Chinese Remainder Theorem : Recent Trends and New Results in Filter Banks Design

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Abstract

Recent advances in the time domain methods have led to many new approaches in filter bank designs. The objective of this paper is to derive a unified theory for these time domain methods, based on the Chinese Remainder Theorem. Topics discussed in this paper include two-channel filter banks, *M*-channel filter banks and 2-D filter banks. Design examples are presented to demonstrate the theory.

1. Introduction

Perfect reconstruction filter banks (PRFB) can be modularized and parametrized using many methods. Among them, a large number of structures presented in literatures share common characertistics. This paper aims at expressing various time domain FB design methods into an algebraic framework using results on polynomial theory already existing in textbooks on Number Theory [1]. The paper simplifies and provide a better understanding of the mathematical structure of time domain methods. It helps to discuss the background of various synthesis techniques of FB that are based on blocks of subfilters and their ladder network representation using the Chinese Remainder Theorem (CRT). By generalizing to multivariable case, concepts and tools of existing one dimensional two-channel FB design methods can be extended to multi-dimensional and multi-channel cases.

The paper is organized as follows : Section 2 discusses basic results in polynomial theory and the connections to the two-channel FB is discussed in Section 3. Results on linear phase solution and zeros at π are derived. Section 4 reviews existing time domain design techniques and their connection to the theorems in Section 2. Results discussed in Section 2 is generalized to multi-dimensional and multi-channel FB using multivariable polynomials and generalized ladder network in Section 5.

2. Reviews of Polynomial Remainder Theorem

We begin by reviewing polynomial remainder theorem results in one dimension. Consider the equation

$$a(z)x(z)+b(z)y(z)=c(z) \qquad (1)$$

where $a(z)$, $b(z)$ and $c(z)$ are 1-D polynomials over a field $\mathbf{R}$, i.e., elements of the ring $\mathbf{R}[z]$. This ring is clearly Euclidean for $\mathbf{R}$ is a field. The equation is called a linear 1-D polynomial equation (also known as Diophantine or Bezout equation) and its solutions are any pairs $\{x(z), y(z)\} \in \mathbf{R}[z]$ satisfying eq(1). The remainder theorem provides the existence condition.

Thm 1 *Eq.(1) has a solution iff* $\gcd(a(z),b(z)) | c(z)$.

Proof ***Only if***. Let $\{x'(z), y'(z)\}$ be a solution of eq(1). Also let

$$a(z)=g(z)\bar{a}(z), \qquad b(z)=g(z)\bar{b}(z). \qquad (2)$$

Then $g(z)(\bar{a}(z)x'(z)+\bar{b}(z)y'(z))=c(z)$, so that $g(z)|c(z)$.

If. Let $\gcd(a(z),b(z))|c(z)$ and denote

$$g(z)=\gcd(a(z),b(z)), \qquad c(z)=g(z)\bar{c}(z). \qquad (3)$$

Since the ring is Euclidean, the remainder theorem implies there exists two polynomials $p(z)$ and $q(z)\in\mathbf{R}[z]$ such that

$$a(z)p(z)+b(z)q(z)=g(z). \qquad (4)$$

Multiplying by $\bar{c}(z)$, we obtain

$$a(z)(p(z)\bar{c}(z))+b(z)(q(z)\bar{c}(z))=c(z). \qquad (5)$$

Hence a solution $p(z)\bar{c}(z), q(z)\bar{c}(z)$ of eq.(1) is constructed. ♦

We are interested in the case of $c(z)=1$ which is closely connected with FIR solutions in FB design.

Thm 2 *The equation* $a(z)x(z)+b(z)y(z)=1$ *has solution iff* $a(z)$, $b(z)$ *have no common zeros.*

Proof Using Thm 1 and the fact that $\gcd(a,b)=1$ iff $a(z)$, $b(z)$ have no common factors. ♦

Any two polynomials satisfying Thm 2 are called coprime polynomials. Since eq.(1) is linear, its general solution can be obtained from a particular solution by

Thm 3 *Let* $\{x'(z), y'(z)\}$ *be a particular solution of eq.(1). Then the general solution is given by*

$$x(z)=x'(z)-\bar{b}(z)t(z); \qquad y(z)=y'(z)+\bar{a}(z)t(z), \qquad (6)$$

where $\bar{a}(z),\bar{b}(z)$ *are defined in eq.(2) and* $t(z)$ *is an arbitrary polynomial* $\in\mathbf{R}[z]$.

Proof By assuming $a(z)x'(z)+b(z)y'(z)=c(z)$, eq(2) implies

$$a(z)(x(z)-x'(z))=-b(z)(y(z)-y'(z)). \qquad (7)$$

The polynomials $\bar{a}(z),\bar{b}(z)$ defined in eq.(2) are coprime and satisfy $a(z)\bar{b}(z)=b(z)\bar{a}(z)$. As a result $\bar{b}(z)|(x(z)-x'(z))$ and $\bar{a}(z)|(y(z)-y'(z))$, that is

$$x(z)-x'(z)=-\bar{b}(z)t(z); \qquad y(z)-y'(z)=\bar{a}(z)t(z), \qquad (8)$$

for a polynomial $t(z)$. To obtain any solution of eq.(8), $t(z)$ ranges over $\mathbf{R}[z]$. ♦

3. Perfect Reconstruction Two-Channel Filter Banks

The output of a two-channel FB with analysis filters $H_i(z)$ and synthesis filter $F_i(z)$ is

$$\begin{aligned} Y(z) &= [H_0(z)F_0(z)+H_1(z)F_1(z)]X(z) \\ &\quad +[H_0(-z)F_0(z)+H_1(-z)F_1(z)]X(-z). \end{aligned} \qquad (9)$$

A biorthogonal system can be obtained by

$$\begin{cases} H_0(z)H_1(-z)-H_1(z)H_0(-z)=z^{-L} \\ F_0(z)=H_1(-z), \quad F_1(z)=-H_0(-z) \end{cases} \qquad (12)$$

Lemma 1: *If the filter pair* $\{\bar{H}_0(z),\bar{H}_1(z)\}$ *is a solution to eq.*(12)*, then the filter pair* $\{H_0(z),H_1(z)\}$ *is also a solution, where*

$$H_0(z)=\bar{H}_0(z)-H_1(-z)t(z) \qquad (13)$$

$$H_1(z)=\bar{H}_1(z)-H_0(-z)k(z) \qquad (14)$$

and $t(z)$, $k(z)$ are *arbitrary polynomials satisfying some linear phase conditions, and* $t(-z)=t(z)$ *and* $k(-z)=k(z)$.

Proof The case of linearphase filters have been proved in [11-13]. In here, we prove the general case. Use Thm 3 with $\{x(z), y(z)\}=\{H_0(z), H_0(-z)\}$ and $\{x'(z), y'(z)\}=\{\overline{H}_0(z), \overline{H}_0(-z)\}$ to prove eq.(13). Eq(14) is proved similarly by using $\{x(z), y(z)\}=\{H_1(z), H_1(-z)\}$ ♦

The symmetric properties of $t(z)$ and $k(z)$ is important to preserve the linear phase property of the solution. There are only two linear phase PR systems that yield good solutions. One of the solution has symmetric and even length filters. Therefore, $t(z)$ and $k(z)$ are required to be symmetric polynomials. Another solution has odd length filters with different symmetric polarities (symmetric/antisymmetric). For instance, if $H_0(z)$ is symmetric, $t(z)$ and $k(z)$ are required to be symmetric and antisymmetric polynomials, respectively. Furthermore, the orders of $t(z)$ and $k(z)$ have to be equal to *L*. Otherwise, additional delays are needed in $\overline{H}_i(z)$ to yield linearphase $H_i(z)$.

Note that although the above analysis is based on linear phase solution, Lemma 1 is also applicable in non-linear phase orthogonal solution. The advantage of Lemma 1 is its formulation in the time-domain which provides an easy way to control the response of the filters. One of the interesting property is the number of vanishing moment at π. By examining Lemma 1, a maximally smooth FB (Daubechies wavelet) can be constructed from the lazy FB ($H_0(z)=1$, $H_1(z)=z^{-1}$) and Bernstein polynomials. By selecting $t_i(z)$ and $k_i(z)$ to be Bernstein polynomials, a maximally smooth wavelet FB can be constructed. Furthermore, by appropriately inserting delay elements as discussed in Lemma 1, linear phase biorthogonal solution can also be constructed.

4. Review of Existing Methods

The application of polynomial theory in filter banks design can be efficiently implemented by ladder network structure as shown in Figure 1 for two-channel FB. The fundamental building blocks of the ladder network are the subfilters $t_i(z)$ and $k_i(z)$. Polynomial theory works directly on the time domain and decomposes a complicated filter banks into modules of subfilters. The advantage of this decomposition is the optimization procedure required in the design is very simple and provides an easy way to control both time and frequency response of the filters.

Ladder network for the design and implementation of PRFB was introduced in [9]. The ladder structure is shown to be robust from coefficient quantization. Furthermore, by deriving the equivalent ladder structure for lattice structure, the structure is shown to be complete. The ladder network, however, does not provide minimal implementation, in spite of the computational advantage over lattice structure by sharing of convolution blocks between subfilters in special cases. There are many variations of the ladder structure including block triangular structure [10] which is a matrix description of ladder network. However, the design of block triangular structure in [10] is emphasized on the prediction property of $k(z)$ in eq.(14). Consequently, the advantage of polynomial theory has not been exploited. [11,12] showed that filter response can be optimized by cascading structure using eq.(14). However, they didn't realize the relationship between the ladder structure and CRT. Consequently, eq.(13) is not used and ladder network with one way communication is constructured which does not fully exploit the advantage of CRT.

On the other hand, [14] used Euclidean algorithm to construct wavelet FB. Euclidean algorithm is derived from CRT and the algorithm iterates eq.(13) and eq.(14). [15] demonstrated the connection between Diophantine equation and two-channel FB, which is essentially eq.(12). [16] foresees the advantage of constructing wavelet using cascade of subfilters and derived the lifting scheme. The simplicity of the lifting scheme is demonstrated by using lazy PRFB as general solution and optimize the filter response using eq.(13) and eq.(14). However, lifting scheme is being explained as interpolation network, and does not realize the relationship between PRFB and polynomial theory. Consequently, the completeness of this scheme (which is actually a complete structure) cannot be shown. Furthermore, linear phase solution is not considered in [16]. Similarly, [22,23] derives the IIR PRFB using eq.(14).

The extension of ladder network to multichannel is first discussed in [9] where three channel FB is constructed. [23] discusses the extension of IIR filter network to three channels. But the real break through comes in [17], where a generalized form of eq.(14) is used to optimize the *M*th filter from *M*-1 filters in *M* channel FB. In Section 5a, the theory and application of the remainder theorem in *M*-channel FB design is derived.

The extension of ladder network to multidimension was first discussed in [20,27] where McClellan transform is used to convert a 1D PRFB to 2D system. McClellan transformation is applied to each subfilters in a 1D PR ladder network. Although the resulting implementation is a ladder network, no advantage of CRT is used [21,22,23] extend CRT to multidimension using polynomial theory in 2D and obtain a simple design procedure and implementation. In Section 5b, 2D polynomial theory will be presented to demonstrate the efficiency of 2D ladder network. Furthermore, a class of 2D wavelet will be designed to show its flexibility in controlling both frequency and time domain responses of the FB.

5a. New Results : Generalization to M-Channel FB

Consider a *M*-channel filter banks with analysis filters $H_i(z)$.

Lemma 2: *If the filters* { $H_0(z)\ldots\widetilde{H}_i(z)\ldots H_{M-1}(z)$ } *is a solution of eq.(15), then* { $H_0(z)\ldots H_{M-2}(z)\ldots H_{M-1}(z)$ } *is also a solution where* $H_i(z)=\widetilde{H}_i(z)-\sum_{k,k\neq i}^{M-1} t_k(z)H_k(z)$ *and* $t_i(z)$ are *arbitrary polynomials satisfying some linear phase conditions.*

Proof Let $\widetilde{H}_i$ be the coprime of the polynomial set { $H_0(z)\ldots H_{i-1}(z), H_{i+1}(z) \ \ldots H_{M-1}(z)$ }. Lemma 2 can be proved by applying the results in Thm 2 and the corresponding symmetric properties of $t_i(z)$. ♦

To exploit the linear phase constraints of $t_i(z)$, consider the analysis polyphase matrix $\widetilde{\mathbf{E}}(z)$ for filter set with $\widetilde{H}_i(z)$

$$\begin{bmatrix} H_0(z) \\ \vdots \\ \tilde{H}_i(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \begin{bmatrix} E_{0,0}(z^M) & & \cdots & & E_{0,M-1}(z^M) \\ \vdots & \ddots & & & \vdots \\ \tilde{E}_{i,0}(z^M) & \cdots & \tilde{E}_{i,i}(z^M) & \cdots & \tilde{E}_{i,M-1}(z^M) \\ \vdots & & & \ddots & \vdots \\ E_{M-1,0}(z^M) & & \cdots & & E_{M-1,M-1}(z^M) \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ z^{-i} \\ \vdots \\ z^{-(M-1)} \end{bmatrix}$$

The polyphase matrix $\mathbf{E}(z)$ of the new filters $H_i(z)$ is

$$\begin{bmatrix} H_0(z) \\ \vdots \\ \tilde{H}_i(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \begin{bmatrix} 1 & 0 & & \cdots & & & 0 \\ 0 & 1 & 0 & \cdots & & & 0 \\ \vdots & & \ddots & & & & \\ t_0(z^M) & \cdots & t_{i-1}(z^M) & 1 & t_{i+1}(z^M) & \cdots & t_{M-1}(z^M) \\ \vdots & & & & \ddots & & \\ 0 & \cdots & & & & & 1 \end{bmatrix} \begin{bmatrix} E_{0,0}(z^M) & & \cdots & & E_{0,M-1}(z^M) \\ \vdots & \ddots & & & \vdots \\ \tilde{E}_{i,0}(z^M) & \cdots & \tilde{E}_{i,i}(z^M) & \cdots & \tilde{E}_{i,M-1}(z^M) \\ \vdots & & & \ddots & \vdots \\ E_{M-1,0}(z^M) & & \cdots & & E_{M-1,M-1}(z^M) \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ z^{-i} \\ \vdots \\ z^{-(M-1)} \end{bmatrix}$$

Note that the first matrix in the right hand side, $\mathbf{T}(z)$ is invertible and has unity determinant. Consequently, the determinant of $\mathbf{E}(z)$ is the same as that of $\tilde{\mathbf{E}}(z)$. The symmetry of $t_k(z)$ changes with the symmetries of $H_k(z)$. Furthermore, the order of $t_i(z)$ has to be equal to the delay of the system. Otherwise delay element has to be multiplied to $E_{i,k}(z)$ such that the resulting filters are linear phase.

Assume that all the delay elements are being absorbed into $t_i(z)$. By repeatedly applying lemma 2 to each newly constructed polyphase matrix, the resulting filter bank is given by

$$\begin{bmatrix} H_0(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \left\{ \prod_{k,i} \mathbf{T}_k(z^M) \right\} \mathbf{E}(z^M) \begin{bmatrix} 1 \\ \vdots \\ z^{-(M-1)} \end{bmatrix} \qquad (15)$$

where $\mathbf{T}_k(z)$ has the same form as $\mathbf{T}(z)$ with $t_{k,i}(z)$ appears on the ith rows. The design problem then reduces to the parametrization of $\mathbf{T}_k(z)$ and the prototype FB. It's relatively easy to design the prototype FB, since one can use the FB constructed by delay chain as prototype system. However, there exist no simple parametrization of the matrix product. By approriate selection of $t_{k,i}(z)$, a class of *t*-matrix product solution can be parametrized by products of permutation matrix and block diagonal invertible matrix [18]. The block invertible matrix is essentially the product of two *t*-matrices acting on adjacent rows.

[19] parametrizes the complete class of biorthogonal LPFB by Hermite reduction. Although it considers a transform matrix acting on columns of $\mathbf{E}(z)$, it is essentially the product of two $\mathbf{T}(z)$ acting on adjacent rows as in eq.(17). The reduction works since eq.(17) is transpose invariant. It is interesting to observe that both [18,19] consider two channels at a time. This is because of LPPR solution has constrained number of symmetric and antisymmetric filters. Working on a pair of filters each time exploits the symmetric properties and simplifies the formulation even though working with one row is sufficient.

5 b. New Results : Generalization to 2D

In 2D, even though the ring $\mathbf{R}[z_1, z_2]$ is not endowed by Euclidean division, the results discussed in Section 1 still stand hold with more restrictive conditions. In fact, we often benefit from thinking of 2D polynomials as of elements of $\mathbf{R}[z_1][z_2]$ or $\mathbf{R}[z_2][z_1]$, i.e., as of 1D polynomials over a Euclidean ring. This enables one to perform Euclidean division among coefficients.

The first difference between 1D and 2D equations materializes when verifying Thm 1. Consider the 2D polynomial

$$a(z_1, z_2)x(z_1, z_2) + b(z_1, z_2)y(z_1, z_2) = c(z_1, z_2). \qquad (16)$$

Thm 4 $\gcd(a, b) | c$ *whenever eq.*(16) *is solvable.*

Proof For a greatest common divisor $d(\mathbf{z}) = \gcd(a(\mathbf{z}), b(\mathbf{z}))$ (and, in fact, for any common divisor at all), the solvability of eq.(16) implies $a(\mathbf{z})x(\mathbf{z}) + b(\mathbf{z})y(\mathbf{z}) = d(\mathbf{z})(\bar{a}(\mathbf{z})x(\mathbf{z}) + \bar{b}(\mathbf{z})y(\mathbf{z})) = c(\mathbf{z})$ so that $d(\mathbf{z}) | c(\mathbf{z})$. ♦

Hence, the divisibility condition remains necessary. However, it is no longer sufficient. Roughly speaking, eq.(16) is solvable provided iff $a(\mathbf{z})$, $b(\mathbf{z})$ are zero coprime, that is every common zero of $a(\mathbf{z})$ and $b(\mathbf{z})$ is a zero of $c(\mathbf{z})$ with the right multiplicity (the Fundamental Theorem of Noether). The solution for 2D Bezout equation is

Thm 5 $a(z_1, z_2)x(z_1, z_2) + b(z_1, z_2)y(z_1, z_2) = 1$ *is solvable if and only if the polynomials* $a(\mathbf{z})$ *and* $b(\mathbf{z})$ *have no zero in common.*

Proof This is a direct consequence of the famous Hilbert Nullstelen-satz. ♦

The Fundamental Theorem of Noether is difficult to inspect practically. For convenience $a(\mathbf{z})$ and $b(\mathbf{z})$ are assumed to be relatively prime, which is the case of an existing 2D PRFB. As in Thm 3, the general solution is given by

Thm 6 Let $a(\mathbf{z})$ and $b(\mathbf{z})$ be relatively prime polynomials and $\{ x'(\mathbf{z}), \ y'(\mathbf{z}) \}$ be a particular solution of eq.(18). Then the general solution is given by

$$x(\mathbf{z}) = x'(\mathbf{z}) - b(\mathbf{z})t(\mathbf{z}); \qquad y(\mathbf{z}) = y'(\mathbf{z}) + a(\mathbf{z})t(\mathbf{z}), \qquad (17)$$

for an arbitrary polynomial $t(\mathbf{z}) \in \mathbf{R}[z_1, z_2]$ satisfying the symmetry condition, $t(-z_1, z_2) = t(z_1, z_2)$ or $t(z_1, -z_2) = t(z_1, z_2)$.

Proof The proof is identical to that of Thm 3.

i. *Perfect Reconstruction 2-D Two-Channel Filter Banks*

The transfer function of 2D two-channel FB is the same as eq.(12), except that the *z*-transform is being replaced with 2D vectors, i.e. $z \leftrightarrow (z_1, z_2)$. Therefore, it is suffices to find 2D filters $H_0(\mathbf{z})$ and $H_1(\mathbf{z})$ satisfying eq.(12). Noticing that eq.(12) in 2D is the 2D polynomial equation, therefore, Lemma 3 below is for the general solutions of 2D FB.

Lemma 3: *If the filter set* $\{ \tilde{H}_0(\mathbf{z}), \tilde{H}_1(\mathbf{z}) \}$ *is a solution to the 2D version of eq.*(12), *then* $\{ H_0(\mathbf{z}), H_1(\mathbf{z}) \}$ *is also a solution where*

$$H_0(\mathbf{z}) = \tilde{H}_0(\mathbf{z}) - \tilde{H}_1(-\mathbf{z})t(\mathbf{z}); \ H_1(\mathbf{z}) = \tilde{H}_1(\mathbf{z}) - \tilde{H}_0(-\mathbf{z})k(\mathbf{z}) \ (18)$$

$t(\mathbf{z}), k(\mathbf{z})$ *arbitrary polynomials satisfying some linear phase and symmetry condition as in Thm 6.*

Proof Use Thm.6. ♦

To achieve linear phase solution, the subfilters $t(\mathbf{z})$ and $k(\mathbf{z})$ must have some symmetric properties. [24,25] show that the number of symmetric and antisymmetric filters must be in pairs, therefore, the polynomials $t(\mathbf{z})$ and $k(\mathbf{z})$ must be symmetric and antisymmetric respectively. Furthermore, the delay of $t(\mathbf{z})$ and

$k(\mathbf{z})$ should be the same as the system delay. Otherwise delay is inserted into $\tilde{H}_0(z)$ and $\tilde{H}_1(z)$ respectively.

ii. Recursive Algorithm to Optimize 2D Filter Banks

Similarly, we can implement 2D filter banks using ladder network. Figure 2 is the ladder network constructed with lazy filters, $t(\mathbf{z})$ and $k(\mathbf{z})$. The recursion is the same as in 1D case. [21] exploits the structure of 2D half band diamond filter and proposes a similar network as in Figure 2, where $t(\mathbf{z})$ and $k(\mathbf{z})$ are selected to be bivariate Bernstein polynomials for constructing maximally smooth wavelet filters. Although the structure is a ladder network, Lemma 3 has not been used. From Lemma 3, it is obvious that FB with higher multiplicity can be constructed by repeatly iterating the structures. Similarly, [22,23] construct a class of 2D FB with essentially the same structure, starting with a lazy FB, and allpass function $t(\mathbf{z})$ and $k(\mathbf{z})$. Thus, the structure is applicable for constructing IIR FB. All the above demonstrate the simplicity of constructing 2D FB using Lemma 3 which allows easy control of vanishing moments at π, frequency response of the filter and the support of the FB. [26] proposes the construction of 2D PR diamond shaped FB from one dimensional filters, where the cascade structure is based on one-dimensional convolution blocks which results in efficient implementation.

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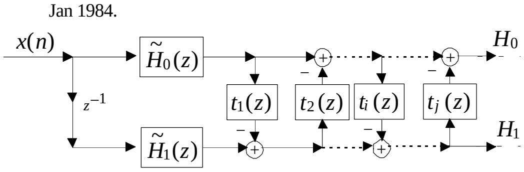

Figure 1. Two-chanel analysis filter bank in ladder structure

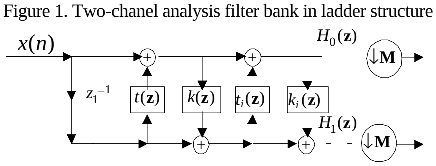

Figure 2. Two dimensional two-channel analysis filter bank in ladder structure, where *M* is the quincunx sampling matrix.

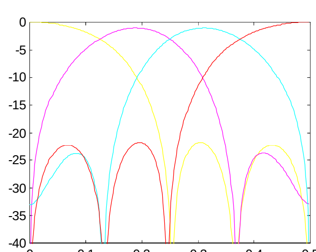

Figure 3a. 4 channel linearphase paraunitary filter banks with length 8 subband filters.

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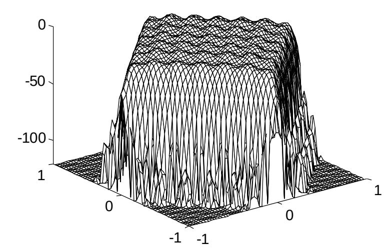

Figure 4. 2D diamond-shaped filter banks constructed by cascade structure using 27x27 maximally smooth linear-phase half band filter cascade with 27x27 linear-phase FIR filter where the first polyphase component is minimum phase.

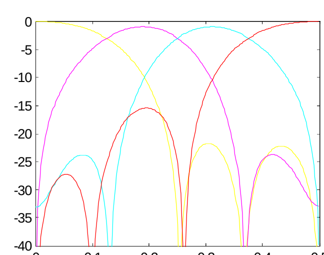

Figure 3b. 4 channel minimum phase linear-phase biorthogonal filter banks with length 8 subband filters by modifing the 4th subband filters as $E_4(z)=\tilde{E}_4(z)-0.001E_1(z)$ $-0.1E_2(z)$ to suppress lowpass components.