AN ENHANCED METHOD FOR THE ESTIMATION OF A DOPPLER FREQUENCY

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Abstract
The enhanced method for the estimation of a Doppler frequency which is dealt with aims at achieving a real time measure of the movements of a vehicle, given an on-board configuration of microwave Radar sensors. The prime idea is that the Doppler frequency can be assimilated to the mean instantaneous frequency of the signal. Then this frequency is estimated using the first moment of a quadratic time-frequency distribution. The enhancing process of the method is involved both in a specific preprocessing of the distribution so as to capture a reliable signal information, and in a weighted rejection of the higher variance components, likely to be meaningless. Simulations, as well as preliminary real tests, show probative results.

1 INTRODUCTION

In order to formulate the signal processing problem, let us introduce the basic concepts which govern the generation of the signal. Given a microwave Doppler sensor and the geometry of its arrangement beneath the vehicle (fig.1), the frequency contribution $f_d i$ in the signal power spectrum, resulting from a $\Theta_i$ incidence angle within the antenna beam, is proportional to the speed $v$ accordingly to the relationship (1).

$$f_{di} = \frac{v}{\lambda} \cos \Theta_i$$  \hspace{1cm} (1)

$\lambda$: freespace wavelength

Assuming propitious working conditions for the Doppler sensor [1], the spectral distribution may be characterized by a gaussian shape centered around $f_d$, and a bandwidth $\Delta f_d$ given by (2).

$$\Delta f_d = \frac{v}{\lambda} \left[ \cos \Theta \frac{\Delta \Theta}{2} - \cos \Theta + \frac{\Delta \Theta}{2} \right]$$  \hspace{1cm} (2)

Moreover, from (1) and (2), it can be derived that the relative bandwidth only depends upon the beamwidth $\Delta \Theta$ and the inclination angle $\Theta$ of the antenna.

$$\frac{\Delta f_d}{f_d} = \tan \Theta \frac{\Delta \Theta}{2}$$  \hspace{1cm} (3)

Such a favourable case, illustrated hereafter by a real power spectrum (fig.2), has given rise to a lot of estimation methods of the Doppler frequency (here denoted "Df" in abbreviated general form). Implicitly the Df is often defined as being representative of the "central frequency" of the lobe.

Within the framework of time methods the Df is derived from the mean number of Zero-Crossings of the signal which occur during a measurement time $T$.

Dealing with parametric methods, the AR model is proved suitable [2]. Further investigations, with regard to the order of the model and the choice of the estimator itself, can lead to a convincing implementation [3].

Unfortunately these methods become unfounded seeing that the power spectrum deviates from the ideal case, as illustrated by the figure 3. Depending upon the scattering surface characteristics (ground material, roughness of the surface, wetness,...) and also possible interferences, the power spectrum may exhibit unexpected sidelobes (fig. 3-a). Moreover, the vehicle vibrations, combined with the proper $1/f$ noise of the antenna, can cause a relatively high power density at low frequencies (fig. 3-b). In such conditions, clearly the estimation of the Df must be derived from the useful lobe of the spectrum, disregarding the undesired regions. Consequently, the estimation is based on the spectral data, and involves a robust preprocessing. These points are developed within the following sections. Section 2 sets the principles of an Instantaneous Frequency based estimation, while the enhancing process is outlined in section 3. The fact that the uncertainty of the estimate both depends upon the observation time and the SNR is common to any method, but this discussion goes beyond the limits of the present paper.

2 BASES OF THE ESTIMATION

Fundamentally, it is postulated that the Df is the mean instantaneous frequency of the signal. Such a definition lends itself to a formal expression, and remains consistent with the original principle, more or less intuitive, which consists in deriving the Df from mean frequency of Zero-Crossings of the signal.

Like the Fourier frequency is associated to the Fourier Transform, the instantaneous frequency IF is associated to
the Hilbert Transform [4]. Let x(t) the real Doppler signal, z(t) its analytic counterpart, and the Parseval equality assumption which is expressed below:

\[ \int_{-\infty}^{\infty} |z(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(v)|^2 \, dv \]

In the time domain the normalized mean instantaneous frequency, denoted \( v_{im} \), conforms to the following relationship:

\[ v_{im} = \frac{1}{\lambda} \int_{-\infty}^{\infty} v(t) \, |z(t)|^2 \, dt \quad (4) \]

On the other hand, in the Fourier domain, the "central frequency" can be defined objectively as the mean Fourier frequency \( v_m \), which therefore produces the variance \( \lambda \):

\[ \lambda = \frac{1}{\lambda} \int_{0}^{\infty} (v - v_m)^2 |X(v)|^2 \, dv \quad (5) \]

Minimizing (5) yields:

\[ v_m = \frac{1}{\lambda} \int_{0}^{\infty} v |X(v)|^2 \, dv \quad (6) \]

The underlying exact relationship between \( v_{im} \) and \( v_m \) is:

\[ v_{im} = v_m \quad (7) \]

Relationships similar to (4) and (6) can be derived for \( N \)-samples discrete signals and discrete spectra [5].

\[ v_{im} = \frac{1}{2\pi} \arg \sum_{n=0}^{N-1} x(n) e^{-j2\pi n} \quad (8) \]

\[ v_m = \frac{1}{2\pi} \arg \sum_{k=0}^{N/2-1} X(k) e^{-j2\pi k} \quad (9) \]

However (7) still holds only if the module of \( z(n) \) is constant, or to a certain extent, slowly time-varying.

We extend these concepts to the definition of a Df at time \( n \), denoted \( f_d(n) \), by substitution in (9) of \( |X(k)| \) by the Short Time Fourier Transform \( X(n,k) \). In that case, it must be noted that \( f_d(n) \) is akin to the smoothed Discrete Instantaneous Frequency estimator which would be derived from the first moment of a Cohen's class Time Frequency Distribution, and especially the Wigner Ville Pseudo distribution [6].

3 ENHANCED ESTIMATION

For further simplicity of notations let

\[ Y(n,k) = |X(n,k)|^2 \]

In practice \( Y(n,k) \) results from an averaging of time-shifted spectrograms (10), consistently with the quasi-stationarity property of the signal, and in accordance with the bias, the variance, and the time resolution required for the Df estimate.

\[ Y(n,k) = \frac{1}{P} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} Y(n-pR, m) e^{-j2\pi k p m} N \quad (10) \]

\( w_r \): real weighting window

The measurement time \( T \) is submitted to a constant product \( \Delta f_d . T \), criterion, and then aims at being adaptive, as induced by (3). Through (10) the time \( T \) can be controlled by the length \( N \) of the window, the number \( P \) of windows, and the shifting parameter \( R \).

Essentially, the enhancing process lies in a filtering operation of the signal, so as to reject in the Time-Frequency representation the low frequency noise and vibrations effects, as well as the emergent sidelobes which interfere with the main one. This means that the Df will be calculated by (11), assuming that an optimal bandwidth, bounded by indexes \( k_L \) and \( k_H \), can be determined beforehand.

\[ f_d(n) = \frac{k_H}{2\pi} \arg \left( \sum_{k=k_L}^{k_H} Y(n,k) e^{-j2\pi k} N \right) \quad (11) \]

Furthermore, we propose to improve the practice of (11) by implementing a weighted rejection of the higher variance components of \( Y(n,k) \). These two concepts are presented hereafter.

3.1 Capture of the useful bandwidth

Accessing indexes \( k_L \) and \( k_H \) proceed from a four step approach, given below:

a) Computation of a smoothed counterpart \( Y_s(n,k) \) of the logarithmic representation of \( Y(n,k) \). Subsequently, every logarithmic quantity is indexed "log".

b) Detection of the peak \( Y_p(n,k_0) \) of \( Y_s(n,k) \).

c) Identification of a low frequency Signal-Noise discriminating threshold \( E_L(n) \).

d) Identification of a high frequency sidelobe discriminating threshold \( E_H(n) \).

Dealing with the smoothing problem, the Wavelet Transform presents an innovating way which we have considered of interest [7,8]. Nevertheless, thresholding of the wavelet coefficients is not easily controllable, and therefore does not allow really robust results as far as \( k_L \) and \( k_H \). As it is, for the present application, a conventional smoothing method, as specified by the relationship (12), is proved more appropriate, all the more the length of the normalized smoothing window \( S \) (i.e. Hamming window) can be made adaptive, thus taking into account (3).

\[ Y_p(n,k_0) = \sum_{l=1}^{N} Y_s(\log(n),l) S(k-1) \quad (12) \]

Searching for the index \( k_0 \) of the peak \( Y_p(n,k_0) \) is a trivial problem which first of all aims at pointing the useful lobe, even if incidentally it provides a rough estimate of \( f_d(n) \) [4]. The possible uncertainty of the measure of \( k_0 \) does not induce shortcomings of the final estimate.
The noise density level at low frequencies is calculated by (13), the upper index \( k_1 \) being previously derived from (3) and (14).

\[
E_L(n) = \frac{1}{k_1} \sum_{k=0}^{k_1-1} Y(n,k) \tag{13}
\]

\[
k_1 = \text{int} \left\{ k_0 \left(1 - \frac{\Delta f_d}{f_d}\right) \right\} \tag{14}
\]

\( \alpha \): coefficient

The discriminating threshold \( E_H(n) \) is based on the detection of a critical change in the slope of \( Y_S(n,k) \). The criterion we introduce is expressed by (15).

\[
C(n,k) = \text{sgn} \left[ \frac{1}{M} \sum_{i=1}^{M} [Y_S(n,k) - Y_S(n,k+i)] \right] \tag{15}
\]

Practically, the parameter \( M \) is adjusted so as to avoid both wrong detections which would be due to the variance of \( Y_S(n,k) \) and missing right detection of interfering emergent sidelobes. The first occurrence of a negative value of \( C(n,k) \) in the searching process gives the index \( k_2 \) and the threshold \( E_H(n) = Y_S(n,k_2) \).

A unique reference discriminating threshold \( E(n) \) is then obtained by the rule (16).

\[
E(n) = \max \{E_L(n), E_H(n)\} \tag{16}
\]

leading to indexes \( k_L \) and \( k_H \), as verifying the best approximation of (17).

\[ Y_S(n,k_L) = Y_S(n,k_H) = E(n) \tag{17} \]

3.2 Rejection of bursts

High scattered samples, indeed meaningless, may be present in the spectrogram, even within the range \( [k_L,k_H] \), which are likely to degrade the accuracy (bias, variance) of the estimate of \( f_d(n) \). Such bursts are detected by comparing the error \( e(n,k) \) to the standard deviation \( \sigma(n) \), which are defined by (18) and (19).

\[
e(n,k) = Y(n,k) - Y_S(n,k) \tag{18}
\]

\[
\sigma(n) = \left[ \frac{1}{k_H - k_L + 1} \sum_{k=k_L}^{k_H} e^2(n,k) \right]^{1/2} \tag{19}
\]

Then a reduction of the incidence of these suspect samples on the estimate of \( f_d(n) \) can be expected seeing that \( Y(n,k) \), in (11), is replaced by a weighted counterpart \( Y_W(n,k) \). The weighting rule is as follows.

\[
Y_W(n,k) = Y(n,k) \text{ if } |e(n,k)| \leq \sigma(n) \tag{20-a}
\]

\[
\Delta(n,k) = \text{sign}(e(n,k)) \left(1 - \lambda \right) \sigma(n) + \lambda \cdot e(n,k) \tag{20-b}
\]

\[
Y_W(n,k) = Y_S(n,k) + \Delta(n,k)
\]

The parameter \( \lambda \), within the range [0,1], controls the weighting operation.

4 RESULTS

An objective assessment lies on the comparison of a Df estimate to a Df reference, as far as this reference can be known. At present, field tests at our disposal do not meet this condition, so we resort to a dedicated test bench using synthetic signals. The test Doppler signal generator which has been achieved conforms the spectral density of an input white noise to a template, characterized by \( f_0 \), \( \Delta f_d \), \( E_L \), and to an interfering sidelobe \( I, \Delta f, E_H \). An additive gaussian white noise assigns the expected Signal-Noise Ratio.

Referring to the section 3, the processing parameters and test conditions are listed below.

- Sampling frequency: 4 KHz
- W: S. Hamming windows
- \( P = 3 \)
- \( R = 400 \)
- \( \Delta f_d/f_d = 0.8 \) (-6dB)

The test conditions, in terms of discriminating thresholds, are not the more adverse possible:

\[ E(n) = E_L(n) = E_H(n) \]

\[ Y_S(n,k_0) - E(n) = 10 \text{ dB} \]

but they keep sense for a comparative analysis between the IF based estimation and a AR estimation (order 2, modified covariance estimator) [3].

A significant set of averaged estimates is written down in the following table.

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<th>SNR</th>
<th>30 dB</th>
<th>15 dB</th>
<th>5 dB</th>
<th>30 dB</th>
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<td>153.3</td>
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</tbody>
</table>

The enhanced IF based method confirms to perform quite right, even at relatively low SNR, and especially if it would be compared to the application of the relationship (9) just as it stands, and a fortiori if it is compared to a basic AR parametric method which besides does not tolerate low SNR.
5 CONCLUSION

This work contributes to prove that the instantaneous frequency concept is really appropriate to the definition of a Doppler frequency. The formal expression of the mean instantaneous frequency, which is based on the first moment of a time-frequency representation, lends itself to taking into account properties inherent to the signal generator, especially to the transducer (i.e. microwave sensor) and its environment. This ability is potentially of great interest for a lot of application fields. In return, compared to parametric approaches, the induced computational cost could be discussed, but the availability of powerful Digital Signal Processors moderates this disadvantage.

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REFERENCES


FIGURES

Figure 1. Geometry of the microwave beam

Figure 2. A basic Doppler signal spectrum

Figure 3. Impairments in the Doppler signal spectrum
(3-a): interference sidelobe
(3-b): high PSD at low frequencies