ABSTRACT
The problem of detecting a known signal in background reverberation with an estimated reverberation spectrum is addressed. In our approach, the prewhitener is a wideband inverse filter estimated from a large data base of reverberation spectra. Simulations and experimental results are presented in the context of detecting a target lying on the seafloor with a wideband transducer. The proposed detector is compared to an AR prewhitener. The results indicate that the proposed detector is well suited for our wideband application.

2. PROBLEM FORMULATION
The problem of detecting a known signal in a background reverberation is addressed. Formally, one has to decide between two hypotheses:

\[ H_0: x_i(n) = n_i(n), \quad i = 1..N \]
\[ H_1: x_i(n) = s(n) + n_i(n) \]

where the variable \( n \) denotes the discrete time, \( s(n) \) is the deterministic and known signal to detect, \( n_i(n) \) the \( i \)th realization of the additive noise and \( x_i(n) \) the \( i \)th received signal.

The detection problem in (1) can be represented in the frequency domain using the Fourier coefficients at frequency \( f_j \):

\[ H_0: X_i(f_j) = N_i(f_j) \]
\[ H_1: X_i(f_j) = S(f_j) + N_i(f_j) \]

We refer to the following assumptions.

A1) The Fourier coefficients of the noise spectrum at different frequencies are complex random variables with zero mean and equal variances.

A2) The coefficients \( X(f_j) = [X_{i1}, X_{i2}, ..., X_{iN}]^T \) are independent in that sense that the repetition
time of the transmitted signal is higher than the duration of the signal \(\frac{T_R}{T} >> 1\).

A3) The time bandwidth product is large enough \(\left(\frac{TB}{>> 1}\right)\), so the Fourier coefficients of the \(i\)th noise spectrum \(X_i(f; H_0) = [X_{i1} X_{i2} \ldots X_{iM}]\) are practically uncorrelated.

The decision rule applied compares the generalized likelihood ratio test (GLRT) \(\lambda(X)\) to a threshold \(\lambda_0\)

\[
\lambda(X) = \frac{p(X/H_1, \hat{\theta}_{ML})}{p(X/H_0, \hat{\theta}_{ML})} \begin{cases} > \lambda_0 \text{ decide } H_1 \\ < \lambda_0 \text{ decide } H_0 \end{cases}
\]  

(3)

where \(\hat{\theta}_{ML}\) are the maximum likelihood (ML) estimates of parameters taking into account the coloration and the variability of the noise.

The performances of the detector depend on the \(i\)th signal-to-noise ratio at time \(n_0\) at the output of the receiver

\[
\rho_i(n_0) = \frac{\sum_1^M H(f)S(f)e^{j2\pi fn_0}}{\sum_1^M |H(f)|^2 \Phi_{Xi}(f; H_0)}, \quad i = 1, \ldots, N_i, \quad (4)
\]

where

\[
H(f) = |H_1(f)|^2 S^*(f)e^{-j2\pi fn_0}, \quad f = \sum_{j_1}^{j_{max}} \sum_{j_2}^{j_{max}} \quad (5)
\]

\(H_1(f)\) is the transfer function of the prewhitener filter, \(S^*(f)e^{-j2\pi fn_0}\) the transfer function of the matched filter and \(\Phi_{Xi}(f; H_0) = (X_i X_i^*) (f; H_0)\) is the power spectrum of the \(i\)th realization of the noise under the \(H_0\) hypothesis.

It appears in (4) that the performances of the detector strongly depend on the adjustment of the coefficients of the prewhitener with respect to the noise variability.

3. WIDEBAND INVERSE FILTER

A set of noise reverberation data given by a wideband stochastic process is considered. We want to design the prewhitener filter that better fitted to the data set in the ML sense.

A basic formulation of the prewhitener coefficients is given by:

\[
e_i(f_j) = |H_i(f_j)|^2 \Phi_{Xi}(f; H_0) \quad (6)
\]

where \(e_i\) is the noise at the output of the inverse filter which ideally should be white with a spectral level \(N_0/2\).

A better approximation of whitening coefficients has to take into account both contributions: variability and coloration of the noise. This is expressed by:

\[
e_i(f_j) = a_i \Phi_{Xi}(f_j) - \frac{N_0}{2} \alpha_i \quad (7)
\]

or using a more convenient form:

\[
e_{ij} = a_{ij} \Phi_{Xi}(f_j) - \frac{N_0}{2} \alpha_i \quad (8)
\]

where \(a_{ij}\) is the coefficient of the inverse filter at frequency \(f_j\), \(\alpha_i\) a compensation term function of the noise coloration and \(e_{ij}\) are zero-mean Gaussian random variables with variance \(\sigma^2\):

\[
p(e_{ij}) = \frac{1}{\sigma \sqrt{2\pi}} \frac{e^{-\frac{e_{ij}^2}{2\sigma^2}}}{2\sigma^2} \quad (9)
\]

Using assumptions A1, A2 and A3, \(e_{ij}\) are independent and identically distributed with respect to both indices. Therefore the pdf at frequency \(f_j\) can be written as:

\[
p(e_{ij}) = \prod_{i=1}^N p(e_{ij}) = (2\pi\sigma)^{-N/2} e^{-\sum_{i=1}^N \frac{e_{ij}^2}{2\sigma^2}} \quad (10)
\]

To find the ML estimate of the prewhitener coefficient at frequency \(f_j\) \((\hat{a}_{ML}(f_j))\), we maximize \(\ln p(e_{ij})\):

\[
\frac{\partial \ln p(e_{ij})}{\partial a_{ij}} = -\frac{1}{\sigma^2} \sum_i \Phi_{Xi}(a_{ij} \Phi_{Xi} - \frac{N_0}{2} \alpha_i) \quad (11)
\]

The resulting estimate is found to be equal to:

\[
\hat{a}_{ML}(f_j) = \frac{N_0}{2} \sum_i \Phi_{Xi}^2 \quad (12)
\]

In a similar fashion, the estimate of \(\alpha\) is found to be equal to:
\[ \hat{\alpha}_{mi} = \frac{2}{MN_0} \sum_{j=1}^{M} a_j \Phi_{xj} \]  
(13)

Assuming we have minimized the part \( \sum_i \left( a_i \Phi_{xj} - \frac{N_0}{2} / \alpha_i \right)^2 \) of the log-likelihood function to produce \( a_i \), then:

\[ \frac{1}{M} \sum_{j=1}^{M} a_j \Phi_{xj} \rightarrow \frac{N_0}{2} \]  
(14)

and (12) can be approached by:

\[ \hat{\alpha}_{ML}(f_j) \approx \frac{N_0}{2} \sum_i \Phi_{xj} \]  
(15)

4. RESULTS

The previous results are applied in the context of detecting a target lying on the seafloor. The experiment was held on Lake Geneva using a wideband sonar system with a constant directivity in the frequency range 20-140 kHz [3]. The transmitted signal is a linear frequency modulated waveform with a large time bandwidth product \( TB = 120 \) and a pulse repetition period to transmitting time \( T_R/T = 1000 \).

The detection of a sphere target on a silty lake bottom is investigated. The data echoes are characterized by the input signal-to-noise ratio. The output signal-to-noise ratio at time \( n_0 \) is computed by applying Eq. (4).

Fig. 1 shows the variability and the coloration of noise power spectra in the case of the silty bottom reverberation. Fig. 1 also compares the noise power spectra to the spectrum of the target to detect.

The performances of two different detectors are compared. Detector 1 (D1) consists of an AR prewhitener and a matched filter. Detector 2 (D2) consists of the wideband inverse filter and a matched filter. Fig. 2 summarized the performances of these detectors in term of signal-to-noise ratio. The amplitude in Fig. 2 is given by the ratio \( p_1(n_0)/\text{snr} \) as a function of the ith signal. snr denotes the linear value of the input signal-to-noise ratio and SNR the corresponding log value. Fig. 3 give an example of experimental data and the output receivers corresponding to D1 and D2.

Fig. 1. (---) Noise power spectra of the silty bottom reverberation. (—) Spectrum of the sphere target to detect.

Fig. 2. Output of detector D1: (---) SNR = 0 dB and (---) SNR = 6 dB. Output of detector D2: (---) SNR = 0 dB and (---) SNR = 6 dB.

5. CONCLUSION

The findings have shown the ability of our wideband receiver to detect a known signal in a colored and variable background reverberation.
The detector has been compared to an AR prewhitener and has shown better performances in term of signal-to-noise ratio. This is because, by controlling the reverberation noise coloration and the variability, the noise at the output of the wideband inverse filter is better whitened.

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6. BIBLIOGRAPHY


Fig. 3. From upper to lower. Experimental data. Output of detector D1. Output of detector D2.