COMBINED MATCHED FILTER/INTERPOLATOR FOR DIGITAL RECEIVERS

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ABSTRACT

Timing adjustment in digital receivers is usually performed by an interpolator following the matched filter. With a root-cosine pulse with rolloff 0.5, linear interpolation with 2 samples per symbol leads to SNR loss. In this paper, it is shown that the receiver structure can be simplified, and that the SNR-loss can be reduced. This is achieved by the construction of novel strictly time-limited root-Nyquist pulses with good spectral properties. Using these pulses, combination of matched filter and interpolator for timing adjustment in digital receivers with negligible SNR-loss up to a BER of 10^{-6} for BPSK is possible at two samples per symbol.

1 INTRODUCTION

A fully digital receiver samples the received signal at instants that are not synchronized with the transmitter clock. The ideal Maximum-Likelihood receiver has to sample the matched filtered data pulse at its exact maximum, assuming that a root-Nyquist pulse is used for transmission. Therefore, the timing error between transmitter and receiver has to be corrected to avoid SNR loss. Solutions of this problem have been studied in [2;3;4] for different cases. The receiver considered there consists, after the sampling unit at rate $1/T$, of a digital matched filter, an interpolator to correct the time-shift, and the decision unit working at symbol rate $1/T_s$, see Fig.1. Perfect timing estimation is assumed there, and throughout this paper, too. For BPSK, the typical case of a root-cosine pulse with rolloff 0.5 and sampling rate of two times the symbol rate, linear interpolation (two filter coefficients) leads to an SNR-loss off 0.2 dB at a BER (Bit-Error-Rate) of 10^{-2} and 0.7 dB at a BER of 10^{-6}, see [2]. Higher order interpolation or a higher sampling rate lead to better performance at expense of higher computation load.

2 THE IDEAL MATCHED FILTER/-INTERPOLATOR

The investigations described in the paper start from the idea that the interpolator can be incorporated in the matched filter, by using a delayed replica: ideally, the by τ delayed and sampled with rate $1/T$ root-Nyquist pulse $rp(nT-\tau)$ is convolved with its time-reversed version and computed at symbol rate $1/T_s$, leading to the result

$$x(nT_s) = T \sum_{k=-\infty}^{\infty} rp(kT+\tau)rp(nT_s+kT+\tau) \quad (1)$$

With the the symbol $\circ\!\!-\!\!\bullet$ denoting as usual the Fourier-Transform

$$s(t) \circ\!\!-\!\!\bullet S(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft}dt \;, \quad (2)$$

the Transform of equation (1) is obtained with the help of Poisson's summation formula (see [5])

$$X(f) = RP(f) \sum_{m=-\infty}^{\infty} RP(\frac{m}{T}-f)e^{j2\pi\tau\frac{m}{T}} \quad (3)$$

It is now clear that for bandlimited pulses sampled at the Nyquist rate and infinite summation, in (3), only the term for $m=0$ remains, that is

$$X(f) = RP(f)RP(-f) := P(f). \quad (4)$$

Hence this filtering operation exactly cancels the delay τ, and $x(nT_s) = p(nT_s)$, where $p(nT_s)$ are the samples of the underlying Nyquist pulse. No intersymbol interference occurs therefore and hence, no additional interpolator is necessary, see Fig. 2.

However, the same problems as for interpolation described in [2,3,4] arise:

- Exactly bandlimited pulses are not realizable, thus (4) is an approximation
- The infinite summation in the discrete correlation (1) is not realizable

Therefore, errors arise, leading to SNR loss. In order to keep these errors small, the following new procedure is chosen.

3 CONSTRUCTION OF STRICTLY TIME-LIMITED ROOT-NYQUIST PULSES

Strictly time-limited Nyquist-pulses, denoted by p_{2m} and corresponding root-Nyquist pulses rp_{2m} are now constructed. In this way, the matched filter has finite length, and the sum (1) needs only a finite number of terms. The spectrum of these pulses decays rapidly in order to avoid high sampling rates. This twofold performance is obtained by chosing the Nyquist pulses as a weighted finite sum of time-shifted B-Splines. B-Splines of order n, denoted $B_n(x)$ are well known [7] and are piecewise polynomial functions.

With the usual notation $sinc(x) = \sin(\pi x)/(\pi x)$ and the relation [7]

$$B_n(x) \circ\!\!-\!\!\bullet\, sinc^n(f) \;, \qquad (5)$$

we look for Nyquist pulses $p_{2m}(x)$ of the form

$$\begin{aligned} p_{2m}(x) &= \sum_{k=0}^{N} a_{2m,k}\{B_{2m}(x+k/2) && (6)\\ &\quad +B_{2m}(x-k/2)\} && (7)\\ &\circ\!\!-\!\!\bullet\; sinc^{2m}(f)\sum_{k=0}^{N} 2a_{2m,k}\cos(\pi k f)\;. && (8)\end{aligned}$$

Note that the splines are shifted by multiples of 1/2. This is important, because if only integer shifts are allowed in the sum (6), it turns out that no solution exists, see [7]. With this special settimg for $p_{2m}(x)$ we look for a finite coefficient sequence $a_{2m,k}$ such that $p_{2m}(x)$ satisfies the well-known Nyquist-condition for $T_s = 1$

$$p_{2m}(kT_s) = \delta(k) \circ\!\!-\!\!\bullet \frac{1}{T_s}\sum_{k=-\infty}^{\infty} P_{2m}(f-\frac{k}{T_s}) \quad = 1. \quad (9)$$

Inserting $P_{2m}(f)$ in this condition at the first step it follows

$$1 = 2\sum_{n=-\infty}^{\infty}\left(\frac{\sin(\pi(f-n))}{\pi(f-n)}\right)^{2M}\sum_{k=0}^{N} a_{2m,k}\cos(k\pi(f-n)) \qquad (10)$$

Using trigonometric identities, this can be transformed in

$$1 = 2\sum_{k=0}^{N} a_{2m,k}\cos(\pi k f)\sin(\pi f)^{2M}\sum_{n=-\infty}^{\infty}\frac{(-1)^{kn}}{(\pi(f-n))^{2M}} \qquad (11)$$

The key point now is that the expression

$$\sin(\pi f)^{2M}\sum_{n=-\infty}^{\infty}\frac{(-1)^{kn}}{(\pi(f-n))^{2M}} \qquad (12)$$

is a trigonometric polynomial of order 2M. This follows from the relations (see[1])

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^{n}}{(\pi(f-n))} = \frac{1}{\sin(\pi f)} \qquad (13)$$

and

$$\sum_{n=-\infty}^{\infty}\frac{1}{(\pi(f-n))} = \frac{\cos(\pi f)}{\sin(\pi f)} \qquad (14)$$

by differentiation. Now, again using trigonometric identities, the right side of (11) can be manipulated into a trigonometric polynomial $T_l(f)$, whose coefficients are linear combinations of the $a_{2m,k}$. Since all coefficients of $T_l(f)$ except the first must vanish, a linear system of equations can thus be extracted and solved. This gives the coefficients $a_{2m,k}$ of a Nyquist pulse. Of course, symbolic mathematics software, like Maple, is of great help for this calculations.

The corresponding root-Nyquist pulse is then be found in the Fourier domain by taking the square-root of $sinc^{2m}(f)$ and performing spectral factorization of the trigonometric polynomial $\sum a_{2m,k}\cos(\pi k f)$ in a causal and an anticausal component, by collecting the zeros of the polynomial inside and outside the unit circle. Denoting the coefficients of the causal root-polynomials by $b_{m,k}$, the resulting root-Nyquist pulse has the form

$$\begin{aligned} rp_{2m}(x) &= \sum_{k=0}^{N/2+1} b_{m,k}\{B_m(x-k/2)\} && (15)\\ &\circ\!\!-\!\!\bullet\; sinc^{m}(f)\sum_{k=0}^{N/2+1} 2b_{m,k}\cos(\pi k f) && (16)\end{aligned}$$

and is thus again a linear combination of shifted B-splines, ensuring easy computation.

The $sinc^m(f)$-Term in $RP_{2m}(f)$ assures fast spectral decay of these pulses. It can be shown by methods similar to those in [6] that the RMS-difference between $x(nT_s)$ and $p_{2m}(nT_s)$ using (1) behaves as T^m, which means that small errors are obtained by higher sampling rates or increased m, which means longer pulse duration.

Two remarks are to be made at this point. In the first place, it can be claimed that root-Nyquist pulses of the type considered here exist for every order m. However, this has not yet been proven. In the second place, a closed analytical formula for the coefficients of the Nyquist pulses would be very desirable. On the other side, for use in digital transmission, only pulses with a moderate number of coefficients are of real interest.

As representative examples, the first three root-Nyquist pulses $rp_4(x)$, $rp_6(x)$ and $rp_8(x)$ are shown together with the corresponding Nyquist-pulses and their Fourier-Transform in Fig. 3 for normalized symbol rate $T_s = 1$. The matched filter needs 3,5 and 7 times T/T_s coefficients, respectively. The coefficients are given in the following table:

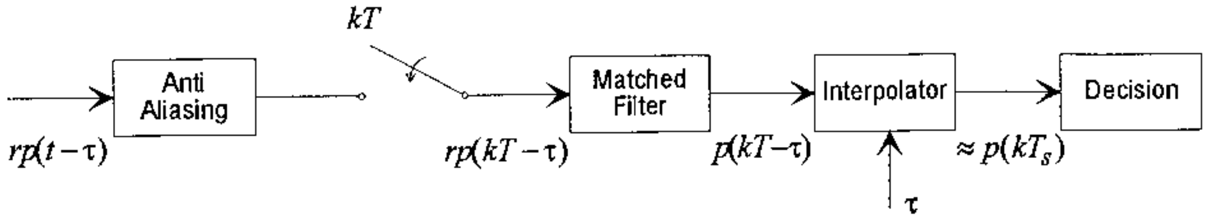

Figure 1: Matched Filter followed by Interpolator

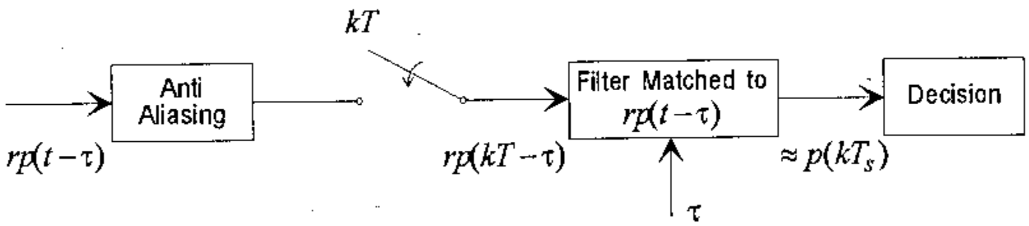

Figure 2: Combined Matched Filter/Interpolator

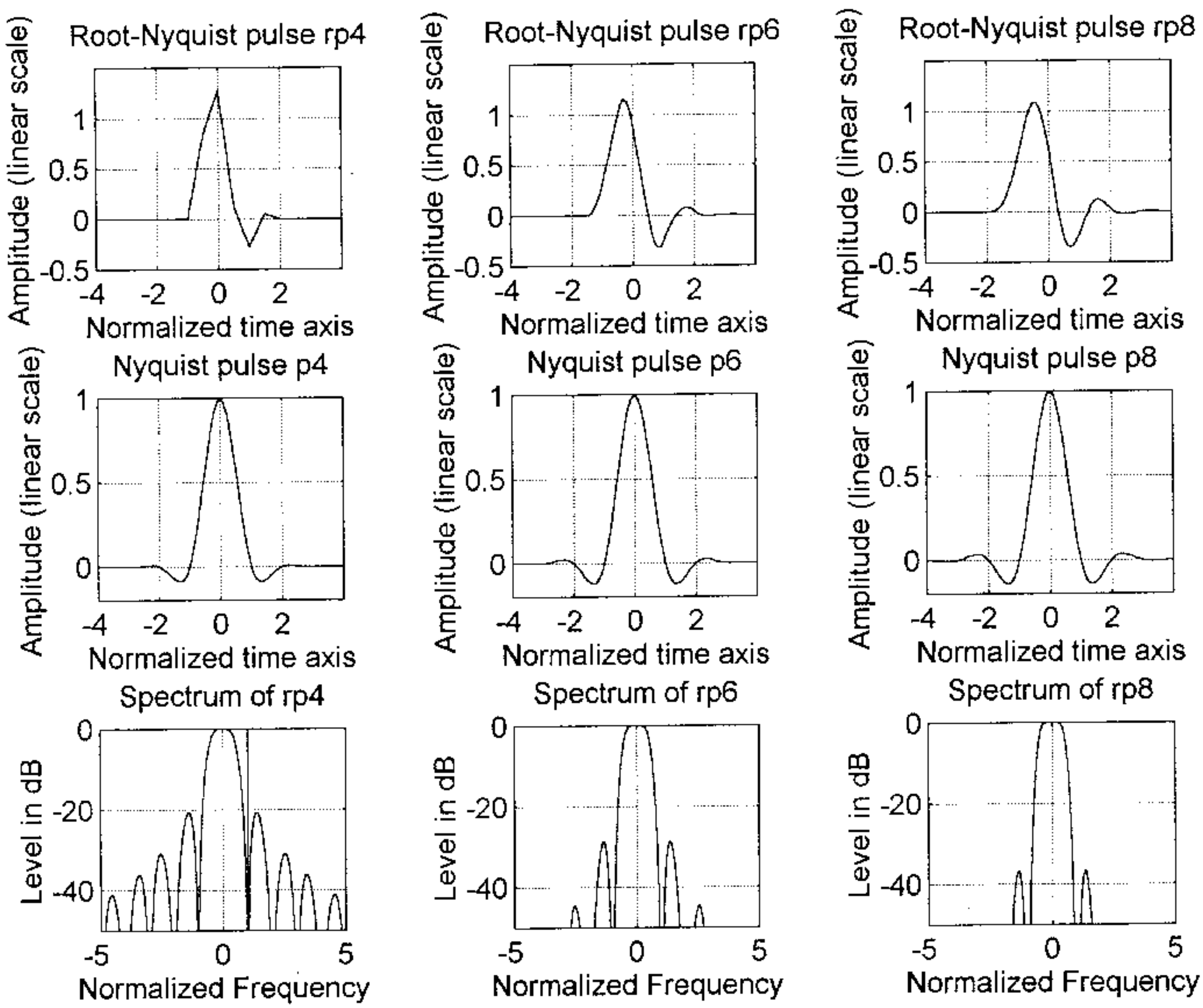

Figure 3: Some Root-Nyquist and Nyquist pulses and Fourier-Transforms

Coeff/Pulse	rp_4	rp_6	rp_8
$b_{m,0}$	1.6578	2.5441	3.9117
$b_{m,1}$	-0.7583	-2.2258	-5.1091
$b_{m,2}$	0.1005	0.7975	3.0013
$b_{m,3}$		-0.1196	-0.9477
$b_{m,4}$		0.0038	0.1532
$b_{m,5}$			-0.0098

4 SIMULATION RESULTS

Simulation results have been obtained for BPSK, assuming additive white gaussian noise and an ideal distortionless channel, except for phase rotation and time delay. The BER has been obtained as an average BER, assuming that the time-shift τ is an identically distributed random variable. The flexible simulation program implements a digital receiver with matched filter, followed by an interpolator, as well a receiver with combined matched filter/interpolator, followed by a decision unit, as in Figures 1 and 2 . Various pulse types can be chosen for transmission. In Figure 4, the average BER for $rp4$ and $rp6$ at sampling rate twice the symbol rate is shown together with the analytical results for BPSK. At sampling rate only twice the symbol rate, rp_4 gives approximately the same performance as a root-cosine pulse with linear interpolator from [2]. Up to a BER of 10^{-6}, for rp_6 and rp_8 no SNR-loss is observed. rp_6 needs less filter coefficients and is therefore superior up to these BERs. For higher BER, analytical and semi-analytical methods are under investigation in order to compute the SNR-loss, because computation time and roundoff-errors make simulation difficult for very high BER.

The conclusion is that special root-Nyquist pulses have been found that allow to incorporate the interpolator in the matched filter for digital ML-receivers, and that they have excellent performance at sampling rates of only twice the symbol rate.

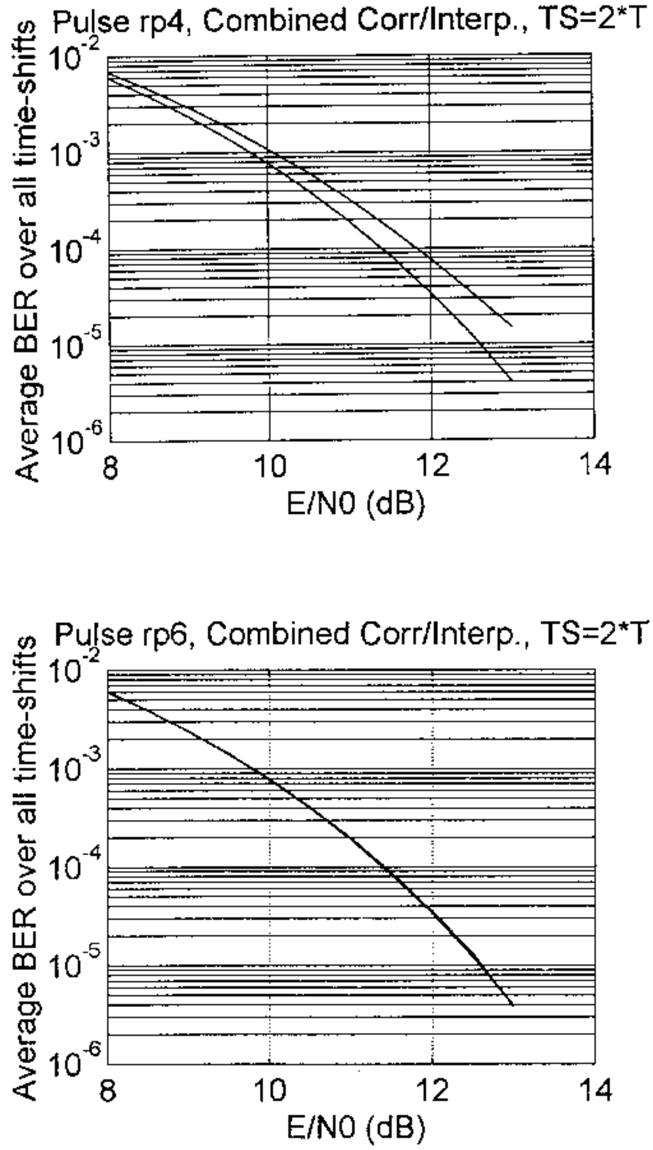

Figure 4: Average BER analytical/simulated

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