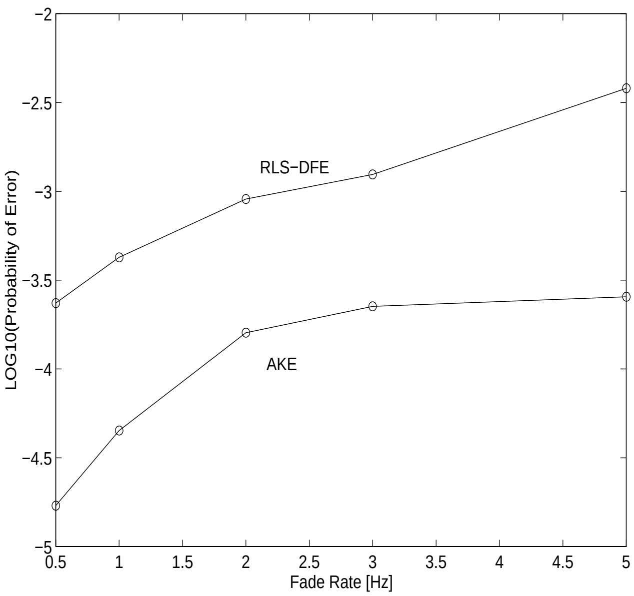

Figure 4: Bit error rate performance against channel fade rates for a signal-to-noise ratio of 30 dB.

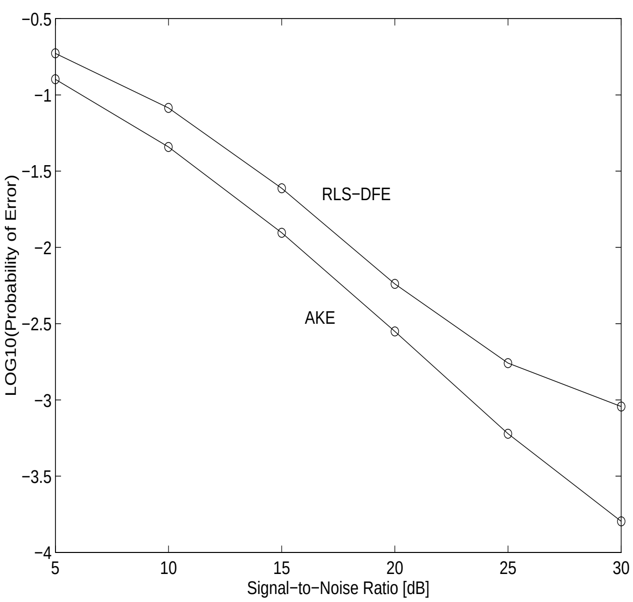

Figure 5: Bit error rate performance against additive noise for a fade rate of 2 Hz.

to the RLS DFE when the additive noise is low as well as when it is high. This is a surprising result, because the DFE feeds back a noise-free output and does not have noise enhancement unlike the AKE. From Figure 5, we deduce that regardless of the SN ratio time variation invokes catastrophic propagation of the decision error and degrades the performance of the DFE, while the AKE retains better performance despite the feedback of the noise-corrupted output.

5 CONCLUSIONS

From the point of view of time variant channel equalisation, Kalman filtering has been investigated and an adaptive Kalman equaliser has been developed. By adopting a channel estimator, coefficients of which are updated by the gradient algorithm with fading memory prediction, the AKE by Mulgrew and Cowan has been modified to suit time variant environments. Computer simulations have demonstrated that the AKE provides a significant improvement related to the RLS DFE in the case of a second order Markov communication channel model.

Acknowledgements

This work was undertaken with sponsorship from the Telecommunication Advancement Foundation in Japan.

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