THE SYNTHESIS OF A HIGH ORDER DIGITAL BANDPASS FILTERS WITH TUNABLE CENTRE FREQUENCY AND BANDWIDTH*)

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ABSTRACT

In this paper, described tunable digital bandpass filters whereby the centre frequency and bandwidth can be independently related to the multiplier coefficients, which permit simple frequency response adjustment by varying the coefficients values. The bandpass filters proposed here have a cascade form and are composed of several second-order recursive bandpass sections with identical characteristics. The methods for the direct computation of the number of second-order filters in the cascade form, adjustable parameters and designing filter bank are shown in this paper. The design equations strate the true parametric tuning ability of the circuit. By cascading a few such circuits, a complete parametrically adjustable digital frequency responsed equalizer may be realized. It does not require precomputing the multiplier coefficient values for all designed equalizer settings.

1 DESIGN CONSIDERATIONS

Design procedures for variable cutoff frequency digital filters are well known [1-6]. This paper presents the method of designing high order digital bandpass filters whereby simply changing the multiplier number values.

Let us calculate the coefficients of a tunable bandpass digital filter in the cascade form with the following specifications: centre frequency ω_0, 3dB bandwidth $\Delta\omega$ and m dB bandwidth $\Delta\omega_m$ for cutoff frequency. The cascade structure of bandpass digital filter is composed of several second-order recursive bandpass sections (filters) with identical characteristics. Let K be the coefficient which characterizes the steep of the transfer function of a cascade bandpass filter and defined as $K = \Delta\omega_m / \Delta\omega$. Obviously, $K > 1$, and an ideal bandpass filter has $K = 1$ for $m \neq 0.5$.

Let $A^2(\omega)$ be a square of magnitude response of a cascade structure digital filter, and $A_s^2(\omega)$ a square of magnitude response of a second-order section, then $A^2(\omega) = [A_s^2(\omega)]^L$, where L is the number of second-order sections in the cascade form of digital filter.

The result of designing will be the specifications of a second-order section, such as quality factor q_s and coefficient of specification form $K_s = \Delta\omega_m / \Delta\omega_s$, and the number L of the second-order sections of the cascade filter structure, where $\Delta\omega_s$ is the bandwidth of second-order section.

2 TUNABLE DIGITAL SECOND - ORDER BANDPASS FILTER

The transfer function of an analog bandpass second-order filter is:

$$H(p) = \frac{(\Omega_0 / Q) p}{p^2 + (\Omega_0 / Q) p + \Omega_0^2} = \frac{\Delta\Omega p}{p^2 + \Delta\Omega p + \Omega_0^2},$$

where Q - quality factor of an analog filter; Ω_0 - bandpass centre frequency; $\Delta\Omega$ - 3dB bandwidth of a bandpass filter; p - Laplace transform variable.

The transfer function of a bandpass digital filter with bandwidth $\Delta\omega_s$ and centre frequency ω_0 using a bilinear transformation $p = 2(1 - z^{-1}) / \Delta t (1 + z^{-1})$ is defined as

$$H(z) = a_0 \frac{1 - z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}, \qquad (1)$$

where Δt - sampling period; a_0, b_1, b_2 are digital filter coefficients, which are defined as:

$$\begin{cases} a_0 = \dfrac{\sin\omega_0\Delta t}{2Q + \sin\omega_0\Delta t}, \\ b_1 = -\dfrac{4Q\cos\omega_0\Delta t}{2Q + \sin\omega_0\Delta t}, \\ b_2 = \dfrac{2Q - \sin\omega_0\Delta t}{2Q + \sin\omega_0\Delta t}. \end{cases} \qquad (2)$$

Let q_s is a quality factor of a digital filter and $Q = q_s = \omega_0 / \Delta\omega_s$, then equation (2) can be written as

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$$\begin{cases} a_0 = \dfrac{\Delta\omega_s \sin\omega_0\Delta t}{2\omega_0 + \Delta\omega_s \sin\omega_0\Delta t}, \\ b_1 = -\dfrac{4\Delta\omega_s \cos\omega_0\Delta t}{2\omega_0 + \Delta\omega_s \sin\omega_0\Delta t}, \\ b_2 = \dfrac{2\omega_0 - \Delta\omega_s \sin\omega_0\Delta t}{2\omega_0 + \Delta\omega_s \sin\omega_0\Delta t}. \end{cases} \quad (3)$$

However, the digital filter which was designed using equations (1)-(3) has a quality factor q_s more bigger than is required, because the frequency response is warped by the bilinear transformation, since the analog and digital frequencies Ω and ω are related by the equation [7]:

$$\Omega = \frac{2}{\Delta t} tg \frac{\omega_s \Delta t}{2}. \quad (4)$$

Denote the bandwidth $\Delta\Omega$ of an analog filter by $\Delta\Omega = \dfrac{\partial\Omega(\omega)}{\partial\omega_0}\Delta\omega$, where $\dfrac{\partial\Omega(\omega)}{\partial\omega_0} = \sec^2\dfrac{\omega_0\Delta t}{2}$, then the relationship between $\Delta\Omega$ and $\Delta\omega_s$ can be written

$$\Delta\Omega = \Delta\omega_s \sec^2\frac{\omega_0\Delta t}{2}. \quad (5)$$

It follows from

$$Q = {\Omega_0}/{\Delta\Omega}, q_s = {\omega_0}/{\Delta\omega_s}, \Omega_0 = \frac{2}{\Delta t} tg {\omega_0 \Delta t}/{2},$$

and also from equation (5), that [4]

$$Q = \frac{\sin\omega_0\Delta t}{\omega_0\Delta t} q_s. \quad (6)$$

We can see from (6), that $Q < q_s$ in all frequency range $\omega\Delta t \in (0, \pi)$.

Finally, using equation (6), by setting to (2), the coefficients of the digital filter are defined as:

$$\begin{cases} a_0 = \dfrac{\omega_0\Delta t}{2q_s + \omega_0\Delta t}, \\ b_1 = -\dfrac{4q_s \cos\omega_0\Delta t}{2q_s + \omega_0\Delta t}, \\ b_2 = \dfrac{2q_s - \omega_0\Delta t}{2q_s + \omega_0\Delta t}. \end{cases}$$

The computations showed that the quality factor error is less then 0.2% for digital filter specifications of $q_s = 200$ and relationship $\omega_q/\omega_0 \in 2{,}5..100$, ω_q is sampling frequency. On the other hand, if digital filter specifications has bandwidth $\Delta\omega_s$, then the coefficients can be calculated as

$$\begin{cases} a_0 = \dfrac{\Delta\omega_s\Delta t}{2 + \Delta\omega_s\Delta t}, \\ b_1 = -\dfrac{4\cos\omega_0\Delta t}{2 + \Delta\omega_s\Delta t}, \\ b_2 = \dfrac{2 - \Delta\omega_s\Delta t}{2 + \Delta\omega_s\Delta t}. \end{cases} \quad (7)$$

Furthermore, if $\Delta t = 2\pi/\omega_q$ is constant then equation (7) directly provides the following result:

$$b_1 = (a_0 - 1)g,$$
$$b_2 = 1 - 2a_0,$$

where

$$g = 2\cos\omega_0\Delta t.$$

Thus, the transfer function of a tunable bandpass second-order digital filter is defined by:

$$H(z) = a_0 \frac{1 - z^{-2}}{1 + (a_0 - 1)gz^{-1} + (1 - 2a_0)z^{-2}}, \quad (8)$$

where the coefficients a_0 and g are defined by only the bandwidth $\Delta\omega_s$ and central frequency ω_0 respectively.

A number $y(n)$ of the output set is obtained from the numbers $x(n)$ of the input set from equation:

$$\begin{cases} w(n) = x(n) - gv(n-1) + 2v(n-2), \\ v(n) = a_0 w(n) + gv(n-1) - v(n-2), \\ y(n) = v(n) - v(n-2). \end{cases} \quad (9)$$

The corresponding circuit is given in figure 1. The circuit has two data memories to store the elements $v(n-1)$ and $v(n-2)$. The calculation of each element of the output set requires two multiplications and five additions.

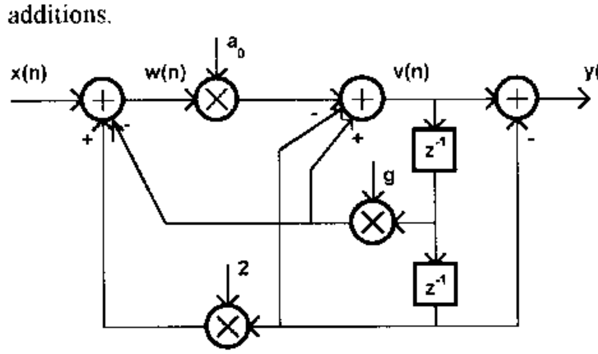

Fig.1. Circuit of the tunable second-order recursive digital filter

Thus, final calculation of the digital filter coefficients are given by the following equations:

$$\begin{cases} a_0 = \dfrac{\omega_0\Delta t}{2q_s + \omega_0\Delta t}, \\ g = 2\cos\omega_0\Delta t, \end{cases} \quad (10)$$

or

$$\begin{cases} a_0 = \dfrac{\Delta\omega_s \Delta t}{2 + \Delta\omega_s \Delta t}, \\ g = 2\cos\omega_0 \Delta t. \end{cases} \tag{11}$$

3 CASCADE STRUCTURE OF A TUNABLE DIGITAL FILTER

The square magnitude response of second-order section is defined as

$$\begin{cases} H(p)H(-p) \xrightarrow{p^2 = -\omega^2} A_s^2(\omega), \\ A_s^2(\omega) = \dfrac{(\Delta\omega_s)^2 \omega^2}{(\omega^2 - \omega_0^2) + (\Delta\omega_s)^2}. \end{cases} \tag{12}$$

Defined the lower and upper passband cutoff frequency ω_1 and ω_2 of a second-order section from the following equations:

$$\begin{cases} \omega_1 \omega_2 = \omega_0^2, \\ \omega_2 - \omega_1 = \omega_s / q_s \end{cases}$$

$$\begin{cases} \omega_1 = \dfrac{\omega_0}{2q_s}(\sqrt{4q_s^2 + 1} - 1), \\ \omega_2 = \dfrac{\omega_0}{2q_s}(\sqrt{4q_s^2 + 1} + 1). \end{cases}$$

Furthermore, defined the lower and upper stopband cutoff frequency ω_3 and ω_4 of a second-order section from the following dependencies:

$$\begin{cases} \omega_3 \omega_4 = \omega_0^2, \\ \omega_4 - \omega_3 = K_s \omega_0 / q_s \end{cases}$$

$$\begin{cases} \omega_3 = \dfrac{\omega_0}{2q_s}(\sqrt{4q_s^2 + K_s^2} - K_s), \\ \omega_4 = \dfrac{\omega_0}{2q_s}(\sqrt{4q_s^2 + K_s^2} + K_s). \end{cases} \tag{13}$$

Using equation (13), by setting to (12), and after some mathematical transformations, the square of magnitude response of a second-order section on the frequencies ω_3 and ω_4 can be calculated as

$$A_s^2(\omega_{3,4}) = \frac{(\Delta\omega_s)^2 \omega_{3,4}^2}{\omega_{3,4}^2 K_s^2 (\Delta\omega_s)^2 + (\Delta\omega_s)^2 \omega_{3,4}^2} = m,$$

then the specification form coefficient of a second-order section becomes

$$K_s = \sqrt{1/m - 1}. \tag{14}$$

Let ω_{11} and ω_{22} be a bandpass cutoff frequency of a cascade structure digital filter, which has a 3dB bandwidth $\Delta\omega = \omega_0 / q$ and a square magnitude response on the ω_{11} and ω_{22} $A^2(\omega_{11,22}) = 0.5$, but the square magnitude response of a second-order section in this frequencies is $A_s^2(\omega_{11}, \omega_{22}) = \sqrt[L]{0.5}$. On the other hand

$$A_s^2(\omega_{11}, \omega_{22}) = \frac{(\omega_0 / q_s)^2 \omega_{11,22}^2}{\omega_{11,22}^2 (\omega_0 / q)^2 + (\omega_0 / q_s)^2 \omega_{11,22}^2} = \sqrt[L]{0.5}$$

and we get

$$q_s = \sqrt{\sqrt[L]{2} - 1}\, q. \tag{15}$$

The mdB bandwidth of a cascade digital filter is

$$\Delta\omega_m = K\Delta\omega = K\omega_0 / q, \tag{16}$$

and on the other hand, a bandwidth of a second-order section on the level $\sqrt[L]{m}$ of a square magnitude response can be denote as

$$\Delta\omega_m = K_s \omega_0 / q_s. \tag{17}$$

Using (15), (16) and (17) we get

$$K_s = \sqrt{\sqrt[L]{2} - 1}\, K. \tag{18}$$

Furthermore, using equation (18), by setting (14), we can write:

$$K = \sqrt{(\sqrt[L]{1/m} - 1) \big/ (\sqrt[L]{2} - 1)}. \tag{19}$$

Remark. The equation (14) and (19) do not depend on the quality factor q and the centre frequency ω_0, and the result (14) is a particular case of (19) for $L = 1$.

Theorem. *The value of a coefficient K is limited for $L \to \infty$, and defined as*

$$K_\infty = \sqrt{\log_2(1/m)}. \tag{20}$$

Proof. Let us $x = 1/L$, then

$\lim\limits_{L \to \infty} K^2 = \lim\limits_{x \to 0} K^2 = \lim\limits_{x \to 0} \dfrac{(1/m)^x - 1}{2^x - 1}$, and after some mathematical transformations, we get

$$\lim_{x \to 0} K^2 = \lim_{x \to 0} \frac{(1/m)^x \ln(1/m)}{2^x \ln 2} = \frac{\ln(1/m)}{\ln 2} = \\ = \log_2(1/m) > 0,$$

and

$$\lim_{L \to \infty} K = \sqrt{\log_2(1/m)}. \bullet$$

From the theorem follows, that we do not get the cascade structure of a digital filter with $K = 1$ for $L \to \infty$, and we do not get the cascade form of a digital filter with K better then $\sqrt{\log_2(1/m)}$.

Thus, the results (19), (20) and (15) give for us number L of a second-order sections in the cascade structure digital filter and the specifications of the basic second-order section. If $K > \sqrt{1/m-1}$, then it is necessary to have only one second-order section for bandpass digital filter, i.e. $L=1$. For the case, when $\sqrt{\log_2(1/m)} < K < \sqrt{1/m-1}$, L second-order sections are required for a bandpass digital filter. The number of L is defined by (19) and rounding to the bigger integer number.

4 THE DESIGN ALGORITHM

The design algorithm as follows:

Step 1. Calculate K_∞ by (20)

and ***if*** $K < K_\infty$, ***then*** we can not get the digital filter with specifications of K and m. **END**.

Step 2. Calculate K_s by (14)

and ***if*** $K > K_s$ ***then*** $L=1$, go to **step 5**.

Step 3. Define the number L of second-order sections in the cascade structure of digital filter by equation (19).

Step 4. Compute the quality factor q_s (15) and bandwidth ω_s of the second-order section

Step 5. Calculate the coefficients of the second-order section by the equations for the tunable digital filter (10 or (11).

Step 6. **END**.

5 EXAMPLE

Design the high-order bandpass digital filter with tunable centre frequency for the following specifications: 3dB bandwidth is 40 Hz; mdB bandwidth is 400 Hz; m-level is 0.0001 (-40 dB); central frequencies: 1700; 1500; 1300; 1100; 900; 700; i.e. calculate the coefficients of the filter bank is composed of six high-order filters with an identical 3dB bandwidth.

Solution: ω_q/ω_0 = 4, K = 400/40=4, K_s = 100, $K_\infty \approx$ 3.7, $K_s > K > K_\infty$, L is 3, $q_s = 21.67 \Rightarrow \Delta\omega_s = 0.0725$, $a_0 = 0.0350$; the coefficient g for different centre frequency (see above) is 0.0, 0.3675, 0.7225, 1.0529, 1.3474, 1.5960 respectively. On the Fig. 2 is illustrated magnitude response of the filter bank for $L=3$.

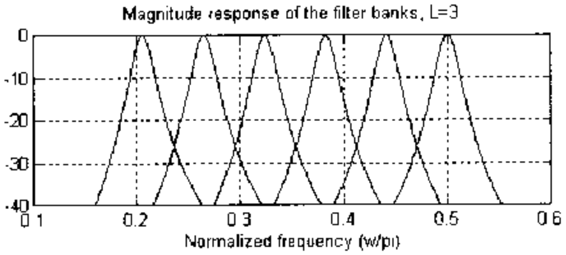

Fig.2 Magnitude response of the filter bank

6 CONCLUDING REMARKS

The design equations (9) - (11) strate the true parametric tuning ability of the circuit. By cascading a few such circuits, a complete parametrically adjustable digital frequency responsed simple equalizer may be realized. By comparison, the designs in [8-9] require precomputing the multiplier coefficient values for all designed equalizer settings. This does not represent a tunable design, and as such has the drawback of requiring excessive coefficient storage. In addition, such filters find application in real-time signal enchancement/correction, cochlea implant speech processor [10-11], etc.

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