PARAMETER IDENTIFICATION OF FREQUENCY-SELECTIVE NOISY FAST-FADING RAYLEIGH DIGITAL CHANNELS VIA NONLINEAR YULE-WALKER-LIKE EQUATIONS

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ABSTRACT

A new procedure is proposed for the identification of data channels affected by randomly time-variant fading. It is based on a set of nonlinear equations employing a minimum number of lags of the observed autocorrelation function (acf), and its solution gives the desired channel fading parameter estimates. Better estimation accuracy is obtained in comparison with the use of classic higher-order Yule-Walker procedure (although this latter employs a linear equation system), in particular for small Doppler spreads and for signal-to-noise ratios not very high.

1 INTRODUCTION

Data transmission over time-varying frequency-selective fading multipath channels is a classic topic in many applicative areas of the communication field such as in HF radio links or in mobile communications [1,Ch.7]. Several strategies have been investigated in the past years to develop efficient detectors in such a difficult environment (see, e.g., [2,3,7]). A common solution is constituted by a channel estimator (CE) which feeds a detector (e.g., a Viterbi decoder) with the current channel estimate. In this context it is generally assumed that the fading process is wide-sense stationary, and that its second-order statistics are known to the CE.

However, in practice this is not the case [2], so that a preliminary identification procedure must be carried out to calculate the channel parameters before tracking the channel and finally entering into the data-detection operative mode.

This work has been supported by the European R&D, ACTS-MOSTRAIN programme.

If the fading cannot be assumed stationary, the identification procedure must be periodically repeated in order to take into account the so-called "long-term effects" of the fading process [2].

2 THE SYSTEM MODEL

The following complex baseband-equivalent discrete-time observation model is assumed:

$$y(i) = \sum_{m=0}^{L-1} g(i;m)\, a(i-m) + v(i), \qquad (1)$$

where a(i) is the transmitted data sequence, generally complex (e.g., PAM, QAM or PSK); v(i) is a complex stationary zero-mean additive white noise sequence with variance N_0; g(i;m), $0 \le m \le L-1$, are L complex discrete-time Rayleigh-distributed fading processes, assumed wide-sense stationary (WSS).

After introducing the L-dimensional complex vectors $G^T(i)$ = [g(i;0) g(i;1) ... g(i;L-1)] and $x(i)^T$ = [a(i) a(i-1) ... a(i-L+1)], eq.(1) can be rewritten as

$$y(i) = G^T(i)\, x(i) + v(i). \qquad (2)$$

The power density spectra of the fading processes can be often adequately modelled by Butterworth-type linear filters of appropriate order and with bandwidths (or Doppler-spreads) $\{B_D^{(m)},\ 0 \le m \le L-1\}$ for the L univariate components {g(i;m)} [1,Ch.6], [4,Tab.I], [3], [5]. According to [1,Sect.6.8.6],[3],[5] and considering the widely employed second-order model, the sequence $\{G(i)\}$ in (2) constitutes a L-variate complex second-order zero-mean Gauss-Markov discrete-time process described by the following homogeneous autoregressive model AR(2):

$$G(i+2) = A\, G(i+1) + B\, G(i) + d(i+2), \qquad (3)$$

where A, B are $L\mathrm{x}L$ real matrices and the complex random sequence d(i) is stationary zero-mean white Gaussian with given covariance matrix $R_d = E\{d(i)\, d^H(i)\}$. The process G(i) is stationary with covariance matrix $R_g(k) = E\{G(i)\, G^H(i+k)\}$.

3 TECHNIQUES FOR CHANNEL IDENTIFICATION

In order to estimate the unknown matrices A, B, $R_g(0)$ and the noise level N_0, a training sequence of length L_t yielding a constant channel-state sequence x(i) = ξ, $1 \le i \le L_t$, is transmitted, so that the received sequence in (2) becomes: $y(i) = \xi^T G(i) + v(i)$, $1 \le i \le L_t$. In this case it can be proved that the acf of the observed process $R_y(k) = E\{y(i)\, y^*(i+k)\}$ is related to the acf of the fading process $R_g(k)$ through the relationships:

$$R_y(k) = \xi^T R_g(k)\, \xi^*,\ k \ge 1, \quad (4)$$

$$R_y(0) = \xi^T R_g(0)\, \xi^* + N_0. \quad (5)$$

The above equations are quite general, and in principle they can be exploited to obtain estimates of the fading model parameters and of the noise level N_0 on the basis of estimates of $R_y(k)$, having assumed an AR model for the fading process. In particular, in the following the AR(2) model of (3) is explicitly assumed.

3.1 The WSSUS fading channel and the high-order Yule-Walker solution

Let us consider now the particular case when the fading processes are not only WSS, but also exhibit "uncorrelated scattering" (US), according to the WSSUS model of [1,Ch.7]. The following assumptions, largely adopted in the literature and deriving from the WSSUS hypothesis, are then stated: 1) A, B and R_d are diagonal matrices with equal elements along the main diagonal, i.e.: $A = \alpha\, I_{LxL}$, $B = \beta\, I_{LxL}$, $R_d = 2\, \sigma_d^2\, I_{LxL}$ with α, β real (I_{LxL} is the $L\mathrm{x}L$ identity matrix); 2) the covariance matrix of G(i) is $R_g(k) = 2\, R(i+k)\, I_{LxL}$, where R(i+k) is the autocorrelation function (acf) of the real and of the imaginary components of the processes g(i;m), valid for every m. In this case eqs.(4,5) become:

$$R_y(k) = 2\, R(k)\, \|\xi\|^2,\ k \ge 1, \quad (6)$$

$$R_y(0) = 2\, \sigma^2\, \|\xi\|^2 + N_0. \quad (7)$$

from which it is observed that the spectrum analysis of y(i) directly gives, apart from a scale factor, the fading spectrum.

An immediate solution to the identification problem is constituted by the classic high-order Yule-Walker (HOYW) equations, obtained by writing eq.(6) for some suitable values of the lag k [6]. The observed acf $R_y(k)$ is estimated from the available L_t observed data on the basic of the classic expression: $\hat{R}_y(k) = L_t^{-1} \sum_{i=1,L_t-k} y(i) y^*(i+k)$ (the symbol ^ denotes an estimate). The acf lags k = 0, 1, 2, 3 and 4 are involved in the HOYW equations.

The HOYW procedure shows an accuracy loss whenever the Doppler spread is small (10^{-3} or less) and when the signal-to-noise ratio (SNR) is not very high (below 20 dB), as it is verified in Section 3.3 (Results). This is perhaps due to the denominators in the HOYW equations, which are very close to zero when the acf exhibits a slow decay (as it happens for small Doppler spread), so that R(1)≅R(2) ≅R(3) ≅R(4). For this reason, alternative solutions have been found, as described below.

3.2 Nonlinear Yule-Walker-like (NLYW) solution for WSSUS fading channel

An alternative solution is obtained by writing (6) for k = 1, 2, 3 and expressing $R_g(1)$, $R_g(2)$, $R_g(3)$ as a function of the unknown parameters α, β and σ^2. This gives a nonlinear algebraic system with three equations. Solving the system gives the unknowns α, β, σ^2, while N_0 is computed from (7). After some algebra a closed-form solution is found as:

$$\hat{\beta} = \left\{1 - \left(\frac{\hat{R}_y(2)}{\hat{R}_y(1)}\right)^2\right\}^{-1} \left\{\left[\frac{\hat{R}_y(3)}{\hat{R}_y(1)} - \left(\frac{\hat{R}_y(2)}{\hat{R}_y(1)}\right)^2\right] - \frac{\hat{R}_y(2)}{\hat{R}_y(1)} \sqrt{\left[1 - \frac{\hat{R}_y(3)}{\hat{R}_y(1)}\right]\left[\left(\frac{\hat{R}_y(2)}{\hat{R}_y(1)}\right)^2 - \frac{\hat{R}_y(3)}{\hat{R}_y(1)}\right]}\right\}, \quad (8)$$

$$\hat{\alpha} = \frac{1}{2}\left\{\frac{\hat{R}_y(2)}{\hat{R}_y(1)} + \sqrt{\left(\frac{\hat{R}_y(2)}{\hat{R}_y(1)}\right)^2 - 4\hat{\beta}(1-\hat{\beta})}\right\}, \quad (9)$$

$$\hat{\sigma}^2 = \frac{\hat{R}_y(1)(1-\hat{\beta})}{2\|\xi\|^2|\hat{\alpha}|}, \quad \hat{N}_0 = \hat{R}_y(0) - 2\hat{\sigma}^2\|\xi\|^2. \quad (10)$$

Another solution can be obtained by writing (6) for k = 1, 3, 5, which leads to the estimates:

$$\hat{\beta} = -\sqrt{\frac{\hat{R}_y^2(3)/\hat{R}_y^2(1) - \hat{R}_y(5)/\hat{R}_y(1)}{1 - \hat{R}_y(3)/\hat{R}_y(1)}}, \quad (11)$$

$$\hat{\alpha} = \sqrt{\hat{\beta}^2 - 2\hat{\beta} + \frac{\hat{R}_y(3)}{\hat{R}_y(1)}}, \quad (12)$$

while σ^2 and N_0 are computed as in (10).

Eqs.(8),(9),(10) or (11),(12),(10) constitute two possible non-linear Yule-Walker-like solutions, denoted as NLYW-1 the first and NLYW-2 the second. Both them implies the estimation of the acf $R_y(k)$ at three lags only (k = 1, 2, 3 for NLYW-1 and k = 1, 3, 5 for NLYM-2), i.e. one lag less than the HOYW procedure, with a consequent computational saving.

3.3 Results

The estimation accuracy of the HOYW and NLYW solutions has been evaluated via computer simulations. Different SNRs and Doppler spreads (expresses as B_DT_s, i.e. normalized to the symbol interval T_s) have been considered for the case of binary antipodal transmission and AR(2) fading process with two-pole Butterworth Doppler spectrum. In this case, the parameters α, β are related to the Doppler spread B_D through the expressions: $\alpha = 2\ e^{-\theta}\cos\theta$, $\beta = -\ e^{-2\theta}$, where $\theta = 1.41\ \pi\ B_DT_s$.

The results are reported in Figs.1,2 in terms of the mean-square-error (MSE) of the estimation of α and σ^2 as a function of L_t. This allows to evaluate the required training sequence length which gives the desired accuracy. The results for β and N_0 are not reported for the sake of brevity, because they are quite similar to those relative to α and σ^2, respectively.

It is observed that the NLYW solutions outperforms the HOYW, perhaps because the denominators in (8), (9) are more robust than those in the HOYW solution against the accuracy loss occuring in the acf estimation. In particular, the NLYW-2 procedure exhibits better performance for any value of SNR and B_DT_s. By fact this latter solution employs large-lag acf samples (up to k = 5), which can be better discriminated in the current case of highly-correlated process, where the acf exhibits a very slow decay.

The effectiveness of the improvement is even better appreciated by considering the case when the goal is to estimate the Doppler spread B_D (which can be calculated from α by means of the expressions reported before), because it is easily checked that small estimation errors for α give rise to large errors for B_D.

4 CONCLUSIONS

Due to the highly-correlated nature of the fading process, classic high-order Yule-Walker linear approach fails to give a satisfactory accuracy, especially for small Doppler spreads and SNR not very large.

Improvements are obtained by employing the proposed nonlinear Yule-Walker like procedure, in particular the NLYW-2 based on eqs.(11),(12),(10), which gives a robust solution to the addressed channel fading identification problem.

In the present analysis only the training operative mode has been considered, assuming a constant channel state which is known at the receiver. In the data-detection operative mode it is possible to employ the decided data to properly adjust the received data samples and resort to the same algorithms. In this way the fluctuations of the fading process can be tracked in real-time, and a very large number of data samples can be employed in the acf estimation for this task. This constitutes an important topic in applications such as in mobile radio communications, where the mobile speed may vary, with consequent variations of the Doppler spread.

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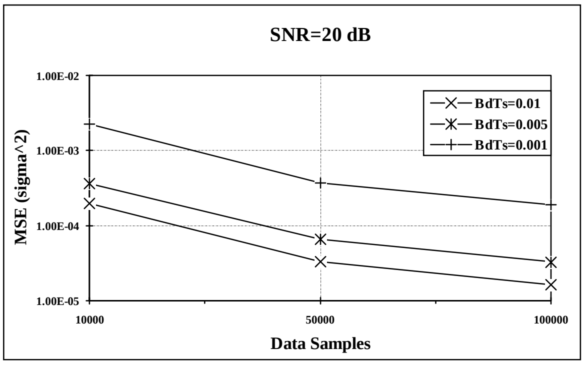

Fig.1 - MSE of the estimation of σ^2 as a function of L_t for SNR = 20 dB and for some different values of B_DT_s. Only one curve family, because the results for the HOYW, NLYW-1 and NLYW-2 procedures practically coincide and do not change significantly with the SNR.

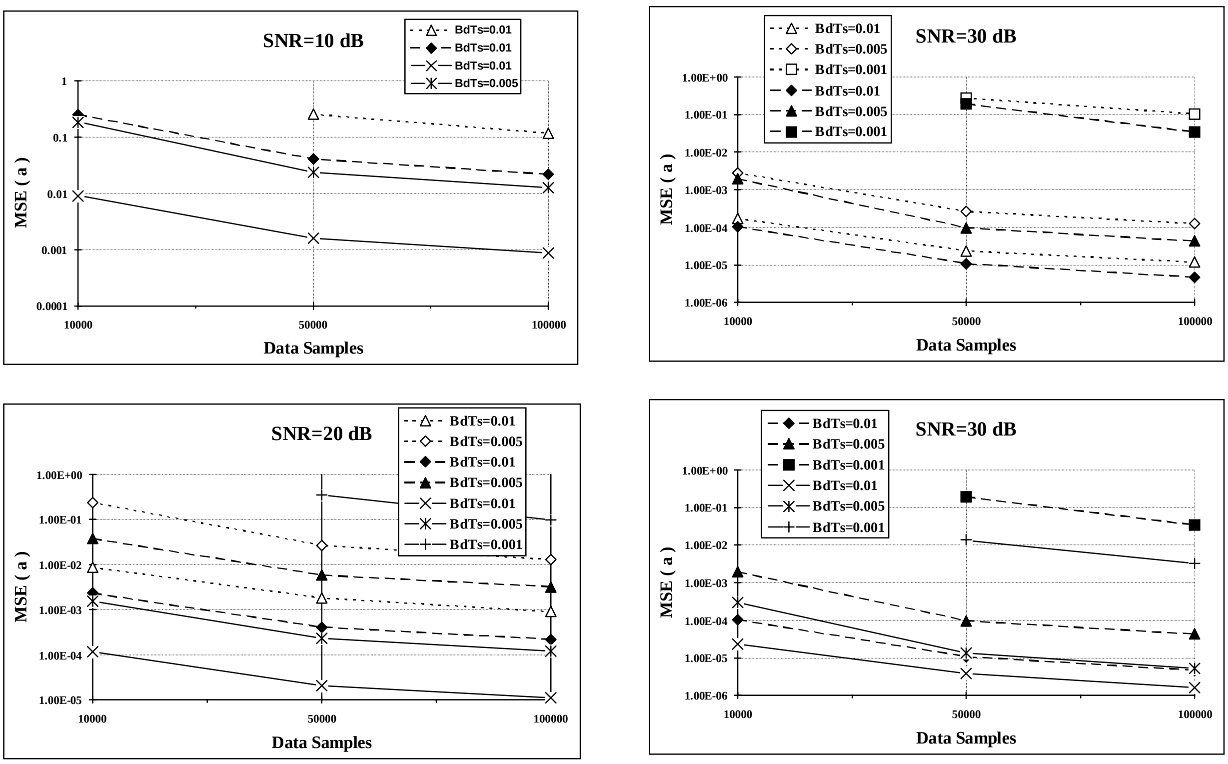

Fig.2 - MSE of the estimation of α as a function of L_t for some different values of SNR and B_DT_s. Dotted line: HOYW procedure; dashed line: NLYW-1 procedure; solid line: NLYW-2 procedure.