WIGNER TRANSFORM INSTANTANEOUS PHASE ESTIMATOR

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ABSTRACT

Computation of an instantaneous phase shift between two real-value signals by means of the Wigner transform is proposed. It is pointed out that the new method is about 37.5% faster than the Fourier transform one while having the similar dynamic accuracy and noise sensitivity for signals with high SNR.

1 INTRODUCTION

The instantaneous phase shift estimation is a crucial problem in different practical areas like telecommunication, power systems and modern measurement techniques. In this case the computational efficiency and dynamic accuracy of the estimators are of great importance. Using Wigner transform for instantaneous phase shift estimation between two real-value signals is proposed in the paper. This approach is more efficient than the initial one [1] utilising the Wigner-Ville transform, since calculation of complex-value analytic signals by means of the Hilbert transform is avoided in it. The new instantaneous phase shift estimation method is also significantly faster than alternative Fourier transform based approach while preserving similar dynamic accuracy for signals with high SNR.

2 PROBLEM DESCRIPTION

Efficient estimation of instantaneous phase shift (IPS)

$$\Delta\phi(n)=\phi_2(n)-\phi_1(n) \tag{1}$$

between two real-value discrete signals

$$x_1(n)=a_1(n)\cos\left(2\pi\, n\, f_c\,/\,f_s+\phi_1(n)\right), \tag{2}$$

$$x_2(n)=a_2(n)\cos\left(2\pi\, n\, f_c\,/\,f_s+\phi_2(n)\right), \tag{3}$$

is addressed in the paper. $a_1(n)$ and $a_2(n)$ denote amplitudes slowly varying in time while f_c and f_s are carrier and sampling frequencies, respectively. $\Delta\phi(n)$ can be estimated using many different methods among which the most popular are Fourier transform ones.

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3 FOURIER TRANSFORM METHOD

The discrete instantaneous phase shift estimate is calculated in the short-time Fourier transform method with the following formulae (m=–M/2,...,0,...,M/2):

$$\Delta\hat{\phi}_{FT}(n)=\hat{\phi}_2(n)-\hat{\phi}_1(n)= \tag{4}$$

$$=\arg\left\{\underset{(f_c)}{\overset{(m)}{\mathrm{DFT}}}\left(w(m)x_2(n+m)\right)\right\}-\arg\left\{\underset{(f_c)}{\overset{(m)}{\mathrm{DFT}}}\left(w(m)x_1(n+m)\right)\right\}$$

$$=\arg\left\{\underset{(f_c)}{\overset{(m)}{\mathrm{DFT}}}\left(w(m)x_2(n+m)\right)\cdot\left[\underset{(f_c)}{\overset{(m)}{\mathrm{DFT}}}\left(w(m)x_1(n+m)\right)\right]^*\right\}$$

where the discrete Fourier transform (DFT) is computed over variable m only for the carrier frequency f_c, $w(m)$ denotes a symmetric time window ($w(m)=w(-m)$, $w(m)\neq 0$ for $|m|\leq M/2$ and $w(m)=0$ elsewhere), "*" represents complex conjugation and "arg" denotes argument of complex data. The detailed algorithm implementing (4) is given in table 1.

4 WIGNER TRANSFORM METHOD

The instantaneous phase shift computation using the short-time Wigner transform is proposed in this paper and it is defined as (m=–M/2,..., 0,...,M/2):

$$\Delta\hat{\phi}_{WT}(n)=\arg\left\{\underset{(2f_c)}{\overset{(m)}{\mathrm{DFT}}}\left(w(m)x_2(n+m)x_1^*(n-m)\right)\right\}. \tag{5}$$

The same notation is used in (4) and (5). The advantage of our present proposition lies in its computational efficiency. In contrary to [1] the analytic signal computation by means of the Hilbert transform as well as complex-value calculations are not necessary at present (see table 2). Features of the estimators (4) and (5) are compared in the following sections.

Table 1

Algorithm of the short-time Fourier transform IPS estimator.

- Calculation of sine/cosine analysis *wavelets* $w_s(m)$ and $w_c(m)$ ($w(m)$ - time window function):

$$w_s(m) = w(m) \cdot \sin\left(2\pi\, m \cdot f_c \,/\, f_s\right)$$
$$w_c(m) = w(m) \cdot \cos\left(2\pi\, m \cdot f_c \,/\, f_s\right)$$

- Convoluting the *wavelets* $w_s[m]$ and $w_c[m]$ with analysed signals $x_1[n]$ and $x_2[n]$:

$$x_1^s(n) = \sum_{m=-M}^{M} x_1(n+m) w_s(m)\,, \quad x_1^c(n) = \sum_{m=-M}^{M} x_1(n+m) w_c(m)$$
$$x_2^s(n) = \sum_{m=-M}^{M} x_2(n+m) w_s(m)\,, \quad x_2^c(n) = \sum_{m=-M}^{M} x_2(n+m) w_c(m)$$

- Calculation of the instantaneous phase shift estimate:

$$\Delta\hat{\phi}_{FT}(n) = \tan^{-1}\left(\frac{x_2^s(n) x_1^s(n) + x_2^c(n) x_1^c(n)}{x_2^s(n) x_1^c(n) - x_2^c(n) x_1^s(n)} \right)$$

Table 2

Algorithm of the short-time Wigner transform IPS estimator.

- Calculation of sine/cosine analysis *wavelets* $w_s(m)$ and $w_c(m)$ ($w(m)$ - time window function):

$$w_s(m) = w(m) \cdot \sin\left(2\pi\, m \cdot 2\, f_c \,/\, f_s\right)$$
$$w_c(m) = w(m) \cdot \cos\left(2\pi\, m \cdot 2\, f_c \,/\, f_s\right)$$

- Convoluting the *wavelets* $w_s[m]$ and $w_c[m]$ with analysed signals $x_1[n]$ and $x_2[n]$:

$$m=-M,\ldots,0,\ldots,M: \quad x_{21}(n,m) = x_2(n+m) x_1(n-m)\,;$$

$$x_{21}^s(n) = \sum_{m=-M}^{M} x_{21}(n,m) w_s(m)\,, \quad x_{21}^c(n) = \sum_{m=-M}^{M} x_{21}(n,m) w_c(m)$$

- Calculation of the instantaneous phase shift estimate:

$$\Delta\hat{\phi}_{WT}(n) = \tan^{-1}\left(\frac{x_{21}^s(n)}{x_{21}^c(n)} \right)$$

Table 3

Comparison of computational complexity of the short-time Fourier and the short-time Wigner instantaneous phase shift estimation methods (number of real-value operations per one phase shift value: multiplications, additions, subtractions, divisions and function arc tangent). N=2M+1.

Operation	**Fourier**	**Wigner**
mult	4N+4	3N
add / sub	4N-2	2N-2
div	1	1
$\tan^{-1}$	1	1

5 METHODS COMPARISON

5.1 Computational Efficiency

Let us compare the computational complexity of both methods: Fourier and Wigner. Number of arithmetic operations required for calculation of one discrete instantaneous phase value using both methods is given in table 3. Taking into account the fact that multiplication and addition (subtraction) are performed in one instruction cycle in contemporary digital signal processors we can conclude that the Wigner method is at least 37.5 % faster than the Fourier one.

5.2 Dynamic Accuracy

The following two signals $x_1(n)$ and $x_2(n)$ have been analysed:

$$x_1(n) = \cos\left(2\pi \frac{f_c}{f_s} n\right), \tag{6}$$

$$x_2(n) = \cos\left(2\pi \frac{f_c}{f_s} n + \frac{17}{18}\pi \cdot \sin\left(2\pi \frac{f_m}{f_s} n\right)\right), \tag{7}$$

where carrier frequency f_c=20 kHz, phase modulation frequency f_m=0÷2 kHz and sampling frequency f_s= 200 kHz (signals length = 2000 samples; 17π/18 rd = 170 deg). As the time envelope $w(m)$ the 13-point Kaiser window [2] with β=6.6 has been used (M=6; N=2M+1=13). Figure 1 presents observed maximal absolute instantaneous errors of estimators (4) and (5) for different values of the phase modulation frequency f_m. The similar dynamic accuracy of both methods is observed.

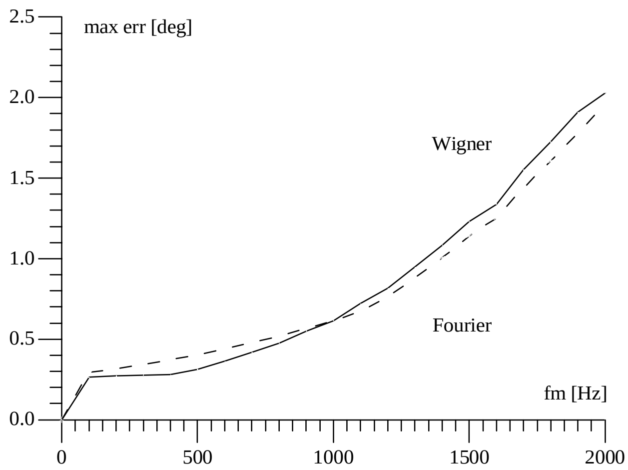

Fig. 1. Dynamic accuracy of the analysed instantaneous phase shift estimators.

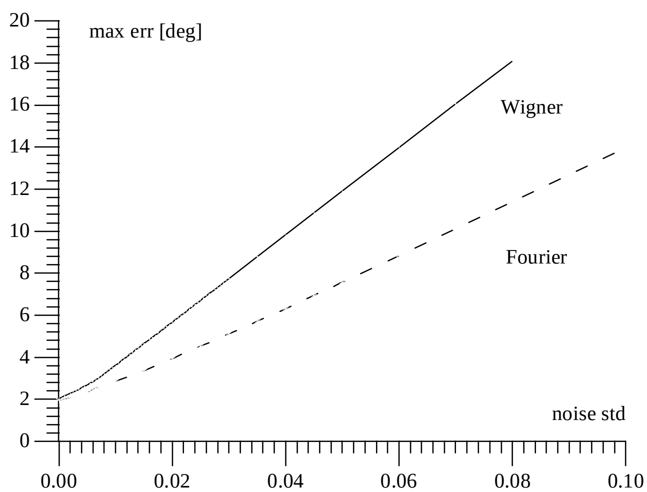

Fig. 2. Noise sensitivity of the analysed instantaneous phase shift estimators.

Increasing both the carrier and sampling frequencies in (6)(7) one can decrease the estimation error for Fourier and Wigner transform methods. For instance for the frequencies: f_m=2 kHz, f_c= 40 kHz, f_s=400 kHz the observed maximum absolute error of the Wigner estimator is equal to 0.6 deg and for frequencies: f_m=2 kHz, f_c= 80 kHz, f_s=800 kHz - only 0.3 deg.

5.3 Noise Sensitivity

In this case both test signals (6) and (7) with modulation frequency f_m=2 kHz have been both embedded in uncorrelated Gaussian noise. Figure 2 depicts the observed maximal absolute instantaneous error of estimation for different standard deviation (std) of corrupted disturbances (only one realisation of noise). Estimation errors for std=0 can be interpreted as pure *dynamic* ones and they are the same as in figure 1.

For different noise realisations the maximum error has varied from 16 to 24 deg for Wigner and from 10 to 16 deg for Fourier transform estimator. In the presence of noise maximum error of the dynamic characteristic (figure 1) have been observed for different modulation frequencies.

5.4 The Other Methods

Next we have extended the area of interest and tested several different IPS methods [1] in the same conditions as before (signals (6)(7) embedded in noise). The achieved results are summarised in figure 3. Their interpretation and discussion can be found in [1]. It can be briefly concluded that there exist IPS estimators with higher dynamic accuracy and lower noise sensitivity than the Wigner transform one but generally they are more complex computationally.

6 CONCLUSIONS

The new Wigner transform method of instantaneous phase shift estimation is about 37.5% faster in numerical implementation than the short-time Fourier transform one. Both methods have similar dynamic accuracy (figure 1) but the Wigner algorithm is more noise sensitive (figure 2). For relatively high SNR its application is still profitable.

REFERENCES

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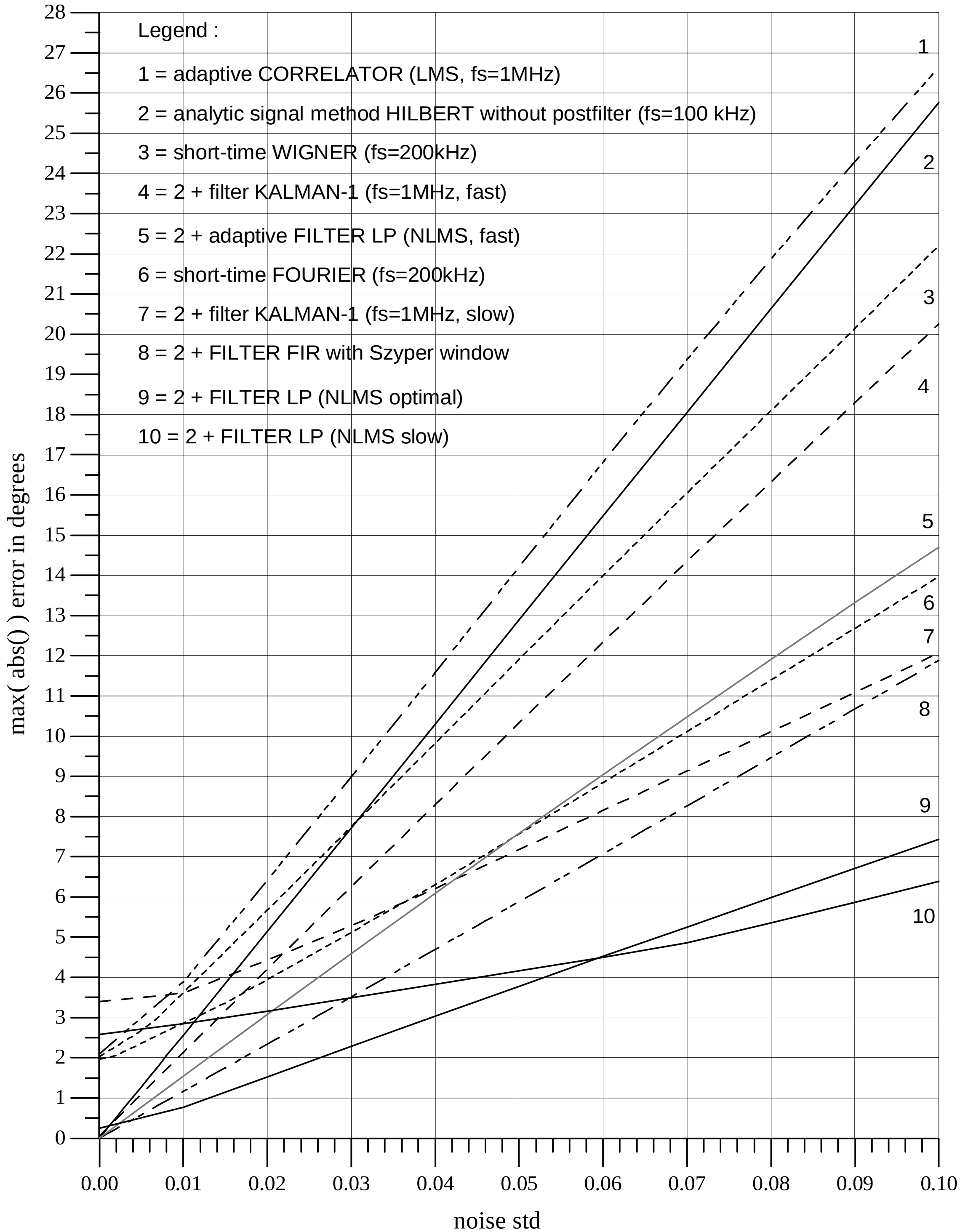

Fig. 3. Noise sensitivity comparison of several IPS estimators: Fourier, Wigner and the others described in [3] (f_S denotes required sampling frequency).