DATA ASSOCIATION AND TRACKING FROM ACOUSTIC DOPPLER AND MAGNETIC MEASUREMENTS

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ABSTRACT

This paper is devoted to the localisation problem of acoustic-magnetic sources moving in straight line at constant speed. Our technique is based on the association of Acoustic Doppler and Magnetostatic Methods. The objective of this study is to achieve localisation with only *one* sensor performing both frequency and magnetic measurements. The set of possible location is shown to be a circle since no angular information is available. The subsequent developments describe an Extended Kalman Filter with a linear observation equation to perform maximum performance in case of poor initialisation. The filter convergence is actually ensured when tested with simulated signals. A small residual bias on the velocity estimate is however noticed due to the non linearity of the prediction equation.

1 INTRODUCTION

In this paper we present a localisation method of acoustic-magnetic sources moving in straight line at constant speed. In next part we introduce briefly Acoustic Doppler and Magnetostatic Matched Filter Techniques that both require straight line and constant speed movement. Then we produce an interesting Extended Kalman Filter that is expected to provide good convergence performances since its measurement equation is linear. Finally we show simulation results and discuss the properties of this data association method.

2 COMPARISON OF DOPPLER BASED AND MAGNETOSTATIC TECHNIQUES

The straight line and constant speed movement is a strong hypothesis that recondition a very ill-posed problem. Indeed magnetic and omnidirectional acoustic measurements have very poor informative content. If no hypothesis can be done, several simultaneous measurements must be available to yield position estimates [1, 2, 3]. In some applications however, the number of sensors cannot be that many and as few as one device may be used. In this case, additional hypothesis on the relative motion between source and sensor are necessary. Then one should consider measured signals as signatures since the informative content is actually due to their relative motion. As a result, the Closest Point of Approach (CPA) plays a determinant role in both treatments.

In the following we will assume that the same device may simultaneously perform acoustic and magnetic measurements. A consequence is that two omnidirectional measurements done at the same site cannot yield any angular information. Therefore, the best expected result is to localise the source in a relative coordinate system.

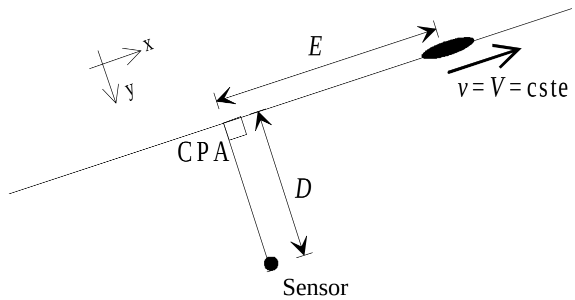


Figure 1: illustration of geometry

2.1 Acoustic Doppler Shifts

Acoustic emissions undergo Doppler shifts due to the relative motion of the emitter and the receiver. In the case of narrow band signals, a common approach [4] involves the use of short-time periodograms to provide Maximum Likelihood estimates of instantaneous frequency. Tracking algorithms such as $\alpha\beta$ filters are then applied to the sequence to eliminate false alarms and to smooth the final estimates. Except when SNR are sufficiently large, frequency rate-of-change is usually not available.

In polar coordinates, the Doppler effect is described by:

$$f = f_0 \frac{C}{C + \dot{r}} \quad \mathbf{(1)}$$

where f_0 denotes the emitted centre frequency, C the wave velocity in the medium and r the polar distance between the emitter and the receiver. In the Cartesian coordinate system proposed on figure 1, geometrical considerations lead to the following equivalent expression:

$$f = f_0 \frac{C r}{C r + x \dot{x}} \quad \text{since} \;\; V = \dot{x} \tag{2}$$

Taking the derivative of this expression and eliminating f_0 by using (2):

$$V^2 y^2 + V x \frac{\dot{f}}{f} r^2 + \frac{\dot{f}}{f} r^3 C = 0 \tag{3}$$

Thus, velocity may be obtained by solving a second order equation. The same kind of second order equation may also be obtained in polar coordinates. However, it is of interest to notice that equation (3) simplifies if a modified Cartesian (MC) coordinate system is used. This new coordinate system is defined by the preceding Cartesian direction with coordinate values given relatively to the velocity V:

$$\left\{ \tfrac{x}{V} \frac{\vec{x}}{\|x\|} ,\ \tfrac{y}{V} \frac{\vec{y}}{\|y\|} \right\} \tag{4}$$

In the modified Cartesian coordinate system, one may easily show that equation (3) becomes:

$$V = \frac{-C r_V^3}{f / \dot{f}\, y_V^2 + x_V r_V^2} \quad \text{where} \;\; (\;)_V = \frac{(\;)}{V} \tag{5}$$

2.2 Magnetostatic physical basis and estimation

Ferromagnetic objects are surrounded by a magnetostatic field and behave as magnetic dipoles, provided that they are far enough from the sensor. Such sensors measure the vectorial perturbation **B** due to this field in the earth field, or its projection along the earth field:

$$\mathbf{B} = \frac{\mu_0}{4\pi r^5} \left[3\mathbf{x}\mathbf{x}^{\mathrm{T}} - r^2 \mathbf{I} \right] \mathbf{M} \tag{6}$$

where **M** denotes the dipolar moment, μ_0 is the air permittivity and **x** denotes the vector between source and sensor.

With a good approximation the signature of a constant speed source may be decomposed on an orthogonal Anderson basis [5] governed by a set of two parameters (cf. figure 1):

$$\{ E/V,\ D/V \} = \{ x/V,\ y/V \} \tag{7}$$

It is therefore possible to perform a 2-D matched filter on these signals [6] yielding an estimate of the former set of parameters as well as detection criteria. A specific difficulty of this technique is that estimates are only available shortly before the CPA.

We underline that *D/V* is unsigned and must be strictly considered as a distance. On the contrary *E/V* is arbitrarily signed since its origin is usually set to the CPA. However, the orientation of the trajectory is arbitrary. It could be strictly set if two sensors were used.

3 DATA ASSOCIATION WITH AN EXTENDED KALMAN FILTER

3.1 Prediction and measurement equations

As we underlined in the two former subsections, there is some interest in both Acoustic and Magnetic Methods for using the modified Cartesian (MC) coordinate system. Therefore we derive an Extended Kalman Filter using these coordinates. As will be shown it provides a linear measurement equation and simplifies the prediction equation by using (5).

In the case of only one sensor recording useful signal, the measurement vector may be written:

$$\mathbf{Z} = \left[E/V \quad D/V \quad f \right]^{\mathrm{T}} = \left[x/V \quad y/V \quad f \right]^{\mathrm{T}} \tag{8}$$

In MC coordinates, the state vector and the prediction equation are given by:

$$\begin{bmatrix} x/V \\ y/V \\ V \\ f \\ \dot{f} \end{bmatrix}_{k+1} = \begin{pmatrix} x_1 + t_s \\ x_2 \\ \dfrac{C\left(x_1^2 + x_2^2\right)^{1.5}}{\frac{x_4}{x_5} x_2^2 + x_1\left(x_1^2 + x_2^2\right)} \\ x_4 + t_s x_5 \\ x_5 \dfrac{\left(x_1^2 + x_2^2\right)^{1.5}}{\left(x_1^2 + x_2^2 + t_s^2 + 2 x_1 t_s\right)^{1.5}} \end{pmatrix} \begin{bmatrix} x/V \\ y/V \\ V \\ f \\ \dot{f} \end{bmatrix}_k + \Gamma w_k \tag{9}$$

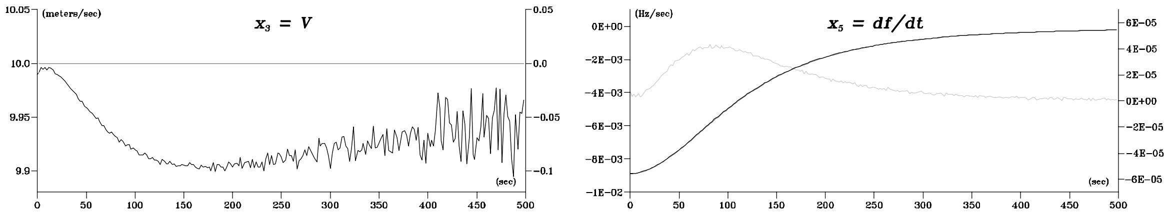
where $(\;).[\mathrm{X}]$ denotes a non linear fonction of the vector X.

The third line of (9) is in fact equation (5): since speed is assumed constant over time, the prediction of velocity between k and k+1 may be replaced by an exact formulation in terms of state vector components. This should allow rapid convergence of velocity estimate in case of poor initialisation.

Because of line three, $\dot{f}$ must be added to the state vector as in (9). Deriving (2), taking this expression at time instant k and k+1, and calculating their ratio yield the fifth line of (9).

The first, second and fourth line of (9) are the simplest prediction equation one can use. On the contrary of the third and fifth state variables they are also measured quantities, what ensure fast convergence in case of poor initialisation.

Our formulation should be considered as a smoothing procedure of measured quantities. This smoothing enables an accurate estimation of the frequency rate-of-change, which in turn yields the velocity estimate. Indeed velocity may be seen as the lacking parameter ensuring data coherency.



Left scale: state estimate
true value
Right scale: error

Figure 2: Noise free simulation initialised at CPA. State variables x_1, x_2 and x_4 are directly measured and estimated without errors. On the left hand side, the state estimate curve hides the error curve since left and right scales are simply shifted. On the right hand side, state estimate and measured quantity are very close. Their difference is shown in light grey and is found to be strongly correlated with the velocity error.

3.2 Initialisation

This tracking is initialised at CPA because magnetic estimates are not available before. However the CPA time is reliable and allows a trustworthy initialisation of velocity, since f_{asympt} may be roughly estimated from the past shifted line frequency:

$$V_{init} = C\left(f - f_{asympt} \right) \Big/ f_{asympt} \tag{10}$$

3.3 Residual bias analysis and asymptotic performance on velocity

When conducting noise free simulations, the three “measured state variables” show unbiased estimates while velocity and frequency rate-of-change exhibit residual bias as can be seen on figure 2. This behaviour is explained by the strong sensibility of the velocity expression in (9) to frequency rate-of-change errors. Indeed the fifth line of (9) gives rise to small errors that cannot be corrected by measured quantities such as frequency.

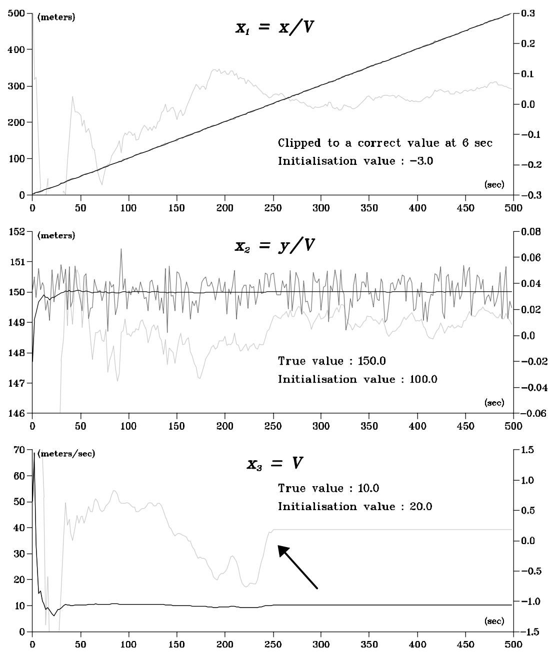
When the frequency tends to its asymptotic value the relative error on the frequency rate-of-change increases. As a consequence, statistical errors on velocity rise. The velocity formulation in (9) should be modified and account for a trade-off between the ability of rapid convergence at the beginning of the tracking and the possibility of using a simpler expression at the end. We propose the following prediction equation:

$$x_{3,k+1} = \left[(1-\xi) x_3 - \xi \frac{C\left(x_1^2 + x_2^2\right)^{1.5}}{\dfrac{x_4}{x_5} x_2^2 + x_1\left(x_1^2 + x_2^2\right)} \right]_k \tag{11}$$

where:

$$\xi = \frac{x_4 - f_{asympt}}{f_0 - f_{asympt}} \tag{12}$$

When ξ becomes smaller than a given threshold, its value is forced to zero to prevent negative frequency rate-of-change. This can be seen on the velocity curve of figure 3 (black arrow).



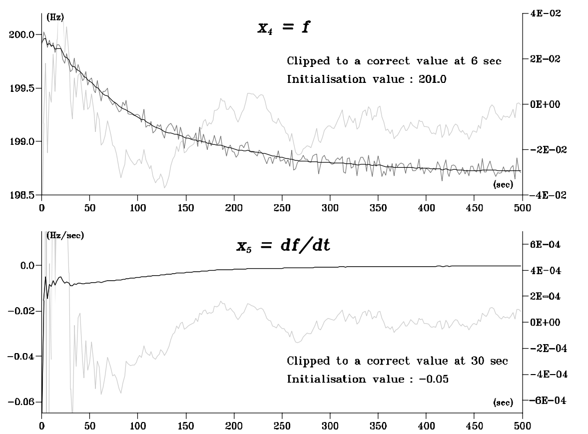


Figure 3: Noisy and poorly initialised simulation. Input and output of the data association process. With the notation of figure 1: *D*=1.5km, *V*=10m/s, the emitted narrow band noise is centered around 200Hz. The sampling rate is 0.5Hz.

Left scale: state estimate
measured quantity
Right scale: error

4 SIMULATION

The undergoing simulation illustrates the convergence ability of the filter. Input signals are plotted in dark grey, estimated (output) signals are black. Both refer to the left hand side scale. Light grey curves show the error between estimated quantities and true values. They refer to the right hand side scale.

The smoothing effect on the measured state vector can be easily seen, except on the first plot because of the scale dynamic.

One can also check that the error on velocity and on the frequency rate-of-change are strongly correlated. At time t=250 sec., the forgetting factor ξ is forced to zero as can be seen on the speed curves. On the same curve, a residual bias of the order of 0.2m/s is reached.

This simulation is poorly initialised. The following values are used:

$$\mathbf{x}_0 = \begin{bmatrix} -3.0 \\ 150.0 \\ 20.0 \\ 201.0 \\ -0.05 \end{bmatrix} \rightarrow \begin{matrix}\text{Relative} \\ \text{errors :}\end{matrix} \begin{bmatrix} NaN & \text{w.r.t.} & 0.0 \\ 50\% & \text{w.r.t.} & 100.0 \\ 100\% & \text{w.r.t.} & 10.0 \\ 77\% & \text{w.r.t.} & 1.3 \\ 44\% & \text{w.r.t.} & -.0888 \end{bmatrix} \qquad \textbf{(13)}$$

The state covariance matrix is set to:

$$\mathbf{P}_{kk} = \mathbf{Id}\left[5.0\ 5.0\ 10^{-2}\ 10^{-1}\ 10^{-3}\right] \qquad \textbf{(14)}$$

Nevertheless, convergence properties are excellent. An other model in usual Cartesian coordinates was tested. This filter is not linear with respect to measured variables and as predicted in [7] shows worse convergence properties: it does not converge in this case.

5 CONCLUSION AND PERSPECTIVES

This data association yields better estimates of localisation and velocity in case of poor initialisation, than formulations using polar or Cartesian coordinates do. No angular information is available through measurements though. Setting the geometry of figure 1 with respect to an absolute direction such as the North direction is therefore impossible. Two techniques may be used to overcome this difficulty. If the sensor number must be kept as low as possible, only one sensor records simultaneously useful signals. However, we may keep the prediction going when exceeding the sensor's range until the source reaches an other sensor. This approach assumes that the straight line and constant speed movement is strictly verified and leads to the intersection of two circles. If the sensor density may be increased, algorithms involving two sensors recording simultaneously useful signals may be investigated. This approach is presented in [9].

6 REFERENCES

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