ABSTRACT
Per-Survivor Processing is a general approach which includes in the survivors of the Viterbi Algorithm trellis, the relative estimation of unknown parameters; this expensive method better approximates the optimum decoder in certain conditions. The method is applied to the case of a typical HF channel and a simplification is proposed, based on a per survivor equalizer, to be employed when selective fading is present. This solution, although increasing the per survivor cost, greatly reduces the number of states of the Viterbi decoder.

1 INTRODUCTION
The Viterbi algorithm (VA) is the well known optimum decoder in the presence of both Gaussian noise and Intersymbol Interference (ISI). To operate, the VA requires the knowledge of the overall channel impulse response (CIR), which is usually computed using an adaptive estimator driven by either least squares (LS) or least mean squares (LMS) algorithms.

Both LS and LMS algorithms are updated by mean of the supposed knowledge of the transmitted data, the usual solution is to take the most probable data at the first step of the VA trellis. This is affected by some errors whose contribution can be negligible if the bit error rate (BER) is low enough. An alternative solution uses the final decision of the VA to drive the channel estimator combined with some kind of prediction to compensate the delay introduced by the decoder [1].

Both solutions are valid when both the BER is low and the channel is affected by a limited Doppler spread. When the channel Doppler spread is large, the LMS estimate algorithm requires a large step size to track the channel variations. As a consequence, the averaging effect of the step size is reduced and even a limited number of errors can lead to the failure of the estimator. Using LS algorithms partially overcomes the problem, but poorer tracking performance results at higher fading rates [2][3].

In all those extreme cases the optimum solution, to approach the theoretical performance, is to introduce a different channel estimator in each survivor of the VA trellis, of course resulting in a computational heaviness. That method, Per-Survivor Processing (PSP), has been independently introduced, recently, by different authors in different application fields [4][5][6].

In this paper, the PSP method is applied to the case of a typical HF channel, affected by both multipath and Doppler spread, and solutions are proposed which considerably reduce the complexity of the PSP decoder. Although the system has been developed for HF communications, it is well suited whenever selective fading is present.

A per-survivor linear equalizer is introduced in the algorithm, to shape the CIR to a shorter Desired Impulse Response (DIR); this increases the complexity associated with the single survivor, but considerably reduces the number of states with minor variations in the noise level.

The structure of the linear equalizer is explained in section 2; section 3 describes the entire modified Viterbi decoder; simulation results are presented in section 4 and, the conclusions are drawn in section 5.

2 THE EQUALIZER
Optimum filtering for data communications is represented by the Nyquist filter, with impulse response $Z=\{z_i\}$, which ensures optimum band occupation of the transmitted signal without ISI, when properly sampled. One half Nyquist filter, with impulse response $Q=\{q_i\}$ in order to have $Q\otimes Q=Z$, ($\otimes$ denotes convolution) is usually positioned on the transmitter, while the other half filter is on the receiver. Considering a channel with CIR $H=\{h_i\}$, the overall impulse response, $F=\{f_i\}$, becomes $F=Z\otimes H$. To compensate for the channel, the receiver is usually equipped with an adaptive linear equalizer, with impulse response $C=\{c_i\}$, such that $C\otimes H=Q$.

When multipath is present, the CIR is the sum of $m$ different independent contributions:

$$H = \sum_{x=1}^{m} A_x$$
where $A_e = \{a_{e,i}\}$ represents the time varying impulse response of the single ionospheric path. The $A_e$ usually present a limited time dispersion and the relatively disperse CIR is mainly due to the mismatching of the overall component $A_e \otimes Z$.

The equalizer, with impulse response $C$, is introduced to compensate for the mismatching, the DIR becoming:

$$\text{DIR} = Z \otimes (\sum_{e=1}^{m} A_e) \otimes C,$$

where the equalizer $C$ attempts to find the best matching filter for all the different ionospheric paths, this approach is similar to the equalizer used in single path fading channels. It must be pointed out that, in the multipath case, where the relative distance between the different paths has no reason to be a multiple of the sampling period; a larger amount of ISI has to be compensated, requiring longer equalizers than the single path case.

It is worth noting that the required equalizer does not attempt to compensate for nulls in the channel transfer function, but only reduces the energy in the tails of the CIR: it is an almost all-pass filter and introduces only minor variations in the noise level. Simulations show that the noise enhancement is in the order of 1 dB. That kind of equalizer, follows the scheme described in [7] and is described hereinafter for better understanding.

In HF communications, due to the presence of periodical training data, the CIR is estimated and analyzed during training phases; different applications could require a continuous monitoring of the CIR to modify the DIR and the equalizer when necessary. The equalizer length is a parameter affecting the overall performance of the system, it also depends on the choice of the most significant part of the CIR. When attempting a consistent reduction of the trellis size, only few significant coefficient of the CIR is selected. In this case, the large amount of residual ISI to compensate requires higher order equalizer and the input signal at the equalizer shows larger eigenvalue spread.

The system has been simulated in the case of two different choices for the significant part of the CIR, using 4, 6 and 8 taps equalizers, to show the dependence of the performance on the equalizer length.

If the signal is properly filtered and sampled at the symbol rate, the complex envelope of the received signal $R = \{r_n\}$ at the input of the decoder is given by:

$$r_n = \sum_{p=0}^{L} f_p(n) x_{n-p} + w_n,$$

where $\{w_n\}$ represents white Gaussian noise of variance $\sigma_w^2$, independent of data and $X = \{x_n\}$ is the 8PSK symbol sequence. The useful signal is:

$$y_n = \sum_{p=0}^{L} f_p(n) x_{n-p},$$

In the usual VA the definition of state is:

$$s_n = [x_n, \ldots, x_{n-L+1}],$$

assuming the overall CIR is known, the useful signal $y_n$ is a function of both the symbol at present time $x_n$ and the state at previous time $s_{n-1}$:

$$y_n = y(x_n, s_{n-1}),$$

and the VA attempts to minimize the overall cost of the sequence:

$$J_n = \sum_{i=0}^{n} [y_i - y_i]^2, n \to \infty.$$
The overall decoder requires the following sequence of operations be executed at each step:

**A.** Filtering of the corresponding input signal for each survivor: \( r_{n,q} = R(n) \otimes C_q(n-1) \)

**B.** Computation of the branch metrics for possible values of the signal set: \( e_{n,q}(x_n) = |y_{n,q}^* - r_{n,q}|^2 \)

**C.** Selection of the least cost survivor for each state: \( s_n \)

**D.** Updating of the channel estimator for each survivor:
\[
G(s_n) = G(s_{n-1}) + \mu e_n X^*(n)
\]

**E.** Updating of the equalizer for each survivor:
\[
C(s_n) = C(s_{n-1}) + \mu e_n R^*(n)
\]

The described procedure, as specified in the previous section, requires the selection of the significant coefficients of the CIR during the initial training, then at step D only the selected coefficients are updated. It is clearly evident that the structure associated with each survivor is exactly like a Decision Feedback Equalizer (DFE).

4 SIMULATION RESULTS

The signal used for simulations consists of i.i.d. data, with periodic retraining, transmitted over a 3kHz voiceband channel, using 8PSK modulation scheme and 2400 symbols per second. At the receiver the signal is supposed to be correctly sampled at the symbol rate. Both the transmitter and the receiver include a square root raised cosine filter, roll-off = 0.2. The channel used in all simulations consists of two independent Rayleigh fading paths, with same average magnitude, 1Hz Doppler spread each and 1ms relative delay. The time dispersion of the considered channel is over 4 samples, but, due to the mismatching of the Nyquist filters, the overall CIR has a significant magnitude over 8-9 symbols.

Figure 1 reports the symbol error rate for different SNR, obtained using a Viterbi decoder with 8^5 states and 8 taps linear equalizer when a first step decision is used for channel estimate and equalizer, compared with the results obtained by the same decoder assuming the perfect knowledge of the CIR.

In the same figure, the curve c shows the results obtained using a Viterbi decoder with 8^3 states and discarding the energy in the initial and final tails.

In figure 2, the curve (a) is the same VA decoder with 8^5 states and 8 taps linear equalizer as in previous figure, the curve (b) reports the results obtained with the modified VA, when the equalizer is introduced to get a DIR length of 4 coefficients and only 8^3 states are used. The curve (c) reports the results obtained when considering DIR length of 5 coefficients, the same trellis as in the previous example (8^3 states), and the 5th coefficient’s contribution is fed back, based on a tentative decision at 4th step of the PSP trellis.

In figure 3, the curve (a) is the same VA decoder with 8^5 states and 8 taps linear equalizer as in previous figure, the curve (b) reports the results obtained with the modified VA, when the equalizer is introduced to get a DIR length of 4 coefficients and only 8^3 states are used. The curve (c) reports the results obtained when considering DIR length of 5 coefficients, the same trellis as in the previous example (8^3 states), and the 5th coefficient’s contribution is fed back, based on a tentative decision at 4th step of the PSP trellis.
All previous PSP decoders have been evaluated using 8 taps equalizer, figure 3 and figure 4 show the results obtained with the two PSP decoders of figure 2, for different equalizer lengths: 4, 6 and 8 taps. In both figures the 4 taps curve is the worst one and the 8 taps curve is the best one.

In the system of figure 3 a large amount of ISI has to be compensated and with the 4 taps equalizer performance is degraded. The equalizer in the system of figure 4 has an easier work. In fact the overall performance with 6 and 8 taps equalizers is always better than in the previous case, 4 taps is still unsatisfactory.

5 CONCLUSIONS
The performed work confirms the effectiveness of the PSP in approximating the optimum solution, when unknown parameters have to be estimated to decode data, like in the case of HF communication.

The proposed structure extends the principle of "Per-Survivor Processing" to both channel estimate and equalization of the incoming signal. The combined use of the preequalizer and channel estimator, although suffering for the intrinsic computational heaviness of the PSP method, reduces the dispersion of the CIR and, consequently, the number of states in the trellis.

The overall performance of the proposed method better approaches the optimum decoder, especially at higher SNR levels, that is when the dominant part of the errors is due to the contribution of the ISI.

REFERENCES