MODIFIED SIGMA FILTER FOR PROCESSING IMAGES CORRUPTED BY MULTIPLICATIVE AND IMPULSIVE NOISE

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ABSTRACT

A new modification of sigma filter is proposed and tested in this paper. This modification is suitable for processing images corrupted by Gaussian multiplicative and impulsive noises and it avoids some typical disadvantages of the standard sigma filter while possessing robust properties. The test images include both simulated and real radar images. It is seen that the proposed modification provides improved speckle suppression efficiency and less bias for homogeneous regions of images.

1 INTRODUCTION

Sigma filter, introduced and investigated by Lee [2], is a useful tool for processing of images corrupted by additive or multiplicative Gaussian noise having relatively small variances σ_o^2 and σ_μ^2, respectively. Its main advantage is the ability to preserve well edges and fine details. Another is that it can be easily implemented in software and hardware, [5]. However, the standard sigma filter when applied to processing of images corrupted by a mixture of Gaussian multiplicative noise (with relative variance $\sigma_\mu^2 > 0.01$) and impulsive noise (spikes) does not perform well because: 1) it gives biased estimates of the levels of homogeneous regions; 2) it is unable to eliminate impulsive noise. These obstacles explain why the basic sigma filter was recommended only for processing of locally active (i.e., corresponding to to edges and fine details) fragments as a component of hard switching adaptive filtering algorithms, [6]. The purpose of this paper is to present a new modification of sigma filter which avoids the undesirable effects mentioned above. First, in Section 2 the main principles of the modified sigma filter operation are described and, then, in the next section the quantitative estimates and comparison results are presented. They prove the efficiency of the proposed approach for both simulated and real radar remote sensing data.

2 MODIFIED SIGMA FILTER

The image and noise model considered is described as follows

$$I_{ij} = \begin{cases} \mu_{ij} I_{ij}^t, & \text{with probability } 1 - p_{imp} \\ I_{ij}^{imp}, & \text{with probability } p_{imp}, \end{cases}$$

where I_{ij} is the ij-th pixel value of the original noisy image, μ_{ij} defines the multiplicative noise component having mean value equal to 1.0 and variance σ_μ^2, I_{ij}^t denotes the true image pixel intensity value, p_{imp} is the probability of spike noise, I_{ij}^{imp} denotes the image value for pixels corrupted by impulsive noise. As it is known, [2], the main idea of sigma filter is to form 2σ–neighborhood of the central pixel value of the scanning window, to determine the corresponding sample element values belonging to this neighborhood and their number N and then to average these N elements. These operations can be expressed in the following way

$$I_{ij}^f = \frac{1}{N_{ij}} \sum_{k=i-K}^{i+K} \sum_{l=i-L}^{i+L} \delta_{kl} I_{kl} \tag{1}$$

$$\delta_{kl} = \begin{cases} 1, & \text{if } I_{kl} \in [X_{ij}^l, X_{ij}^u] \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$X_{ij}^l = I_{ij}(1 - 2\sigma_\mu); X_{ij}^u = I_{ij}(1 + 2\sigma_\mu) \tag{3}$$

$$N_{ij} = \sum_{k=i-K}^{i+K} \sum_{l=i-L}^{i+L} \delta_{kl}, \tag{4}$$

where I_{ij}^f is the filter output, X_{ij}^l and X_{ij}^u denote the lower and upper bounds of 2σ–neighborhood interval, $N_A = (2K+1)(2L+1)$ defines the size of the scanning window aperture which is supposed to be rectangular, $N_{ij} \le N$. Naturally, instead of 2σ–neighborhood one can use $\alpha\sigma$–neighborhood, but the decrease of α results in worse noise suppression efficiency and for $\alpha > 2$ the filtering algorithms edge/fine detail preservation properties become poor. That is why $\alpha = 2\sigma$ is usually considered to be the best trade-off. It is seen from (3) that the greater I_{ij} is the wider is the 2σ–neighborhood. Just this obstacle leads to the fact that for the case of multiplicative noise the bias of image homogeneous regions average level for basic sigma filter output occurs. It can reach 1dB for σ_μ approaching to 0.1. The other disadvantage which stimulated our interest to sigma filter performance improvement is its rather poor efficiency of noise suppression as well as an unability to eliminate spikes. One more interesting phenomenon peculiar to the basic sigma filter is that increasing the scanning window size does not lead to noise improvement of suppression efficiency proportional to N_A. This improvement appears to be of lower order.

The proposed modified sigma filter applies the algorithm described by (2), (3), (4) at the first step, but the averaging (1) is not performed during this stage. The next step is to compare N_{ij} with a predetermined integer value M e.g. 2 or 3. If $N_{ij} \leq M$, it means that, probably, there is the spike in the central pixel. Thus, in this case some robust small aperture filter should be used. Certainly, if $N_{ij} \leq M$ the corresponding pixel can belong to a high contrast edge, but, as it will be seen from following considerations, the mistake resulting from such simplified procedure of "spike detection" does not result in performance failure of the proposed algorithm. Now, for the case in paper [3] we proposed to calculate the median value for eight pixels surrounding the central one. Here we suggest to compute $med(\text{x-}med_{ij}, +\text{-}med_{ij}, I_{ij})$, where x and + indicate the configurations of two 5-element cross-shape subapertures. We consider that this variant is characterized by better edge and detail preservation properties while being only a little less robust than the median. It is also obvious that the median-type filters described above are edge/detail preserving as well, thus, the problem of mistake in "classification" does not matter much. If N_{ij} happens to be greater than M, another algorithm is proposed. One has to count the numbers K_g and K_l of sample (window) pixel values (belonging to the primary 2σ–neighborhood) which are respectively greater and less than I_{ij}. Then, if $K_g < K_l$ one is to find the maximal value max_{ij} among all the pixel values belonging to the scanning window and the primary neighborhood determined by (3). After this the new neighborhood is derived as

$$X_{ij}^{ln} = max_{ij}(1-2\sigma_\mu)/(1+2\sigma_\mu); X_{ij}^{un} = max_{ij}.$$

Otherwise, minimal value min_{ij} is to be computed in the similar way and one gets the new neighborhood as

$$X_{ij}^{ln} = min_{ij}; X_{ij}^{un} = min_{ij}(1+2\sigma_\mu)/(1-2\sigma_\mu).$$

Then again the elements in the new neighborhood are found and averaged. The corresponding formulae are simple and analogous to (2), (4) and (1). In fact the relationship between K_g and K_l for homogeneous image regions indicates to what side the neighborhood interval is to be shifted in order to better deal with the central part of gaussian distribution. Obviously, in any case max_{ij} is not greater than X_{ij}^u, $min_{ij} \geq X_{ij}^l$ and the average differences $|X_{ij}^u - max_{ij}|$ and $|X_{ij}^l - min_{ij}|$ usually increase when the difference $|I_{ij} - I_{ij}^t|$ becomes greater. It occurs because, for a homogeneous region characterized by Gaussian distribution the sample values are placed more sparcely in distribution tail areas. Therefore, the greater $|I_{ij} - I_{ij}^t|$ is, the greater (in statistical sense) should be the shifting of the new neighborhood to the center of distribution in respect to primary neighborhood. Because of the expansion of the neighborhood and its shift to the center (mean value for homogeneous regions) the algorithm essentially decreases the remainder bias and the fluctuation variance (approximately by 2–5 and 2–3 times, respectively).

It may seem that the proposed modification is much more complicated than the basic one and this algorithm can be classified as hard switching adaptive one, [6], where N_{ij} plays the role of adaptation parameter and M implies the threshold. While this is true it is also known that data-dependent adaptive image filters perform better than other ones, [5]. Another benefit is that the computational speed of modified sigma filter can be improved by: 1) use of look-up tables for calculation of $X_{ij}^u, X_{ij}^l, X_{ij}^{un}, X_{ij}^{ln}$; 2) use of running data sorting algorithms - in fact, all the scanning window image pixel values which belong to the primary neighborhood belong to the new one as well, and so it is necessary to take into consideration only few values that belong to the interval $[X_{ij}^{ln}, X_{ij}^l]$ if $Kg < K_l$ and $[X_{ij}^u, X_{ij}^{un}]$ if $Kg \geq K_l$; for already sorted (ordered) sample it is very easy to do; 3) computations of median-type filter output being accepted as I_{ij}^f when $N_{ij} \leq M$ can be done in parallel; fast hardware and/or software implementations of these kinds of filters can be easily designed as analogs of FIR median hybrid filter implementations, [4].

3 FILTERING ALGORITHM PROPERTY ANALYSIS

We analyse some statistical properties of the proposed filter by using simulated images in order to get quantitative estimates of the behavior of the filter. The properties analyzed include the efficiency of multiplicative noise reduction and the edge preservation ability. They were compared to the basic sigma filter as well as to linear, median, and local statistic Lee filter, [1]. Typical representation of images as 8-bit integer value 2-D arrays was used, therefore, simultaneous analysis of round-off error influence on filters properties could be performed.

First the study was carried out for simulated homogeneous region that had the size 256×256 pixels and $I_{ij}^t = 128$ for all i, j. The multiplicative noise parameter σ_μ varied in the range $0.05 \ldots 0.3$. The results of the noise suppression efficiency of the filtering algorithm, characterized by the ratio of noise relative variances σ_f^2 and σ_μ^2 respectively after and before processing, are pre-

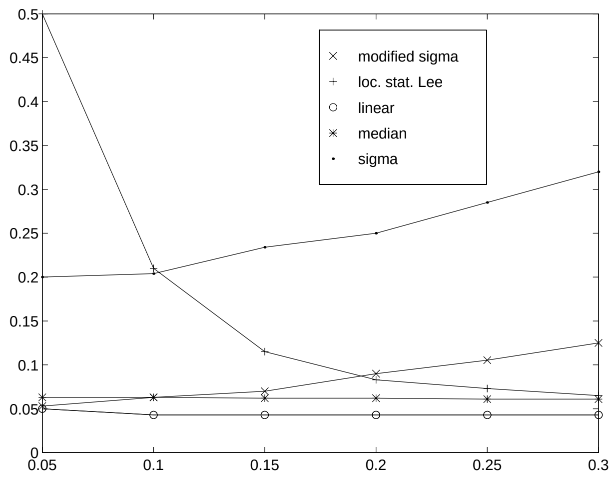

Figure 1: Noise suppression efficiencies in terms of σ_f^2/σ_μ^2 in homogenious image regions.

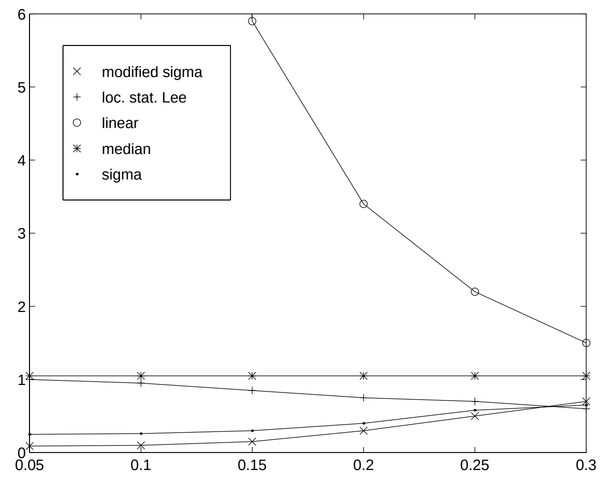

Figure 2: Edge preservation abilities in terms of $\delta_f/\delta p$ in a synthetic strip image.

sented in Figure 1 (5×5 pixels scanning window aperture was used for all the filters). It is seen that 1) due to quantization and round-off errors even linear filter does not reach the predicted efficiency of noise suppression $\sigma_f^2/\sigma_\mu^2 = 1/N_A$; 2) the proposed modified sigma filter outperforms essentially its basic counterpart and local statistic Lee filter; for relatively small σ_μ values its noise suppression efficiency can be comparable to median filter.

The ability of filters to preserve edges was investigated by using horizontal 4–element width strip image with contrast of the average values of the strips being equal to 3 ($I_{ij}^{t1} = 128, I_{ij}^{t2} = 43$). Obviously, when the scanning window aperture size is 5×5 pixels it contains the elements of different strips for any position. In other words, this kind of test data is favourable for median filter. In many other situations its edge preservation ability is more poor. In Figure 2 we give the ratios of parameters $\delta_f/\delta p$ defined as

$$\delta_f = \frac{1}{N_{Im}} \sum_{i=1}^{I_{max}} \sum_{j=1}^{J_{max}} (I_{ij}^f - I_{ij}^t)^2$$

$$\delta_p = \frac{1}{N_{Im}} \sum_{i=1}^{I_{max}} \sum_{j=1}^{J_{max}} (I_{ij} - I_{ij}^t)^2,$$

where $N_{Im} = I_{max} J_{max}$ denotes the total image size. These are, in fact, the integral variances (discrepances) of considered images and the true one after and before image processing. The argument axis corresponds to root mean square value σ_μ (for Figures 1 and 2).

It is easy to see that it is expedient to use modified sigma filter for rather wide range of variation of multiplicative noise variances: $\sigma_\mu^2 = 0.01 \ldots 0.06$ which is typical for radar images.

Some results of different filtering algorithm application to radar remote sensing data are presented below. Figure 3 demonstrates the image obtained by the airborne side-look radar with operation wavelength 3 cm. Homogeneous region histogram analysis has shown that the multiplicative noise distribution was Gaussian, $\sigma_\mu^2 = 0.02$. The output of median (5×5) filter is presented in Figure 4. It smoothed large size object sharp corners and almost eliminated the details (the roads between agricultural fields) accepting them as several spikes. Figure 5 shows the result of basic sigma filter application and Figure 6 presents the output of modified sigma filter. Their visual comparison shows that both filters are characterized by almost the same edge/detail preservation properties but the modified one has higher efficiency of noise suppression for homogeneous regions. It was also proved by results of image local statistical analysis, they match well with theoretical ones. Note, that all the images have been gamma corrected in order to make the details more visible.

Finally, we want to emphasize the importance of the robust properties of the filter. They are useful while processing real radar data which is often corrupted by spikes because of coding/decoding errors arising from image transfer from airborne or satellite carrier to terrestrial processing. For radar test image spikes were not observed because it was formed and recorded just on board of the aircraft and then sampled and processed by laboratory computer.

References

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Figure 3: Original radar image.

Figure 5: Sigma filtered image.

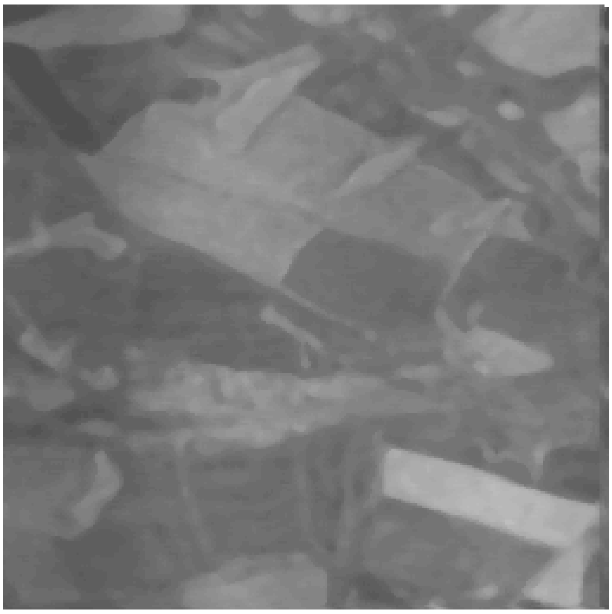

Figure 4: Median filtered image.

Figure 6: Modified sigma filtered image.

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