COLOR IMAGE FILTERING USING GENERALIZED COST FUNCTIONS

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ABSTRACT

The concept of cost function (CF) in the context of image filtering is put under investigation in this work. Optimal behaviour of the resulting filters in respect with noise attenuation and edge preservation is sought through the minimization of these functions. This behaviour can be controlled by proper adjustment of certain parameters in some cases. Function combinations are also considered. Finally, the proposed schemes are tested on real images and objective as well as subjective results are reported.

1 INTRODUCTION

Multichannel signal processing has drawn the attention of scientists lately and has led to the development of efficient techniques in this area. These techniques often benefit from the correlation that exists between the channels by treating them as vector components in order to improve their performance. The R,G,B channels of color images posses an inherent correlation that makes them ideal for vector processing methods.

An application area of great importance for these methods is image filtering, since various kinds of noise, commonly degrade image quality. At the same time it is required that certain characteristics important to the human eye, like details, color and edges, are kept intact. Vector median and other multivariate ordering-based filters [1] as well as distance-weighted filters [3,4] have been developed and studied for this purpose. These filters are produced by a minimization process related to the following cost function:

$$E_{eu}(\mathbf{X}) = \sum_{i=1}^{N} \|\mathbf{X} - \mathbf{X}_i\|, \qquad \mathbf{X} \in R^3 \qquad (1)$$

where $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$,... $\mathbf{X}_N$ is a set of vectors and ||.|| is the L_2 norm. A plot of this function for the set of 4 2D vectors: {(50,25), (30,11), (11,20), (10,30)} is in Figure 1. If closeness is assumed, so **X** can only be selected from the set of $\mathbf{X}_i$s, the well known Vector

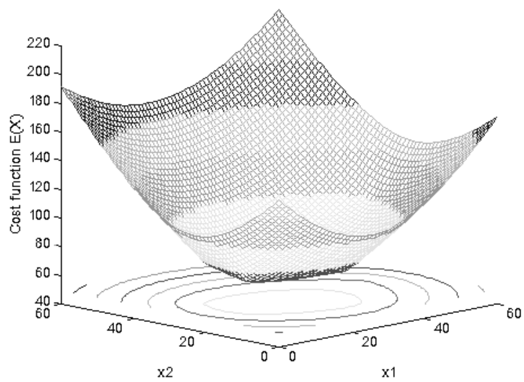

Figure 1. Plot of cost function (1).

Median filter is produced. This function is coming from the Maximum Likelihood Estimation theory and guarantees that the selected vector **X** based on the observation data $\mathbf{X}_i$, with i=1,... N, is the optimum in the case of biexponential noisy conditions. If the constrain of closeness is dropped the global minimum, given by the root of equation:

$$\sum_{i=1}^{N} \frac{\mathbf{X} - \mathbf{X}_i}{\|\mathbf{X} - \mathbf{X}_i\|} = 0 \qquad (2)$$

leads to much better noise suppression, and this may be the desired outcome in certain cases [5]. Another cost function-based approach to filtering can be found in [2], where a generalization of (1) is attempted by means of the Minkowski distance and the incorporated parameters. Other distance metrics can be treated as CFs, too [8]. The city block distance is one of them. In this work we are extending the concept of cost function by introducing generalized formulas that can act as such functions and show that they may be effectively used for the processing of images. The

filter's implementation may be based either on ordering or on weighting as in [3] or [4] or even using a fuzzy logic scheme [6]. Its behaviour can be adjusted to yield optimal results, in the sense of noise reduction and edge preservation, through proper selection of a given parameter.
The following sections are describing the proposed functions and examine their capabilities in the context of filtering by testing them on a natural image.

2 THE GRADIENT FUNCTION

Since we are talking about vector functions' minimization the gradient is taken to consideration first. Its use in the problem of estimation of a function's extrema is well known in the numerical analysis theory. The gradient conveys considerably useful information. It is a vector which through its components' signs indicates if the CF is increasing or decreasing and its magnitude indicates the steepness of the slope at a specific point. It also points towards the direction of maximum change of CF's value. At a minimum or maximum it becomes zero.
In our case the function is (1). This is a smoothly varying convex function with only one minimum. Its gradient at point $\mathbf{X}$ is given by the formula:

$$E_{gr}(\mathbf{X}) = \nabla E_{eu}(\mathbf{X}) = \sum_{i=1}^{N} \frac{\mathbf{X}-\mathbf{X}_i}{\left\|\mathbf{X}-\mathbf{X}_i\right\|} \qquad (3)$$

A plot of this gradient's magnitude is given in Figure 2.

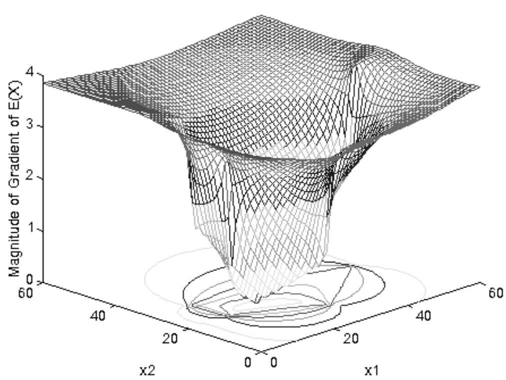

Figure 2. Plot of magnitude of (3)

The use of the gradient is multiple. It can be used to find the true minimum of (1) by employing a method from the numerical analysis toolbox. But the gradient's magnitude can serve also as a cost function itself to derive coefficients for a weighting average scheme [3,4] or to just locate the vector for which it is zero.

3 POTENTIAL FUNCTIONS

The idea of Potential Functions (PF) as the means of estimating the probability is employed in this section.
Briefly, a potential function is formed as a sum of individual potentials produced around each data point $\mathbf{X}_i$, like the electrical potentials around each point charge [7]. Similarly, it's a common way to estimate a probability distribution function of a set of samples by constructing a standard distribution function centered at each sample and sum the results. The following two formulas are widely used as Pfs:

$$E_{PF1}(\mathbf{X}) = \sum_{i=1}^{N} \frac{\sigma^n}{\sigma^n + \left\|\mathbf{X}-\mathbf{X}_i\right\|^n} \qquad (4)$$

and

$$E_{PF2}(\mathbf{X}) = \sum_{i=1}^{N} \exp\left[-\frac{1}{2\sigma^n}\left\|\mathbf{X}-\mathbf{X}_i\right\|^n\right] \qquad (5)$$

where ó and n are adjustable parameters (usually n=2).
It is worth noticing that the maximum of the PF corresponds to the minimum cost or otherwise the inverse of PF could be used as cost function. It is well known that the value of parameter ó plays a decisive role on the shape of the PF. In the case where n=1 the maximum of (5) is given as the solution of the following equation:

$$\sum_{i=1}^{N} \exp\left[-\frac{1}{2\sigma}\left\|\mathbf{X}-\mathbf{X}_i\right\|\right] \frac{\mathbf{X}-\mathbf{X}_i}{\left\|\mathbf{X}-\mathbf{X}_i\right\|} = 0 \qquad (6)$$

If $\left\|\mathbf{X}-\mathbf{X}_i\right\| / 2\sigma$ <<1, (6) is approximating (2) i.e. the root's value coincides with the Vector Median value. In the opposite case $\left\|\mathbf{X}-\mathbf{X}_i\right\| / 2\sigma$ >>1, the PF has as many local maxima as the number of the input vectors. σ can be made adaptive by setting it equal to a percentage of the standard deviation (SD) of $\mathbf{X}_i$ s. In our case to 1/4 of SD.
The PFs are used in this study as a means of finding the maximum from a set of vectors. This leads to outlier rejection and edge enhancement of images.

4 FUNCTION COMBINATIONS

4.1 Magnitude and Angle function combinations

The various CFs exhibit, in general, extrema at different points and naturally this influences the behaviour of the filters that employ them. It is, however, possible to combine CFs in order to combine their characteristics. We can think of numerous combinations but we shall focus in specific ones to be described herein after.

Since color pixels can be regarded in the RGB space as vectors we can define for them angles and magnitudes. So, instead of using (1) we form two new costs functions, one based on the angle $a(\mathbf{X}_i\text{-}\mathbf{X}_j)$ between vector pairs:

$$E_a(\mathbf{X}) = \sum_{i=1}^{N} a(\mathbf{X} - \mathbf{X}_i) \qquad (7)$$

and one based on the magnitude differences:

$$E_m(\mathbf{X}) = \sum_{i=1}^{N} \left\| \|\mathbf{X}\| - \|\mathbf{X}_i\| \right\| \qquad (8)$$

The motivation for this choice emanates from the fact that the angle is connected to the cromaticity and the magnitude to the luminance, which are separable quantities. Equations (7) and (8) can then be combined directly to form a ‘total’ cost function or can be used to calculate coefficients which in turn will be combined. The combinations used were either the product:

$$E_{tot1}(\mathbf{X}) = E_a(\mathbf{X}) * E_m(\mathbf{X}) \qquad (9)$$

or the mean:

$$E_{tot2}(\mathbf{X}) = \left[E_a(\mathbf{X}) + E_m(\mathbf{X})\right]/2 \qquad (10)$$

In the later case normalization should precede the summation.

4.2 Norm combination

Until this point we considered only the aggregate distance, expressed in the form of the L_2 norm, of vector $\mathbf{X}$ to the set of vectors $\mathbf{X}_i$. Alternatives to this distance can be combined with the aggregate to yield new CFs. One such alternative is the distance from the mean vector $\mathbf{X}_{ave}$. The proposed combination is as follows:

$$E_{nc}(\mathbf{X}) = \alpha\, E_{eu}(\mathbf{X}) - \beta\, \|\mathbf{X} - \mathbf{X}_{ave}\| \qquad (11)$$

α and β are properly selected parameters. Through this combination blurring introduced by averaging might be reduced.

5 RESULTS

The cost functions which were presented in this work have been used for the development of filters, which in turn were applied on real images, in order to verify their noise reduction and edge enhancement capabilities. In Table I we present the objective results, using the Normalized Mean Square Error measure, of filtering the image ‘Lenna’ with several filters after corrupting it with various noise types. Noise was gaussian or correlated impulsive, with variable standard deviation (SD) and different percentages respectively, or combination of them. Windows of size 3x3 and 5x5 were used. All filters are using a cost function to select a pixel which minimizes it. The Vector Median filter (VMF), as is well known, uses (1) as CF to select one of the input vectors as its output. Filter 1 uses the same CF but it outputs the vector that truly minimizes this CF without the restriction that it should belong to one of the inputs. The magnitude of (3) is the CF for Filter 2. Finally, Filters 3 and 4 use formulas (9) and (10) as CFs. All the tested filters remove impulsive and Gaussian noise effectively while at the same time they keep edge blurring at low levels.

As was already mentioned in section 3 the PFs (4) and (5) have also been used in similar filtering schemes. These filters have edge enhancement properties, which makes objective evaluation difficult, so the NMSE is inappropriate in their case. In Figures 3 and 4 noisy image ‘Lenna’ (gaussian with SD=15 and 2% correlated impulses) is shown after it was filtered with filters using the (3) and (5) CFs.

Finally, it should be noticed that this study is just presenting the idea of using several vector functions in a variety of ways and produce filters with specific properties. The optimum selection of CF depends on the particular application and the objectives set by the desired output.

Acknowledgements

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6 References

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Table I

NMSE ($x10^{-2}$) for tested filters applied on image 'Lenna' which was corrupted with noise.

Noise	VMF		Filter 1		Filter 2		Filter 3		Filter 4	
	3x3	5x5	3x3	5x5	3x3	5x5	3x3	5x5	3x3	5x5
Gauss SD=30	1.50	.985	.770	.561	1.50	.999	1.62	1.07	1.69	1.10
Gauss SD=15	.521	.495	.304	.361	.529	.507	.551	.513	.566	.518
Gauss SD=15 & Imp 1%	.550	.508	.326	.368	.564	.522	.591	.534	.603	.535
Gauss SD=15 & Imp 2%	.579	.522	.347	.377	.598	.541	.629	.559	.604	.559
Impul 2%	.166	.305	.147	.288	.176	.319	.176	.317	.177	.315
Noiseless	.143	.296	.130	.282	.154	.308	.149	.302	.147	.298

Figure 3

Figure 4