4 ESTIMATION OF THE TRICORRELATIONS

4.1 Cyclostationary case

It is well known that a cyclic correlation, say for example $m^n_{x,2,1}(\underline{\tau}_1)$, can be estimated from a portion $x(t)$ of finite length by:

$$\widehat{m}^n_{x,2,1}(\underline{\tau}_1) = \frac{1}{T}\int x(t)x^*(t-\tau_2)\exp(-2i\pi\frac{n}{T_b}t)dt \ . \quad (22)$$

A similar form is obtained for the moment-based cyclic tricorrelation $\widehat{m}^n_{x,4,2}(\underline{\tau})$. Consequently, the cyclic tricorrelation for $n = 0$, $c^0_{x,4,2}(\underline{\tau})$, can be estimated by replacing in (10) all cyclic moments by their estimation of the form (22). There is no difficulty in computing (22), because the cyclostationary signal $x(t)$ is at our direct disposal in any sample path. We performed the estimation for 4-PSK and 16-QAM signals (4096 symbols, $T_b = 16$), both with the particular conditions (19) and without noise. When computing (10), the sum over k was limited to $|k| \leq 5$, because contributions for $k > 5$ are neglectable. The results perfectly fit the theoretical shapes presented in fig. 1.

4.2 Stationary case

As for estimating the stationary tricorrelation, we wish to create first $\widetilde{x}(t)$ from the signal $x(t)$ that is actually at hand. In fact, it is not necessary, because $\widetilde{x}(t)$ is stationary and ergodic (cycloergodicity has been destroyed by phase randomization), and consequently, its moments can be estimated through any single sample path, namely $x(t)$. So, the estimation of $C_{\widetilde{x},4,2}(\underline{\tau})$ will be performed from a portion $x(t)$ of finite length by:

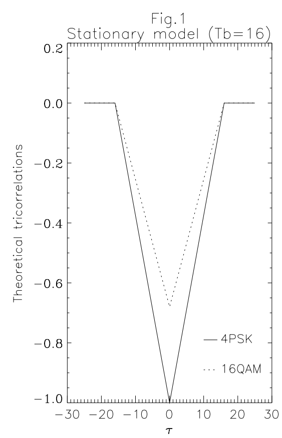
$$\begin{aligned}\widehat{C}_{\widetilde{x},4,2}(\underline{\tau}) = \widehat{M}_{x,4,2}(\underline{\tau}) &- \widehat{M}_{x,2,1}(\underline{\tau}_1)\cdot\widehat{M}_{x,2,1}(\underline{\tau}_2)\\ &-\widehat{M}_{x,2,1}(\underline{\tau}_3)\cdot\widehat{M}_{x,2,1}(\underline{\tau}_4)\\ &-\widehat{M}_{x,2,2}(\underline{\tau}_5)\cdot\widehat{M}_{x,2,0}(\underline{\tau}_6)\end{aligned} \quad (23)$$

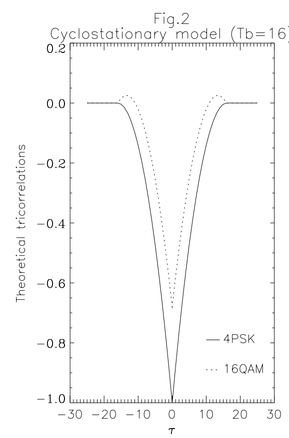
and $\widehat{M}_{x,4,2}(\underline{\tau}) = \frac{1}{T}\int x(t)x(t+\tau_1)x^*(t-\tau_2)x^*(t-\tau_3)dt$; equivalent estimators are built for each $\widehat{M}_{x,2,.}()$. Comparing (10) and (12), it can be noticed that $C_{\widetilde{x},4,2}(\underline{\tau})$ may be estimated through the same procedure as $c^0_{x,4,2}(\underline{\tau})$, except that each sum over k in (10) must be restricted to a single term corresponding to $k = 0$. This remark is important for the interpretation given in Section 5. We performed the estimation for normalized 4-PSK and 16-QAM signals (4096 symbols, $T_b = 16$), both with the particular conditions (19) and without noise. The results perfectly fit the theoretical shapes presented in fig. 2.

5 CONCLUSION AND INTERPRETATION

We have shown, theoretically and by simulations, that fourth-order cyclic statistics yield proportional functions for QAM signals, whereas the fourth-order stationary statistics yield non-proportional functions. Let us forget for a while the opposition cyclic model *vs.* stationary model, and just think that $C_{\widetilde{x},4,2}(\underline{\tau})$ and $c^0_{x,4,2}(\underline{\tau})$ are two different ways to "observe" $C_{s,4,2}$, $M_{s,4,2}$, and $M_{s,2,.}$ in the signals. As already pointed out above, $C_{\widetilde{x},4,2}(\underline{\tau})$ could be seen as a $c^0_{x,4,2}(\underline{\tau})$, for which the cumulant operation has not been fully completed; more precisely, in the expression of the cumulant-based cyclic tricorrelation (10), not only products of second-order cyclic moments at the cycle frequency zero are substracted, but also all products of second-order cyclic moments whose cycle frequencies sum to zero. In other words, although $C_{\widetilde{x},4,2}(\underline{\tau})$ is defined with a cumulant, it is not a real one, and contains not only pure fourth-order characteristics (namely $C_{s,4,2}$), but also some squared second-order characteristics ($M_{s,2,.}\cdot M_{s,2,.}$).

Since QAM signals have the same pure fourth-order statistics, it is impossible to classify them with a real cumulant (*i.e.* $c^0_{x,4,2}(\underline{\tau})$). Therefore, a function that properly mixes pure fourth-order and second-order characteristics must be found: we have shown that $C_{\widetilde{x},4,2}(\underline{\tau})$ achieves this requirement.





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will be different from $c^0_{x,4,2}(\underline{\tau})$ provided this sum is non-zero; it is sufficient to show it for a particular $\underline{\tau}$, say $\underline{\tau}=(0,-\tau,0,\tau)$. For this particular lag-vector,

$$\begin{aligned} &m^k_{x,2,1}(\underline{\tau}_1)\cdot m^{-k}_{x,2,1}(\underline{\tau}_2) = \\ &\left(\tfrac{1}{T_b}\right)^2 \int_{T_b} E\left[x(t_1)x^*(t_1)\right]\exp(-2i\pi\tfrac{k}{T_b}t_1)dt_1 \\ &\times\int_{T_b} E\left[x(t_2-\tau)x^*(t_2-\tau)\right]\exp(-2i\pi\tfrac{k}{T_b}t_2)dt_2 \\ &\qquad = \exp\left(2i\pi\tfrac{k}{T_b}\tau\right)\left|m^k_{x,2,1}(0)\right|^2, \end{aligned} \tag{14}$$

and similarly

$$m^k_{x,2,1}(\underline{\tau}_3)\cdot m^{-k}_{x,2,1}(\underline{\tau}_4) = \left|m^k_{x,2,1}(\tau)\right|^2, \tag{15}$$

and

$$m^k_{x,2,2}(\underline{\tau}_5)\cdot m^{-k}_{x,2,0}(\underline{\tau}_6) = \left|m^k_{x,2,2}(-\tau)\right|^2. \tag{16}$$

Under the condition $m^k_{x,2,.}(\underline{\tau}_i)\neq 0$ for some $k, k\neq 0$, or, equivalently, if $x(t)$ contains cycle frequency of order two, the sum over k, $k\neq 0$, of the terms (15), (16), (17) does not vanish. Consequently, unless every second-order cyclostationarity has been removed before processing (through a particular filtering for example) the cumulant-based cyclic tricorrelation at cycle frequency zero cannot be equal to the cumulant-based stationary tricorrelation, whereas the corresponding moment-based tricorrelations are equal (*cf.* (13)). This conclusion can be extended to any order $k\geq 2$, considering that a kth-order cumulant can be expressed as a function of products of lower-order moments. However, at order two, it is usual to admit that cyclic autocorrelation for $\alpha=0$ and stationary autocorrelation are equivalent because cyclostationarity of order 1 seldom occurs in practical cases.

3 APPLICATION TO QAM SIGNALS CLASSIFICATION

3.1 Cyclostationary modelling

The general expression for a QAM baseband signal is given by:

$$x(t) = \sum_k s_k q(t-kT_b) \tag{17}$$

where $\{s_k = a_k + i.b_k\}_{k\in\mathbf{Z}}$ is a complex-valued, zero-mean, and *i.i.d.* symbol sequence, T_b is the symbol duration, and $q(t)$ is the real-valued pulse function with duration not exceeding T_b. This kind of baseband signal belongs to the class we emphasized in the previous Section, *i.e.* the class of cyclostationary signals in the strict sense, whose cycle frequencies are all multiples of $\frac{1}{T_b}$. In (17), the only random components are the symbols s_k, and using the multi-linearity property of the cumulant and the independence of the s_k, it can be readily shown that the cyclic tricorrelation of $x(t)$ is given by:

$$\begin{aligned} c^n_{x,4,2}(\underline{\tau}) = \tfrac{C_{s,4,2}}{T_b}\int q(t)q(t+\tau_1)q(t-\tau_2)q(t-\tau_3) \\ \cdot\exp(-2i\pi\tfrac{n}{T_b}t)dt \end{aligned} \tag{18}$$

where $C_{s,4,2}$ is the stationary fourth-order cumulant of the random sequence $\{s_k\}$: $C_{s,4,2} = Cum\left[s_k, s_k, s_k^*, s_k^*\right]$. It is easily seen that the shape of this function, with respect to $\underline{\tau}$ or n, depends only on the pulse function $q(t)$. Thus, different types of QAM signals will exhibit the same cyclic tricorrelation (affected by different factors $C_{s,4,2}$). So, it will be impossible to recognize a QAM signal by estimating (10), unless the power of the signal is known.

As an application, consider the following pulse function and lag-vector:

$$q(t) = \begin{cases} 1 & \text{if } t\in[0,T_b[\\ 0 & \text{elsewhere}\end{cases} \quad ;\underline{\tau}=(0,-\tau,0,\tau)\ . \tag{19}$$

For cycle frequency zero, (18) is given by:

$$c^0_{x,4,2}(\tau) = C_{s,4,2}\cdot\Lambda_{T_b}(\tau) \tag{20}$$

where $\Lambda_{T_b}(\tau) = \frac{T_b-|\tau|}{T_b}$ if $|\tau|<T_b$ and $\Lambda_{T_b}(\tau)=0$ elsewhere.

Theoretical shapes of (20) for normalized 4-PSK ($C_{s,4,2}=-1$) and 16-QAM ($C_{s,4,2}=-0.68$) signals with symbol duration $T_b=16$ are presented in fig. 1.

3.2 Stationary modelling

The calculation of the tricorrelation of the stationary signal defined in (1) is much more tedious than for $x(t)$ because the $Cum\left[\cdot\right]$ operator (and so the expectation operations $E\left[\cdot\right]$) must be applied not only to the random sequence $\{s_k\}$, but also to the random time-translation θ. Some steps of the calculation can be found in [2], and the final result for the conditions specified in (19) is given by:

$$\begin{aligned} C_{\tilde{x},4,2}(\underline{\tau}) = (C_{s,4,2}+(M_{s,2,1})^2)\cdot\Lambda_{T_b}(\tau) \\ -(M_{s,2,1}\cdot\Lambda_{T_b}(\tau))^2 \end{aligned} \tag{21}$$

where $M_{s,2,1} = E\left[s_k s_k^*\right]$. $M_{s,2,1}$ is the power of the transmitted signal, and so, in a normalized context, $M_{s,2,1}=1$ whatever the QAM modulation type. Besides, $C_{s,4,2}$ can be shown to be a function of the number of states of the QAM constellation. Consequently, applying (21) to different types of QAM signals will result in non-proportional functions of τ, depending on the different values of $C_{s,4,2}$. So, as pointed out in details in [2], classification of QAM signals is possible when a stationary modelling is adopted, even if the power of the signal is unknown (by using matched filters). Theoretical shapes of (21) for normalized 4-PSK and 16-QAM signals with symbol duration $T_b=16$ are presented in fig. 2.

In practice, since the stationarization operation is just a theoretical trick, and since we can only handle sample paths that are cyclostationary, how is it possible to deal with the stationarized signal $\tilde{x}(t)$? This question is solved when the proper way to estimate (10) and (11) is investigated.

terms, the tricorrelation can be written:

$$C_{x,4,2}(t;\underline{\tau}) \triangleq Cum\left[x(t), x(t+\tau_1), x^*(t-\tau_2), x^*(t-\tau_3)\right] \quad (2)$$

where $\underline{\tau}= (0,\tau_1,\tau_2,\tau_3)$ represents the set of lags applied to each element of the product, and $C_{x,n,m}(t;\underline{\tau})$ stands for a nth-order correlation defined with m non-conjugated terms and $n-m$ conjugated terms. We may rewrite (2) as:

$$\begin{aligned} C_{x,4,2}(t;\underline{\tau}) = M_{x,4,2}(t;\underline{\tau}) & \\ -M_{x,2,1}(t;\underline{\tau}_1)\cdot M_{x,2,1}(t;\underline{\tau}_2) & \\ -M_{x,2,1}(t;\underline{\tau}_3)\cdot M_{x,2,1}(t;\underline{\tau}_4) & \\ -M_{x,2,2}(t;\underline{\tau}_5)\cdot M_{x,2,0}(t;\underline{\tau}_6) & \end{aligned} \quad (3)$$

where $M_{x,4,2}(t;\underline{\tau})$ and $M_{x,2,.}(t;\underline{\tau}_i)$ are the moment-based tricorrelation and correlation respectively, and where

$$\begin{aligned} \underline{\tau}_1 = (0,\tau_2)\,, \quad \underline{\tau}_2 = (\tau_1,\tau_3)\,, \quad \underline{\tau}_3 = (0,\tau_3)\,, \\ \underline{\tau}_4 = (\tau_1,\tau_2)\,, \quad \underline{\tau}_5 = (0,\tau_1)\,, \quad \underline{\tau}_6 = (\tau_2,\tau_3)\,. \end{aligned} \quad (4)$$

Since $x(t)$ is fourth-order cyclostationary, its tricorrelation (2) depends periodically on t with period T_b: $C_{x,4,2}(t;\underline{\tau}) = C_{x,4,2}(t+T_b;\underline{\tau})$. Thus, it is possible to define Fourier coefficients at harmonics of the cycle frequency $1/T_b$:

$$c^n_{x,4,2}(\underline{\tau}) = \frac{1}{T_b}\int_{T_b} C_{x,4,2}(t;\underline{\tau})\exp(-2i\pi\frac{n}{T_b}t)dt\ . \quad (5)$$

where the exponent n is purely notational and is related to the cycle frequency $\alpha = \frac{n}{T_b}$. We propose to call $c^n_{x,4,2}(\underline{\tau})$ *cyclic tricorrelation* (by analogy with order two). Similar relations could be written for $M_{x,4,2}(t;\underline{\tau})$ and $M_{x,2,.}(t,\underline{\tau}_i)$, and thus, we can define $m^n_{x,4,2}(\underline{\tau})$ and $m_{x,2,.}(\underline{\tau}_i)$ as the moment-based cyclic tricorrelation and correlations, respectively. For example,

$$m^n_{x,2,1}(\underline{\tau}_2) = \frac{1}{T_b}\int_{T_b} E\left[x(t+\tau_1)\,x^*(t-\tau_3)\right] \cdot\exp(-2i\pi\frac{n}{T_b}t)dt\ . \quad (6)$$

Now, let $\overline{C}_{x,4,2}(\alpha;\underline{\tau})$, $\overline{M}_{x,4,2}(\alpha;\underline{\tau})$, and $\overline{M}_{x,2,.}(\alpha;\underline{\tau}_i)$ be the Fourier transforms with respect to t of the corresponding temporal moments or cumulants. $\overline{C}_{x,4,2}(\alpha;\underline{\tau})$ is related to the cyclic tricorrelation by:

$$\overline{C}_{x,4,2}(\alpha;\underline{\tau}) = \sum_n c^n_{x,4,2}(\underline{\tau})\delta\left(\alpha-\frac{n}{T_b}\right) \quad (7)$$

and similar relations can be written for the couples $(\overline{M}_{x,4,2}(\alpha;\underline{\tau}), m^n_{x,4,2}(\underline{\tau}))$ and $(\overline{M}_{x,2,.}(\alpha;\underline{\tau}_i), m^n_{x,2,.}(\underline{\tau}_i))$. Taking the Fourier transform of both sides of (3) leads to:

$$\begin{aligned} \overline{C}_{x,4,2}(\alpha,\underline{\tau}) = \overline{M}_{x,4,2}(\alpha;\underline{\tau}) & \\ -\overline{M}_{x,2,1}(\alpha;\underline{\tau}_1) * \overline{M}_{x,2,1}(\alpha;\underline{\tau}_2) & \\ -\overline{M}_{x,2,1}(\alpha;\underline{\tau}_3) * \overline{M}_{x,2,1}(\alpha;\underline{\tau}_4) & \\ -\overline{M}_{x,2,2}(\alpha;\underline{\tau}_5) * \overline{M}_{x,2,0}(\alpha;\underline{\tau}_6) & \end{aligned} \quad (8)$$

where "$*$" stands for convolution product with respect to α. Applying (7) and the corresponding relations for the moments in (8), it can be easily shown that:

$$\begin{aligned} c^n_{x,4,2}(\underline{\tau}) = m^n_{x,4,2}(\underline{\tau}) - \textstyle\sum_{i+j=n} m^i_{x,2,1}(\underline{\tau}_1)\cdot m^j_{x,2,1}(\underline{\tau}_2) \\ - \textstyle\sum_{i+j=n} m^i_{x,2,1}(\underline{\tau}_3)\cdot m^j_{x,2,1}(\underline{\tau}_4) \\ - \textstyle\sum_{i+j=n} m^i_{x,2,2}(\underline{\tau}_5)\cdot m^j_{x,2,0}(\underline{\tau}_6)\ . \end{aligned} \quad (9)$$

At cycle frequency zero, the cyclic tricorrelation of $x(t)$ becomes:

$$\begin{aligned} c^0_{x,4,2}(\underline{\tau}) = m^0_{x,4,2}(\underline{\tau}) - \textstyle\sum_k m^k_{x,2,1}(\underline{\tau}_1)\cdot m^{-k}_{x,2,1}(\underline{\tau}_2) \\ - \textstyle\sum_k m^k_{x,2,1}(\underline{\tau}_3)\cdot m^{-k}_{x,2,1}(\underline{\tau}_4) \\ - \textstyle\sum_k m^k_{x,2,2}(\underline{\tau}_5)\cdot m^{-k}_{x,2,0}(\underline{\tau}_6)\ . \end{aligned} \quad (10)$$

2.3 Tricorrelation of the stationarized signal

The tricorrelation of the stationary signal (1) is simply given by:

$$\begin{aligned} C_{\widetilde{x},4,2}(\underline{\tau}) = M_{\widetilde{x},4,2}(\underline{\tau}) \quad -M_{\widetilde{x},2,1}(\underline{\tau}_1)\cdot M_{\widetilde{x},2,1}(\underline{\tau}_2) \\ -M_{\widetilde{x},2,1}(\underline{\tau}_3)\cdot M_{\widetilde{x},2,1}(\underline{\tau}_4) \\ -M_{\widetilde{x},2,2}(\underline{\tau}_5)\cdot M_{\widetilde{x},2,0}(\underline{\tau}_6) \end{aligned} \quad (11)$$

where the same notations as in the previous paragraph are used. We may rewrite $C_{\widetilde{x},4,2}(\underline{\tau})$ in terms of moment-based correlations and tricorrelation at cycle frequency zero of $x(t)$. Specifically:

$$\begin{aligned} C_{\widetilde{x},4,2}(\underline{\tau}) = m^0_{x,4,2}(\underline{\tau}) - m^0_{x,2,1}(\underline{\tau}_1)\cdot m^0_{x,2,1}(\underline{\tau}_2) \\ -m^0_{x,2,1}(\underline{\tau}_3)\cdot m^0_{x,2,1}(\underline{\tau}_4) \\ -m^0_{x,2,2}(\underline{\tau}_5)\cdot m^0_{x,2,0}(\underline{\tau}_6)\ . \end{aligned} \quad (12)$$

Proof: Let $p_\theta(\omega)$ be the density probability function of θ. Since θ is independent of $x(t)$, we may write (using (5) applied to the moments):

$$\begin{aligned} & M_{\widetilde{x},4,2}(\underline{\tau}) = E\left[\widetilde{x}(t)\widetilde{x}(t+\tau_1)\widetilde{x}^*(t-\tau_2)\widetilde{x}^*(t-\tau_3)\right] \\ & = \int_{\mathbf{R}} E\left[\begin{array}{c} x(t+\omega)x(t+\omega+\tau_1)... \\ ...x^*(t+\omega-\tau_2)x^*(t+\omega-\tau_3) \end{array}\right] p_\theta(\omega)\,d\omega \\ & = \frac{1}{T_b}\int_{T_b} E\left[\begin{array}{c} x(t+\omega)x(t+\omega+\tau_1)... \\ ...x^*(t+\omega-\tau_2)x^*(t+\omega-\tau_3) \end{array}\right] d\omega \\ & = \frac{1}{T_b}\int_{T_b} M_{x,4,2}(t+\omega;\underline{\tau})d\omega = m^0_{x,4,2}(\underline{\tau})\ . \end{aligned} \quad (13)$$

Similarly, it can be shown that

$$M_{\widetilde{x},2,.}(\underline{\tau}_i) = m^0_{x,2,.}(\underline{\tau}_i)\ \forall i = 1,..,6,$$

which completes the proof for (12).

2.4 Comparison between the two representations

Our purpose now is to show that in the general case $C_{\widetilde{x},4,2}(\underline{\tau}) \neq c^0_{x,4,2}(\underline{\tau})$, whereas $M_{\widetilde{x},4,2}(\underline{\tau}) = m^0_{x,4,2}(\underline{\tau})$ (*cf.* (13)). Relation (12) shows that $C_{\widetilde{x},4,2}(\underline{\tau})$ is entirely contained in $c^0_{x,4,2}(\underline{\tau})$, and the difference $C_{\widetilde{x},4,2}(\underline{\tau}) - c^0_{x,4,2}(\underline{\tau})$ can be expressed as the sum of the three terms $\left\{\sum_{k\neq 0} m^k_{x,2,.}(\underline{\tau}_i)\cdot m^{-k}_{x,2,.}(\underline{\tau}_{i+1})\right\}_{i=1,3,5}$. Then $C_{\widetilde{x},4,2}(\underline{\tau})$

HIGHER-ORDER STATISTICS FOR QAM SIGNALS: A COMPARISON BETWEEN CYCLIC AND STATIONARY REPRESENTATIONS

Pierre Marchand *† *and Denys Boiteau* **
* CEPHAG-ENSIEG URA CNRS 346, BP 46, 38402 Saint Martin d'Hères Cedex, France
† TEAMLOG, "Le Grand Sablon", 4 av. de l'Obiou, 38700 La Tronche, France
** CESTA, 37 av. du Général de Gaulle, 35170 Bruz, France

ABSTRACT

For a cyclostationary signal, the cumulant-based cyclic tricorrelation (fourth-order correlation) at cycle frequency zero should not be confused, in the general case, with the cumulant-based tricorrelation of the same signal after stationarization. The reasons for this unusual assertion are detailed; as an illustration, we show that if QAM signal classification is impossible using their fourth-order cyclic statistics, classification is however possible if a stationary modelling is adopted. Remarks on the estimation of both cyclic and stationary temporal cumulants are provided and consequently, the skip between the cyclic and the stationary models is enlightened. Theoretical expressions of cyclic and stationary tricorrelations are derived and computer simulations confirm the results.

1 INTRODUCTION

The use of (second- or higher-order) cyclostationarity property of time series has already proved to be very helpful in many applications, especially to characterize telecommunication signals [4]. For example, it has been shown [1] that M-PSK (M$\geq$ 4) carrier-modulated signals can be differentiated thanks to higher-order cyclic statistics, whereas they have the same second-order cyclic representations. Although it could be concluded that taking into account cyclostationarity property *always* provides more useful information about signals, we describe and analyse in this paper a QAM modulation classification task where a cyclostationary representation of baseband signals destroys some discriminating characteristics which are exhibited in the cumulant-based tricorrelation of the stationarized signals. These considerations will lead us to stress the difference between the higher-order cyclic correlation at cycle frequency zero, and the higher-order correlation of the stationarized signal, called here *higher-order stationary correlation*. The paper in organized as follows: in Section 2, we give the general expression of the cumulant-based cyclic tricorrelation of a cyclostationary signal. At cycle frequency zero, this expression is shown to be different from the cumulant-based stationary tricorrelation. In Section 3, theoretical expressions for both models are detailed in the case of QAM signals. The last Section presents some useful remarks on the estimation procedures, and results of computer simulations are provided. To conclude, we propose a practical interpretation that overtakes the debate cyclostationary model *vs.* stationary model.

2 CYCLOSTATIONARY AND STATIONARY MODELS

2.1 The two signals of interest

Let $x(t)$ be a random, complex-valued and zero-mean process that is cyclostationary in the strict sense ,*i.e.* its probability density function is periodic with respect to t with the single period T_b, or, equivalently, its moments are periodic with respect to t whatever the order.

A stationarized version of $x(t)$ is obtained by:

$$\widetilde{x}(t) \triangleq x(t+\theta) \tag{1}$$

where θ is a random time-translation, independent of $x(t)$, whose probability density function is uniformly distributed over $[0, T_b[$. The probability density function of $\widetilde{x}(t)$ is invariant through any given time-translation, and so are the moments of $\widetilde{x}(t)$ whatever the order.

For simplicity, we have considered that the period of cyclostationarity T_b is unique, but the following developments can be extended to any *stationarizable* random process, such as almost- or quasi- cyclostationary processes (stationary signals are obtained by choosing other probability density functions for θ, see [3] for details).

2.2 Cyclic tricorrelation of the cyclostationary signal

To extend the theory of cyclostationarity from order 2 to order k ($k > 2$), several approaches have been developed (see [4] for example). We choose the usual probabilistic framework, which is helpful in defining cyclic moments and cumulants for complex-valued time series [1]. For simplicity, the expressions are derived at order 4, but could be easily extended whatever the order. Suppose the tricorrelation of $x(t)$ exists (this implies generalization to higher orders of the well-known harmonizability (order 2) [1]). Choosing a definition with two conjugated