SUBBAND DECOMPOSITION BASED ON THE HILBERT TRANSFORM APPLIED TO RADAR IMAGING

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ABSTRACT

In this paper, we propose an approach to improve high-resolution frequency estimation for narrow-band planes. This approach is based on a signal preprocessing combined with a high-resolution method to increase the accuracy of frequency estimation. The preprocessing step is a Subband Decomposition Based on the Hilbert Transform (SDBHT) [1] for one and two-dimensional signals. This improvement is achieved by using an empirical criterion to determine the number of waves of the signals derived from the SDBHT technique. Simulation examples show the performances of this criterion. Then, we apply SDBHT method and empirical criterion to radar imaging.

1 INTRODUCTION

In numerous applications such as radar or sonar, signals are analysed by high-resolution methods. However, high-resolution frequency estimation is not accurate enough when the signal-to-noise ratio (SNR) is low or in case one has only few points of the signal. In this paper, we report the combination of the SDBHT preprocessing and a high-resolution technique to improve frequency estimation. This preprocessing is a spectral subband decomposition which can be applied to one-dimensional (1D) and two-dimensional (2D) signals. This decomposition based on the Hilbert transform [1, 2] generates signals we down-sample by two to increase the accuracy of frequency estimation.
An additional problem in frequency analysis is the determination of the number of waves. Indeed, most high-resolution methods need the knowledge of the number of frequency components. In order to estimate this parameter, we propose an empirical criterion suited to the signals derived from a SDBHT preprocessing.
We first report a review on the SDBHT technique. Secondly, we develop an empirical criterion for number of waves determination. Then we provide simulation examples which illustrate its performances. Finally, we present an application of the SDBHT preprocessing and the empirical criterion to radar imaging.

2 SUBBAND DECOMPOSITION BASED ON THE HILBERT TRANSFORM

The purpose of this preprocessing is to subsample by two the signal to increase the accuracy of frequency estimation.

2.1 ONE-DIMENSIONAL SIGNALS

We consider the noisy exponential data model as follows:

$$\begin{cases} x(n) = \sum_{i=1}^{M} a_i \exp\left(j\phi_i + b_i(n)\right) + v(n) \\ b_i(n) = \alpha_i n + j2\pi f_i n \end{cases} \quad (1)$$

where $v(n)$ is a white Gaussian noise process.
We split up the spectral domain of the signal into two parts, the positive and the negative frequency intervals, by a subband decomposition based on the Hilbert transform **[step A]**. However, this decomposition does not suit to a signal with a DC component, because this latter belongs to both positive and negative frequency intervals. So, if a criterion [1] based on the entropy [3] of the Fourier transform detects a DC component, the signal is frequency modulated to be zero mean.
Fig. 1 presents the operational diagram of the step A.

imag[s]	imaginary part of s
HT[s]	Hilbert transform of s
analytic[s]	analytic signal of s
conj	conjugate operator
T	transpose operator
M	frequency modulation (if necessary)

Table 1. Legend of Fig. 1 and 3.

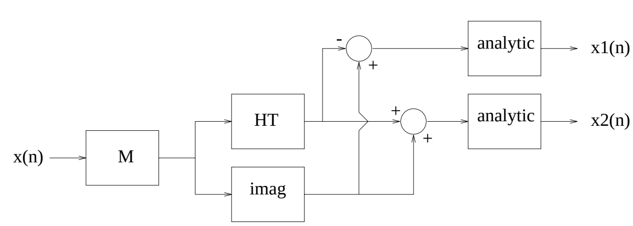

Fig. 1. x1 and x2 are related to the negative and positive spectrum intervals from x.

We have therefore generated two signals defined on the normalized frequency band [0;0.5]. They are down-sampled after a suitable modulation to center their spectra **[step B]**.
Then, they are analysed by a high-resolution method and their estimated frequencies are put back in their original spectral band **[step C]**.

2.2 TWO-DIMENSIONAL SIGNALS

The noisy exponential data model is now the following:

$$\begin{cases} x(n,m) = \sum_{i=1}^{M} a_i \exp\left(j\phi_i + b_i(n,m)\right) + v(n,m) \\ b_i(n,m) = \alpha_{1i} n + \alpha_{2i} m + j2\pi(f_{1i} n + f_{2i} m) \end{cases} \quad (2)$$

where $v(n,m)$ is a white Gaussian noise process.
The 2D preprocessing is composed of three steps (A,B and C), as for the 1D case.
In the step A, the signal spectrum is split up into four quadrants, as reported in Fig. 2. To avoid some signal components belong to several quarter planes, a criterion [1] based on the entropy of the Fourier transform determines if frequency components are located on the axis. If necessary, the signal is modulated to cancel this configuration.
Thus, in order to extract the four quadrants, we apply to the image the routines given in Fig. 3. We notice that the 2D Hilbert transform, easily obtained from the 1D Hilbert transform, operates on a single frequency axis, i.e. the vertical axis here.

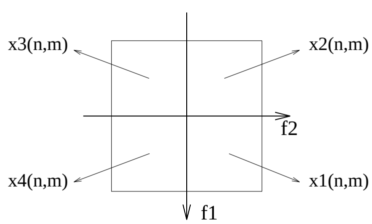

Fig. 2. The four quadrants to extract in the 2D case.

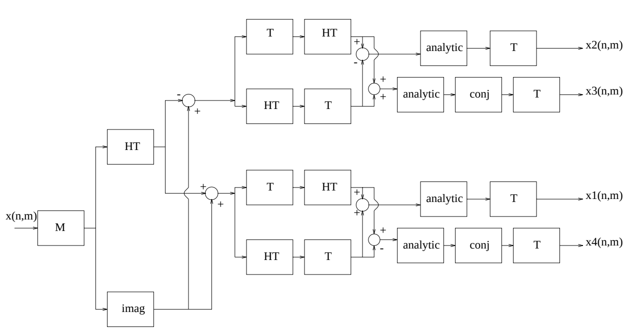

Fig. 3. Operational diagram to extract x1, x2, x3 and x4 from x.

Finally, the two steps B and C described in section 2.1 are directly extended to the two-dimensional case.

3 ESTIMATION OF THE NUMBER OF FREQUENCY COMPONENTS

The subband decomposition generates several secondary signals. A criterion such as MLM [4] gives good results in estimating the original signal number of components. However, it is inefficient in case of secondary signals, due to filtering with the Hilbert transformer [1, 2]. So we propose an empirical criterion for 1D and 2D signals derived from a SDBHT preprocessing.

Let define $\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_L$ as the eigenvalues of the estimated covariance matrix. These eigenvalues are ordered as follows:

$$\widehat{\lambda}_1 > \widehat{\lambda}_2 > \ldots > \widehat{\lambda}_L. \quad (3)$$

K. Konstantinides [5] developed a family of empirical criteria such as:

$$\frac{\widehat{\lambda}_M}{\widehat{\lambda}_1} > \delta > \frac{\widehat{\lambda}_{M+1}}{\widehat{\lambda}_1}, \quad (4)$$

where M is the number of components and δ a certain threshold.
We introduce the vector V defined by:

$$V = \left[\widehat{\lambda}_1\ \widehat{\lambda}_2\ \ldots\ \widehat{\lambda}_L\right] / \widehat{\lambda}_1, \quad (5)$$

and $\bar{v}$, mean of V.
We propose the following empirical choice of δ:

$$\delta = \bar{v}. \quad (6)$$

4 SIMULATIONS

4.1 ONE-DIMENSIONAL CASE

The procedure of the first simulation is defined by Table 2. The percentages of number of waves detection by MLM and empirical criteria (see section 3) are plotted in Fig. 4 for the original signal (a) and the associated secondary signals (b,c).

frequencies	-0.25	-0.24	0.36	0.37
damping factors	0.008	-0.0009	0.005	0.002
magnitudes	5	7	4	2
initial phases (rd)	1.7	2.3	5.1	0.7
montecarlo runs: 100				
SNR: -5 to 45 dB by step of 1 dB				
size of the data matrix: 32 × 32				

Table 2. First simulation.

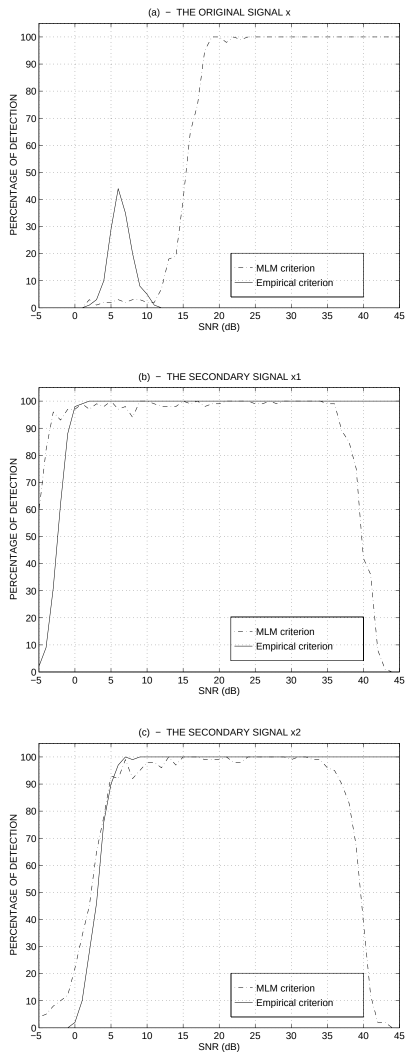

Fig. 4. Estimation of the number of frequency components (1st simulation).

MLM operates better than the empirical criterion for the original signal, but the empirical criterion gives the better results for the signals derived from the SDBHT preprocessing. Moreover, we notice the more reliable estimation of the number of frequency components is obtained by using the SDBHT technique with the empirical criterion.

The second simulation conditions are listed in Table 3.

frequencies: f_1=0.2, f_2=0.4 and f_3: 0.21 to 0.39 by step of 0.01
damping factors: $\alpha_1 = \alpha_2 = \alpha_3 = 0$
magnitudes: a_1=a_2=a_3=1
initial phases (rd): $\phi_1 = \phi_2 = \phi_3 = 0$
montecarlo runs: 100
SNR: -5 to 45 dB by step of 1 dB
size of the data matrix: 32×32

Table 3. Second simulation.

The number of components of the signal x is estimated by MLM criterion and results are given in Fig. 5 (a). Its secondary signal x1 has no frequency components. So the empirical criterion described in section 3 is applied only to the secondary signal x2, and Fig. 5 (b) presents the percentages of detection.

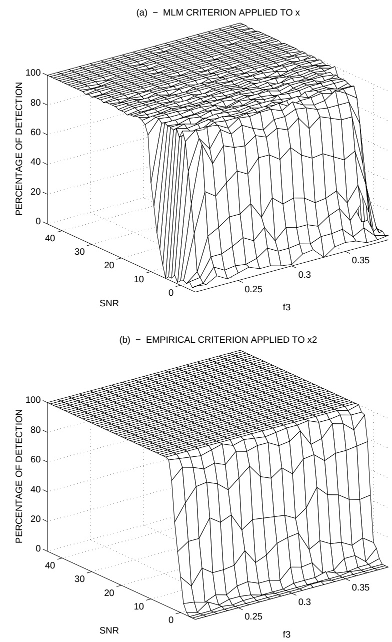

Fig. 5. Estimation of the number of frequency components (2nd simulation).

The SDBHT preprocessing and the empirical criterion increase the percentages of number of components detection obtained with the MLM criterion.

4.2 TWO-DIMENSIONAL CASE

Due to short space available, we will not report 2D results, but we draw the same conclusions from 1D and 2D simulations. The best way of estimating the number of frequency components is the association of the SDBHT preprocessing with the empirical criterion.

5 APPLICATION TO RADAR IMAGING

The target is a simplified missile, as represented in Fig. 6.

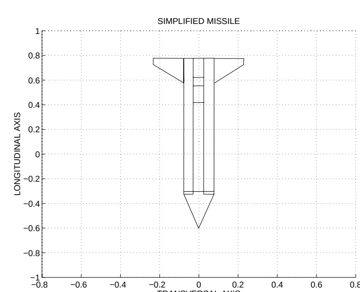

Fig. 6. Target geometry.

First, the Fourier transform of the radar signal is plotted in Fig. 7.

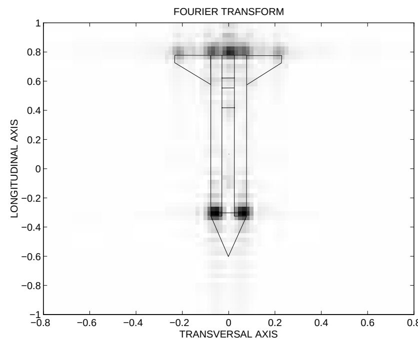

Fig. 7. Fourier transform of the radar image.

Then, we estimate the frequencies of the radar signal by a high-resolution technique which is the MEMP method [6]. This method needs the knowledge of the number of waves.
So, we process the radar signal by two approaches:
- MLM criterion and MEMP method,
- SDBHT preprocessing, empirical criterion and MEMP method.

Results are given in Fig. 8.
When the SBBHT preprocessing is used, we notice that radar signal is frequency modulated to have no frequency components located on the axis. After this modulation, only two quadrants own components, so the MEMP method is applied to only two secondary signals.

The preprocessing improves the high-resolution analysis of the radar signal. Furthermore, it shows up some frequency components which are not clearly detected by the Fourier transform.

6 CONCLUSION

We presented in this paper a method to improve high-resolution frequency estimation. This method begins with a data preprocessing by the Subband Decomposition Based on the Hilbert Transform. Thus we obtain several signals. We developed an empirical criterion to determine the number of frequency components of these signals. Finally, we analyse them by a high-resolution technique.
This method increases the accuracy on frequency estimation and number of waves determination. Furthermore it improves the resolution of spectral analysis.

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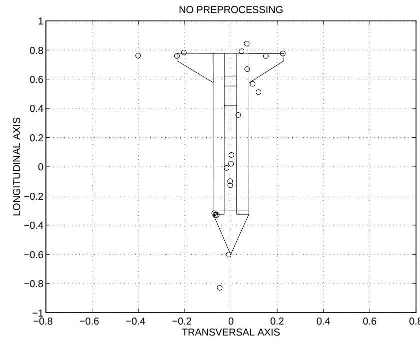

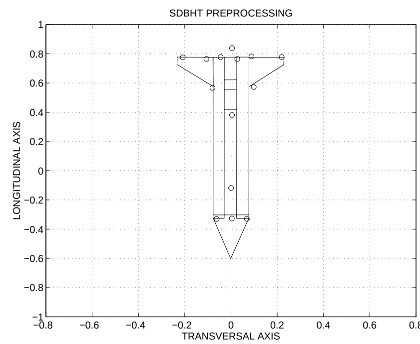

Fig. 8. Frequency estimation of the radar image by the MEMP method.

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