CURVED SURFACE RECONSTRUCTION USING MONOCULAR VISION

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ABSTRACT

In monocular vision, a priori knowledge is necessary to perform 3D reconstruction. This paper describes how to evaluate two out of six external parameters of a camera in order to project an image on a curved surface (generalized cylinder). The final aim consists of reconstructing the model of the surface. Afterwards, with this model we can derive a flat representation of the scene without any distortions due to the projective geometry. In this work based on one projected view of the scene, we develop two methods to detect the projection of the revolution axis of the curved surface. With this axis, we can then extract the external parameters of a camera. The first one is based on the derivation of a polynomial function and the second one is based on the detection of the common normal between curves.

1 Introduction

Generally, picture understanding consists of reconstructing the observed 3D universe. Frequently, this universe is composed of a scene with complex surface objects. In this work, we limit our study to the case when the surface object has a curvature different than zero in only one direction. We prove how to locate an image on a 3D surface and how to project it in the space. Six external parameters are needed to project an image on a 3D surface. Three parameters correspond to a rotation and three to a translation between the camera coordinate system and the curved surface. The originality of this work consists of using certain a priori knowledge of the surface. We consider known the parallels which are projected in the image plane. In our approach, based on the projections of the parallels we detect the projection of the revolution axis in the image. From this projection we derive 2 out of 6 external parameters. This work is applied to art paintings, especially to mural paintings and pictures on columns.

Similar work has been done in domain of monocular vision. The inflexion points of curves are used to interpret the perspective projection of these curves [Richetin 87]. Using matching between elliptic contours on the image and circular frontiers of the model, Dhome proved how to determine the spatial attitude of circular objects. Interpreting a triplet of image lines as the perspective transformation of a triplet of linear ridges of the object model, this determination has been found as well [Dhome 89].

2 Finding the projection of the revolution axis

We want to find the projection of the revolution axis of a surface in the image. It is possible to find a revolution axis by using mathematical morphology [Brady 83], finding local symmetries [Ponce 89] or by using a method based on expectation-maximization [Glachet 91].

2.1 Definition of parallels

On the object surface, curves with a constant altitude are called parallels. Parallel curves are detected in the image plane corresponding to the projection of parallels belonging to the 3D surface. The position of the parallel curves consists of our a priori knowledge. The localization of the revolution axis projection is based on these curves from the image. In the first method presented in Section 2.2 we present how to approximate the curves by parabolic functions, and afterwards how we detect the extrema of these functions. In our second method presented in Section 2.3, we search for the common normal of two curves. The results of these methods will provide points situated on the projection of the revolution axis. These are necessary in order to match the image on the 3D surface.

2.2 Derivative method

We assume that curves are approximated by parabolic functions. Each curve is described by an polynomial equation of second degree $z = a_0 + a_1 y + a_2 y^2$. In our method, we use parallels with the same curvature direction on the surface.

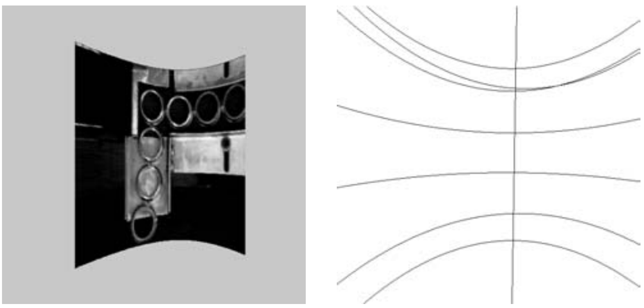
Using convexity properties and the derivative of these polynomials we obtain one point of the axis on each curve. So, for N detected curves, we obtain N polynomials :

$$y_i(x) = a_{i0} + a_{i1}x + a_{i2}x^2,\ i \in [1, N] \qquad (1)$$

Computing the derivative of these polynomials we obtain extrema corresponding to the axis points. So, we have :

$$\frac{dy_i(x)}{dx} = a_{i1} + 2a_{i2}x = 0,\ i \in [1, N] \qquad (2)$$

We obtain N points with $x_i = -\frac{a_{i1}}{2a_{i2}}$ *and* $y_i = y_i(x_i),\ i \in [1, N]$.



(a) (b)

Figure 1: *Detection of an axis using the derivative of functions with a priori knowledge.*

Parallel straight lines present the scene illustrated in Figure 1.(a) are approximated by parabolic curves shown in Figure 1.(b). The projection of the revolution axis presented in Figure 1.(b) is obtained from the detection of extremum on each parabolic curves.

2.3 Computation of the curves common normal

This approach is based on the analysis of the curve shape resulting from the projection of parallels existing on the curved surface. Let us suppose that two curves C_1, C_2 in the image plane correspond to the projection of two parallels. Thus, in this approach we first identify the common normal P_1P_2 of the two curves C_1 and C_2, as shown in Figure 2. As described in [Puech 95b] we use an iterative method. In the initialization phase, we select a point M_1 on the curve C_1. M_2 is defined as the intersection of the curve C_2 and the normal line to C_2 passing through M_1. Afterwards, we determine a new point $M_1^{'}$ on the curve C_1, where the normal line to C_1 passes through M_2. Next, we iterate the method successively for the curves C_1 and C_2. This method stops when two successive points on the same curve are very close to each other.

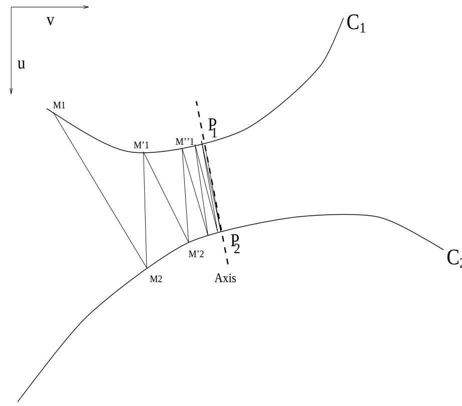


Figure 2: *Common normal for a curved surface.*

The main routine to find the common normal described in the algorithm Figure 3 is :

<u>Main routine :</u>
while $(|x_1 - x_{1prec}| > threshold)$ and $(|x_2 - x_{2prec}| > threshold)$
‖ **calculate** $y_1 = f_1(x_1)$
‖ **search** normal to P_2 passing through M_1 :
‖ ‖ $x_2 = norm(M_1, f_2, 0, x_{max}, threshold)$
‖ **calculate** $y_2 = f_2(x_2)$
‖ **search** normal to P_1 passing through M_2 :
‖ ‖ $x_1 = norm(M_2, f_1, 0, x_{max}, threshold)$
end_while

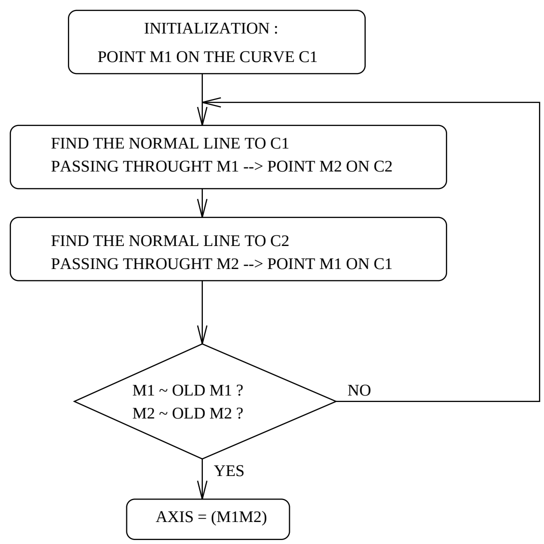


Figure 3: *Algorithm to find the common normal.*

To use the main routine we need to find the normal line to a curve passing through a point M as shown Figure 4. This algorithm is described in the following :

Search : the normal line to f passing through M
norm(M, f, interval x_1 and x_2, threshold)

if $(|x_2 - x_1| < threshold)$ **then**
  return($middle(x_1, x_2)$)
else
  calculus of the points
    $x_3 = middle(x_1, x_2)$
    $y_i = f(x_i)$, $i \in \{1, \ldots, 3\}$
  calculus of the derivative vectors
    $\vec{u_i} = (1, y_i')$ to N_i, $i \in \{1, \ldots, 3\}$
  calculus of the scalar products
    $s_i = \vec{N_i M}.\vec{u_i}$, $i \in \{1, \ldots, 3\}$
  if $(sign(s_3) = sign(s_2))$ **then**
    the point is between N_1 and N_3
    return($norm(M, f, x_1, x_3)$)
  else
    the point is between N_3 and N_2
    return($norm(M, f, x_3, x_2)$)
  end_if
end_if

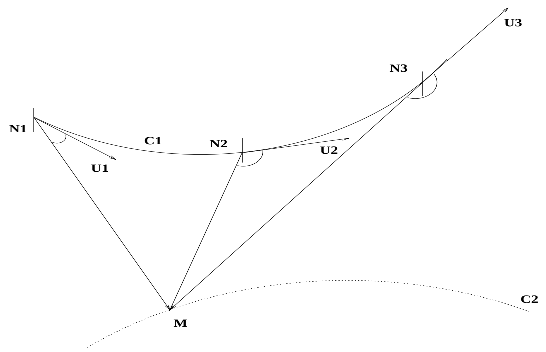


Figure 4: The normal line to a curve passing through a point.

In the following we analyse the convergence of the proposed method. Let us define d_n as the distance between M_1 and M_2, and d_{n+1} the distance between M_2 and the new point M'_1. By searching the normal line passing through these two points we built this string having $d_{n+1} < d_n$. We obtain then $d_{n+1} < d_n$ because $|M_2M'_1|$ minimizes the distance between M_2 and the first curve. So the distance decreases and this proves that the algorithm converges.

2.4 Interpretation

For a convex surface, in the 3D space, the detected axis corresponds to the set of points which are at maximal distance from the viewpoint, Figure 5 (a). Indeed, by perspective projection, if a pair of points of two parallels of the scene are farther away from the camera, these points are closer each of the other in the image. The computation of the axis in the image provides us with information about the rotation of the focal axis and also about a translation. Consequently, we can locate the image on the 3D surface [Puech 95a].

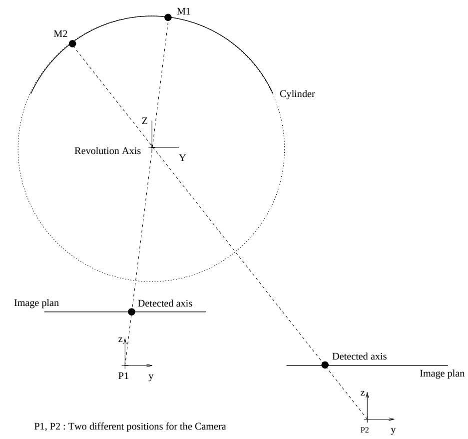


Figure 5: *Position of the image on the 3D surface.*

3 Results

In this Section we illustrate the result of our method on two different examples. The first example is a view of a bottle with a label and the second one is a painting on a vault.

(a) (b)

Figure 6: *3D Reconstruction with only one image.*

In Figure 6 (b) we display the picture after reconstruction by using the parameters detected with the second method. It is possible to compare this image with the original one, shown in Figure 6 (a). In this fist example, Figure 6 (a), the parallels of the surface used to perform our method are the opposite borders of the label.

(a) (b)

Figure 7: *Visualization from other viewpoints*

Moreover, we can obtain other views from different viewpoints. Figure 7 (a) illustrates the result after a vertical rotation, and Figure 7 (b) after an horizontal rotation. In Figure 8 we display the resulting image after 3D reconstruction and back projection on a plane in order to obtain an image without any distortion due to the projective geometry.

Figure 8: *The view without distortion after flattening.*

The second example is a picture of a painting on a vault presented in Figure 9. To find the projection of the revolution axis position we detect two curves as shown in Figure 10. These two curves are supposed parallel in the 3D scene. With this localization we can perform the back-projection on the curved surface and flatten it in order to obtain an image without any distortion due to the projective geometry as shown in Figure 11.

Figure 9: *Mural painting.*

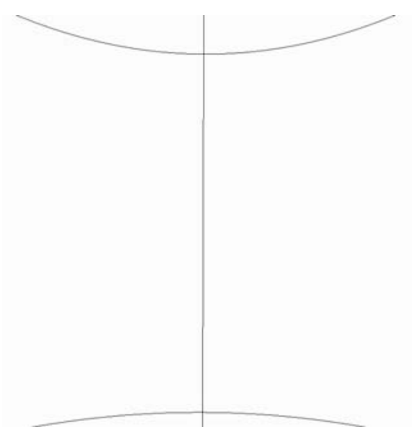


Figure 10: *Parallel curves detection and localization of the revolution axis.*

Figure 11: *Flattened picture after back projection.*

4 Conclusions

In this study we have shown how to perform 3D surface reconstruction from monocular vision using a priori knowledge about the object geometry. Two hypotheses have been proposed: considering that the scene contains parallels and that the picture is projected on a surface characterized by only one curvature axis. Two methods have been proposed to perform such a reconstruction. Real-life examples have illustrated each method. New projections of the reconstructed image from various viewpoints have been presented as well in this study.

References

[Brady 83] M. Brady. *Criteria for Representations of Shape.* Human and Machine Vision, A. Rosenfeld and J. Beck, New York, 1983.

[Dhome 89] M. Dhome, M. Richetin, J.-T. Lapresté, and G. Rives. Determination of the Attitude of 3D Objects from a Single Perspective View. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 11, no. 12, pp. 1265–1278, Dec. 1989.

[Glachet 91] R. Glachet, M. Dhome, and J.T. Lapresté. Finding the Perspective Projection of an Axis of Revolution. *Pattern Recognition Letters*, vol. 12, no. 11, pp. 693–700, Nov. 1991.

[Ponce 89] J. Ponce, D. Chelberg, and W. Mann. Invariant Properties of Straight Homogeneous Generalized Cylinders and their Contours. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 11, no. 9, pp. 951–966, Sep. 1989.

[Puech 95a] W. Puech and J.-M. Chassery. Détection d'axe sur surfaces courbes en vision monoculaire. *In Proc. 15th. GRETSI*, vol. 2, pp. 877–880, Juan Les Pins, France, 1995.

[Puech 95b] W. Puech, P. Schott, and J.-M. Chassery. Discrete Method of Calculus Using Common Normals for Curved Surface Reconstruction. *In Proc. 5th. Discrete Geometry for Computer Imagery*, pp. 83–92, Clermont-Ferrand, France, 1995.

[Richetin 87] M. Richetin, M. Dhome, and J.T. Lapresté. *3D Attitude of Curves From Projection Images. AFCET Reconnaissance des Formes et Intelligence Artificielle*, pp. 671–686, Antibes, France, 1987.