COHERENCE ESTIMATION OF INTERFEROMETRIC SAR IMAGES.

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ABSTRACT

Usual coherence estimation in SAR interferometry is a time consuming task since an accurate estimation of the local frequency of the interferometric fringes is required. In this paper a fast algorithm for generating coherence maps, mainly intended to data browsing, is presented. The proposed estimator is based on the speckle similarity of coherent SAR data and is, thus, independent of the fringes frequency. Advantages with respect to the usual estimates are achieved in terms of computational costs (up to 100 times lower), robustness (the estimator presented is not affected by possible local frequency estimation errors) and flexibility (the estimator can be applied both to complex and to detected images). The statistical properties of the frequency independent estimator are given in the stationary case. A preprocessing technique that reduces the degradions due to non-stationarities is then shown.

1 INTRODUCTION

The interferometric SAR coherence is an indicator of the achievable interferograms quality (the higher the coherence the better the interferogram quality) [1, 2, 3, 4]. Thus, in order to select those SAR pairs that can be actually exploited for interferometric applications, the analysis of coherence should be routinely carried out on those pairs that show suitable perpendicular baseline[5]. With the huge amount of interferometric SAR data that are and will be available in the near future from satellite SAR missions (i.e. ERS-1, ERS-2, RADARSAT, SIR-C/X-SAR III), the computing time to generate interferograms and coherence maps in a semi-operative scenario should be kept as short as possible.

It is well known that the usual coherence estimation is a very time consuming task as an accurate estimation of the local frequency of the interferometric fringes is required. In this paper a fast algorithm for generating coherence maps (e.g. to be exploited for interferometric data browsing and to speed up the images registration step) is presented. It will be shown that the proposed technique is much faster than the usual techniques (at the cost of a reduced statistical confidence) as it is independent of the local frequency of the fringes. Moreover it is more robust as it is not affected by possible local frequency estimation errors.

2 THE USUAL COHERENCE ESTIMATOR

The coherence γ between two SAR images $v_1(n,m)$ and $v_2(n,m)$ (here regarded as random processes) can be theoretically defined on a pixel basis as the correlation coefficient between two zero-mean, complex gaussian random variates $v_1 = v_{1r} + j\, v_{1i}$, and $v_2 = v_{2r} + j\, v_{2i}$:

$$\gamma = \frac{|E\left[v_1 \cdot v_2^*\right]|}{\sqrt{E\left[|v_1|^2\right] E\left[|v_2|^2\right]}} \tag{1}$$

In general the coherence changes from area to area (or even from pixel to pixel) along the imaged scene. The actual coherence, however, cannot be estimated on a pixel basis from two SAR images only. Nonetheless, it can be estimated on stationary regions where ensemble averages in (1) can be substituted with space averages (i.e. by assuming mean ergodicity in a small estimation area of say $N \times M$ pixels). It should be noted, however, that even if the ergodicity hypothesis holds (i.e. all the scatterers within the estimation area are independent with identical statistical properties) the two images differ by a deterministic phase (i.e. the interferometric phase $\phi(n,m)$) that should be estimated and compensated. Thus, the following usual coherence estimation $\hat{\gamma}$ holds:

$$\hat{\gamma} = \frac{\left|\sum_{n,m} v_1(n,m) \cdot v_2^*(n,m) \cdot e^{-j\phi(n,m)}\right|}{\sqrt{\sum_{n,m} |v_1(n,m)|^2 \sum_{n,m} |v_2(n,m)|^2}} \tag{2}$$

From a computational point of view the estimation of the interferometric phase $\phi(n,m)$ (even if it can be assumed to be constant in both directions) is very time consuming. On the other hand, if it is not compensated the coherence estimate $\hat{\gamma}$ would be biased by the local frequency of the fringes that is proportional to the topography by means of the interferometer baseline (i.e. with large baselines even a rolling topography would have a strong biasing effect). Computational efficiency can be improved by using efficient frequency estimation techniques (e.g. MUSIC) instead of the accurate but inefficient periodogram technique. However, much better results can be obtained if a fringes frequency independent estimator is adopted.

3 A FREQUENCY INDEPENDENT COHERENCE ESTIMATOR

It is well know that detected SAR images are affected by the so called speckle noise [6]. In SAR interferometry, however, speckle should be considered useful signal rather than noise. Thus, speckle differences between two interferometric images can be used to estimate their coherence γ as it will be shown in the following.

Let us consider the normalized cross-correlation ρ of the detected SAR images $|v_1|^2$ and $|v_2|^2$:

$$\rho = \frac{E\left[\left(|v_1|^2\right)\cdot\left(|v_2|^2\right)\right]}{\sqrt{E\left[|v_1|^4\right]E\left[|v_2|^4\right]}} \tag{3}$$

The cross-correlation ρ can be expressed as a function of the complex coherence γ by exploiting the following well known property that holds for zero mean, normal random variables:

$$\begin{aligned} E\left[x_1x_2x_3x_4\right] &= E\left[x_1x_2\right]E\left[x_3x_4\right]+ \\ &+E\left[x_1x_3\right]E\left[x_2x_4\right]+E\left[x_1x_4\right]E\left[x_2x_3\right] \end{aligned}$$

If we pose $x_1 = v_1, x_2 = v_2^*, x_3 = v_1^*, x_4 = v_2$, then:

$$E\left[|v_1|^2|v_2|^2\right] = E\left[|v_1|^2\right]E\left[|v_2|^2\right]+E\left[v_1v_2^*\right]E\left[v_1^*v_2\right],$$

whereas, for $v_1 \equiv v_2$, we obtain $E\left[|v_1|^4\right] = 2E\left[|v_1|^2\right]^2$. If (3) is rearranged by combining the expressions above, we get the following relation between ρ and γ:

$$\rho = \frac{\overline{|v_1|^2|v_2|^2}+\left|E\left[v_1v_2^*\right]\right|^2}{2\overline{|v_1|^2|v_2|^2}} = \frac{1+\gamma^2}{2} \tag{4}$$

Thus, the absolute value of the complex coherence can be estimated as follows:

$$\hat{\gamma}_M = \begin{cases} \sqrt{2\hat{\rho}-1} & \hat{\rho} > \frac{1}{2} \\ 0 & \hat{\rho} \le \frac{1}{2} \end{cases} \text{; where} \tag{5}$$

$$\widehat{\rho} = \frac{\sum_{n,m}|v_1(n,m)|^2\cdot|v_2(n,m)|^2}{\sqrt{\sum_{n,m}|v_1(n,m)|^4\sum_{n,m}|v_2(n,m)|^4}} \tag{6}$$

The estimation of γ by means of the (5) and (6) does not need any fringes frequency estimation and thus it is much more efficient than the estimator $\hat{\gamma}$ in (2) (up to 400 times if an accurate local frequency estimation technique is adopted). Moreover it can be directly used with single look detected images.

4 A COMPARISON OF THE TWO ESTIMATORS

Fig. 1.a shows the value of the estimators $\widehat{\gamma}$ and $\hat{\gamma}_M$, averaged over 400 simulations, for different numbers of independent complex samples L within the estimation window (i.e. $L \simeq N\cdot M$ for full bandwidth images). From this figure it is clear that both estimates are unbiased only for high values of L and/or γ (i.e. $L \gg \frac{1}{2\gamma^2}$).

As far as the estimators' variances are concerned, the variance of the estimator $\widehat{\gamma}$ can be approximated for high values of L and/or γ as follows [4]:

$$\sigma^2_{\widehat{\gamma}} = \frac{(1-\gamma^2)^2}{2L}; \tag{7}$$

whereas the variance of the estimator $\hat{\gamma}_M$ can be can be approximated for large values of L, by a first order series expansion of (6,5), to get the following expression [7]:

$$\sigma^2_{|\hat{\gamma}_M|} = \frac{1}{8L\gamma^2}\left(\gamma^8+6\gamma^6-12\gamma^4+2\gamma^2+3\right)$$

The relative accuracy of the usual estimator $\hat{\gamma}$ is higher than that of the proposed estimator $\hat{\gamma}_M$. Namely they are:

$$\begin{aligned} \frac{\sigma_{\widehat{\gamma}}}{\gamma} &= \frac{1}{\gamma}\frac{(1-\gamma^2)}{\sqrt{2L}} \\ \frac{\sigma_{\hat{\gamma}_M}}{\gamma} &= \frac{1}{\gamma^2}\frac{\sqrt{\gamma^8+6\gamma^6-12\gamma^4+2\gamma^2+3}}{2\sqrt{2L}} \end{aligned}$$

As a consequence the number of independent samples L that should be used to get an accurate coherence estimate is larger for $\hat{\gamma}_M$ than for $\widehat{\gamma}$. If we indicate with L_1 and L_2 the number of independent samples used to estimate the coherence with the estimators $\widehat{\gamma}$ and $\hat{\gamma}_M$ respectively, we get the same estimation accuracy if the following ratio is maintained:

$$\frac{L_2}{L_1} = \frac{\gamma^8+6\gamma^6-12\gamma^4+2\gamma^2+3}{4\gamma^2(1-\gamma^2)^2} \tag{8}$$

The ratio between L_2 and L_1 is plotted in Fig. 1.b as a function of γ. For values of coherence greater than 0.8, $L_2 \simeq 3L_1$, for $\gamma = 0.5$, $L_2 = 5L_1$ and for $\gamma \le 0.3$, $L_2 \ge 10L_1$.

On the other hand, the frequency independent estimator $\hat{\gamma}_M$ is faster, more robust and simpler than $\hat{\gamma}$ in many respects:

(*i*) no stationarity of the local interferometric phase is required; (*ii*) it is independent of any focusing phase error that do not affect speckle; (*iii*) it can be directly applied to detected images (standard products); and finally (*iv*) it is independent of any fringes frequency estimation error.

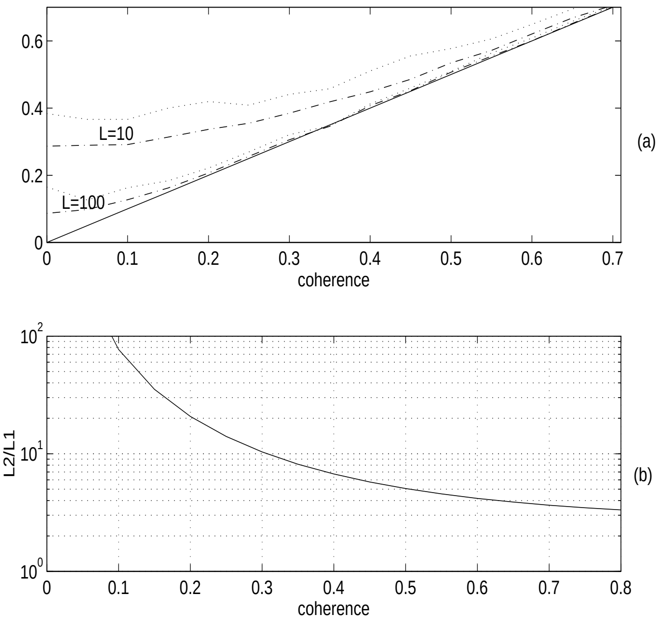

Figure 1: (a) Bias of the estimators $\hat{\gamma}$ (dashdot line) and $\hat{\gamma}_M$ (dotted line). (b) Ratio between the number of real samples L_2 and complex samples L_1 to get the same estimation accuracy.

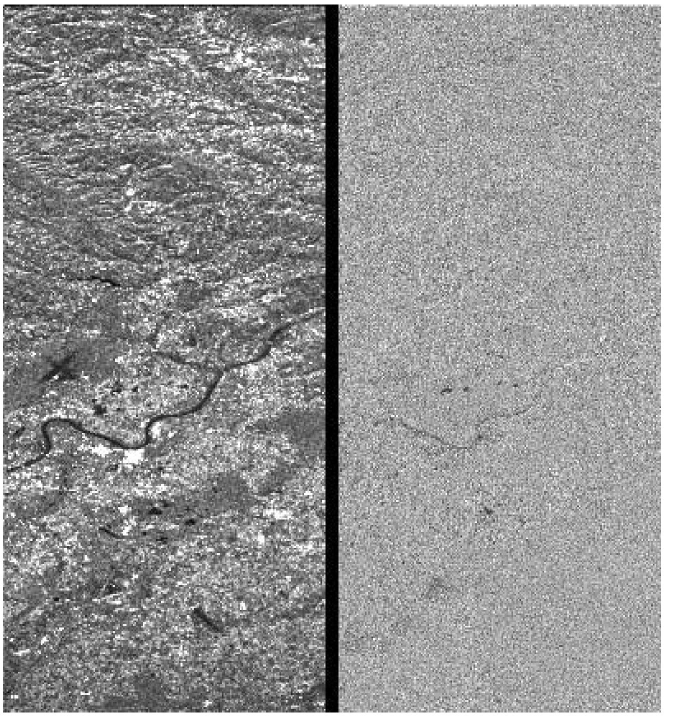

Figure 2: Detected image of the area of Bonn before and after the AGC application.

5 NON STATIONARY SCENES

Up to now, we have discussed the stationary case for the sake of simplicity. In practice, however, the effect of non stationary absolute values within the estimation window should be taken into consideration. Whenever a SAR image contains non stationary absolute values (e.g. due to different features in the area imaged, like in correspondence of mountainous areas), the proposed coherence estimator $\hat{\gamma}_M$ is strongly biased by the absolute values envelope.

As an example, let us consider the case of two stationary detected interferometric signals x and y with a given spatial correlation coefficient ρ_{xy} , the same power $\sigma^2$and mean μ .Let us now suppose to multiply both signals by a coefficient a_1 in the first half of the estimation window (here assumed one dimensional for the sake of simplicity) and a_2 in the second half. It can be shown that the spatial correlation coefficient of the resulting signals u and v has the following expression:

$$\rho_{uv} = \frac{k+\rho_{xy}}{1+k}; \text{ where } k = \frac{(a_1-a_2)^2\mu^2}{2\left(a_1^2+a_2^2\right)\sigma^2} \qquad (9)$$

From (9) it can be seen that even if the signals x and y are uncorrelated (i.e. $\rho_{xy} = 0$) the correlation coefficient of the signals u and v is greater than zero whenever $a_1 \neq a_2$. Thus, in order to reduce this effect on non-stationary SAR images, a sort of automatic gain control (AGC) should be applied to both detected images before computing the coherence estimate $\hat{\gamma}_M$.

A possible way to perform the AGC in practical cases is the following:

i - Two detected images are averaged, to reduce speckle. Averaging is performed by looks processing, or directly in the space domain by means of low-pass filtering (a 3×3 mask has been seen to be enough in all the examined cases).

ii - A small constant (white light) is added to the filtered image.

iii - Both detected images are divided by the image carried out in the first two steps.

It should be pointed out that the cost of the AGC is not negligible compared to the coherence map generation (it takes about 25% of the total computing time). Nonetheless the overall computing time is again much smaller than that required by the usual coherence estimator $\hat{\gamma}$. The effect of the AGC can be visually evaluated in Fig. 2, that shows a SAR amplitude image before and after the compensation: the speckle is still alive whereas the absolute values non stationarities have been almost completely eliminated.

6 EXPERIMENTAL RESULTS

The almost stationary ERS-1 scene of the area of Bonn shown in Fig. 2 has been used to get a visual com-

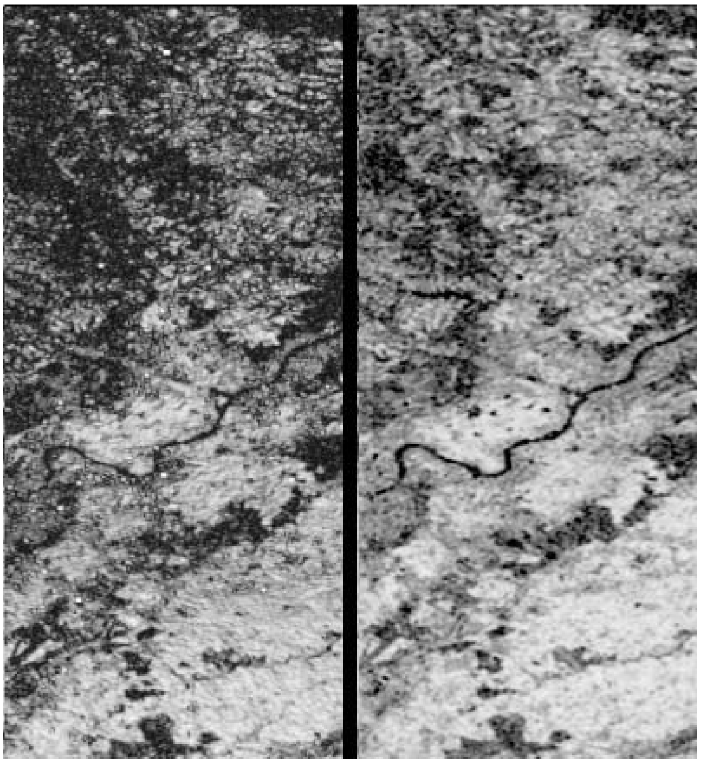

Figure 3: Coherence map carried out with the usual estimator $\hat{\gamma}$ (left); and with the proposed estimator $\hat{\gamma}_M$, and the AGC (right).

parison of the coherence maps carried out with the two estimators $\hat{\gamma}$ and $\hat{\gamma}_M$. The coherence maps generated with the estimators $\hat{\gamma}$ and $\hat{\gamma}_M$ are shown in Fig. 3. The used estimation window size was 11×11 pixels for the $\hat{\gamma}$ estimator, and 21×21 pixels for estimator $\hat{\gamma}_M$. The higher variance of the second estimator has thus been partially compensated by the larger windows. The coherence structure is well identified in both images. On the other hand, it should be pointed out that on the same workstation the first coherence map has been generated in about 30 minutes whereas the second one in about 9 seconds. Moreover, it should be pointed out that the coherence map got from the estimator $\hat{\gamma}$ (see Fig. 3) might be affected by possible errors of the estimated local frequency.

The AGC was applied to get the estimate of $\hat{\gamma}_M$ in Fig. 3: without that compensation the coherence map obtained by the estimator $\hat{\gamma}_M$ would contain several structures that resemble quite well the shape of the intensity of the detected image.

7 CONCLUSIONS

A new coherence estimator based on the similarity of the speckle signal of SAR coherent images has been presented. The new coherence estimator is much more efficient from a computational point of view with respect to the usual estimators despite of a reduced statistical confidence. It looks to be a good compromise between quality and efficiency and it could be usefully exploited to routine data browsing for generating catalogs of usable interferometric SAR images. Finally it should be pointed out that in some cases when the estimation of the local frequency of the interferograms could be problematic (e.g. in mountainous areas with large baselines), the proposed estimator might be more reliable than the usual one.

8 ACKNOWLEDGMENTS

The authors would like to thank Mr. G. Quario for the generation of the experimental results. Our gratitude goes to Prof. F. Rocca for the useful discussions and helpful hints. Finally, we would like to thank the European Space Agency (ESA/ESRIN) for the constant support and ACS for partly financing this work.

References

[1] Prati, C., Rocca, F. (1992) Range resolution enhancement with multiple SAR surveys combination. *IGARSS'92*. pp.1576-1578.

[2] Rodriguez, E., Martin, J. M. (1992). Theory and design of interferometric synthetic aperture radars. *IEE Proceedings-F*, 139(2), pp. 147-159.

[3] Gatelli, F., et. al. (1994) The wavenumber shift in SAR interferometry, *IEEE Transactions Geoscience and Remote Sensing*, Vol. 32(4), pp.855-865.

[4] Hagberg, J. O., Ulander, L. M. H., Askne, J. (1995) Repeat-pass SAR interferometry over forested terrains, *IEEE Transactions Geoscience and Remote Sensing*, Vol. 33(2), pp.331-340.

[5] Solaas G. A., *ERS-1 Interferometric Baseline Algorithm Verification*, ESA rep. n. ES-TN- DPE-OM-GS02, Aug. 1994

[6] Prati, C., Rocca, F. (1990) Limits to the resolution of elevation maps from stereo SAR images. *International Journal of Remote Sensing.* Vol.11(12), pp.2215-2235.

[7] Monti Guarnieri, A., Prati, C. SAR Interferometry: a "Quick and Dirty" Coherence Estimator for Data Browsing submitted to *IEEE Trans. on Geosci. and Remote Sensing* Jan. 1995.