A NEW FREQUENCY DOMAIN EQUALIZER FOR CHANNELS WITH LONG IMPULSE RESPONSE

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ABSTRACT

In this paper a recently introduced block Decision Feedback Equalizer (DFE) is further studied and developed. Moreover it is shown that the new technique is particularly suitable for channel equalization in applications involving channels with medium up to long impulse response. The new equalizer, which is totally implemented in the frequency domain, offers remarkable savings in computational complexity as compared to the conventional time domain DFE. Moreover the new technique results in a Symbol Error Rate which is always lower (or much lower) with respect to that of the existing frequency domain linear equalization techniques.

I INTRODUCTION

An important issue in digital communication aplications involving adaptive filters is the required amount of computational complexity. The issue is even more important in cases where the involved channel has a relatively long impulse response. Microwave Line-Of-Sight Links, Multipoint Microwave Distribution Systems, High Speed Communications over Digital Subscriber Loops, Underwater Digital Communications, and Indoor Propagation in Domestic Environment Systems are some typical applications of the kind.

In most of the above applications the major cause of performance degradation is the so-called multipath reception. In many cases the introduced intersymbol interference (ISI) due to multipath propagation may have a catastrophic effect on the received signal. The impulse response of a multipath channel tends to be of discrete form and may span a relatively long time interval (up to $50\mu s$). Taking into account that the symbol rate is usually high (up to several tens of Mbauds) we deduce that the duration of the impulse response may be as long as several hundreds of symbol periods.

In order to reduce the introduced ISI very long equalizers are required at the receiver's end. A possible way to cope with the increased computational complexity is to use block adaptive linear equalizers implemented in the frequency domain [2], [4], [6]. Unfortunately quite often linear equalizers perform very poorly due to the fact that the inverse channel may have non-negligible values for thousands of symbol time intervals and the corresponding channel spectrum may be characterized by very deep nulls. It is well known [1] that the ISI due to multipath propagation can be effectively rejected using Decision Feedback Equalizers (DFE). Several techniques have been proposed in the literature aiming to eliminate multipath ISI by using efficient variations of the time domain DFE (e.g. [7]). However time domain DFEs require a large amount of computational complexity and their implementation in today's hardware turns out to be quite inexpedient.

Our motivation was to develop an algorithm combining both the performance superiority of a DFE structure with the computational advantages of block adaptation in the frequency domain. As it was explicitly stated in [5], no frequency-domain DFE for multipath ISI cancellation had been reported in the literature. The suggested technique is built upon the results of the very recent work in [3]. The equalizer proposed there is further developed and several new issues are investigated. In Section II the particular characteristics of a typical multipath channel are briefly discussed. In Section III the Frequency-Domain DFE is presented and its suitability for the applications of interest is justified. Finally, in Section IV some indicative simulation results are given.

II MULTIPATH CHANNEL PROPERTIES

The so-called multipath phenomenon consists in the reception of multiple signals originating from a single transmitted signal. Depending on the specific application there are several factors giving rise to multiple paths (e.g refractive and reflective nature of the physical media, mountains, large buildings, moving objects etc). Each received multipath component is a scaled, delayed and phase shifted version of the original transmitted signal. For the applications of interest in this paper the multipath environment is assumed slowly varying.

Let us assume that the original signal propagated through the $k-th$ path undergoes a scaling by α_k and a time delay by τ_k. The impulse response of the multipath channel is then given by

$$h(t) = \delta(t) + \sum_{k=1}^{p} \alpha_k \delta(t - \tau_k) \qquad (1)$$

Obviously $h(t)$ has a discrete form and consists of a linear compination of dirac functions. The dirac function at time $t = 0$ corresponds to the original signal while the rest p terms correspond to the undesired echoes. Note that α_k is in general complex with $|\alpha_k| < 1$ and τ_k may be positive (postcursor echo) or negative (precursor echo). A typical impulse response is shown in Fig. 1 (upper part). Notice that the precursor part of the channel's impulse response is in general much shorter and of less energy as compared to the postcursor part.

Assuming, without loss of generality, that the involved delays τ_k are integer multiples of the sampling time interval T_s we can derive after some lengthy algebra an analytic expression for the inverse channel impulse response. This expression consists of terms of the form

$$\frac{n!}{n_1!n_2!\ldots n_p!}(\alpha_1^{n_1}\alpha_2^{n_2}\ldots\alpha_p^{n_p})\delta(t-n_1\tau_1-n_2\tau_2-\ldots-n_p\tau_p)$$

where $n = 0, 1, 2, \ldots$ and $n_1, n_2, \ldots n_p$ are any nonnegative integers satisfying $n_1+n_2+\ldots+n_p = n$ for every n. The larger the n the more the terms we take into account in constructing the infinite impulse response of the inverse channel. From the above it can be verified that this response is mostly extended to the postcursor direction spanning a time interval much longer than the respective part of the channel's impulse response. Especially if strong far post-echoes are present in the channel then the postcursor part of the inverse channel has nonnegligible values for thousands of symbol time intervals. Also in the latter case the corresponding channel spectrum exhibits very deep nulls.

III FREQUENCY DOMAIN DFE

The previous discussion regarding multipath channel implies that linear equalizers are disquilified from being used in the applications at hand (of course under normal conditions they work acceptably). On the contrary DFEs turn out to be tailor-made for channels of the form described above. The predominant part of ISI is caused by the long causal part of the impulse response; this ISI can be perfectly cancelled by a long Feedback (FB) filter. On the other hand the small amount of ISI due to the anticausal part may be effectively reduced using a relatively short Feedforward (FF) filter.

The new Frequency-Domain DFE (FD-DFE) consists of two distinct parts, i.e. the filtering part and the updating part, both operating on a block by block basis. To derive the two block schemes we start from the time domain DFE equations properly formulated, i.e.

$$\begin{aligned} y(n) &= \boldsymbol{a}_M^T(n)\boldsymbol{x}_M(n+M-\tau) + \boldsymbol{b}_N^T(n)\boldsymbol{d}_N(n-L) \\ d(n) &= f\{y(n)\} \end{aligned} \tag{2}$$

and

$$\begin{aligned} e(n) &= d(n) - y(n) \\ \boldsymbol{a}_M(n+1) &= \boldsymbol{a}_M(n) + 2\mu^a\boldsymbol{x}_M(n+M-\tau)e(n) \\ \boldsymbol{b}_N(n+1) &= \boldsymbol{b}_N(n) + 2\mu^b\boldsymbol{d}_N(n-L)e(n) \end{aligned} \tag{3}$$

Eqs. (2) and (3) correspond to the filtering and the updating part respectively. $\{x\}$ and $\{d\}$ denote the equalizer's input and decision sequences and vectors $\boldsymbol{a}_M(n)$ and $\boldsymbol{b}_N(n)$ denote the $M-th$ order FF filter and the $N-th$ order FB filter respectively. The FB filter is delayed by L symbol intervals, where L is a small integer and preferably power of two (i.e. 8 or 16). The FF filter is shifted to the postcursor part by τ symbol intervals in order to cover the small portion which is left uncovered by the FB filter (i.e. $\tau \geq L$). The relative positioning of the FF and FB filters with respect to the current time instant ($t = 0$) is shown in Fig. 1. Note that in most typical cases M is much less than N.

For the filtering part an FFT based scheme was developed in [3] and the reader is refered there for details. It is worth mentioning that this scheme was designed so as to involve a low number of operators and hence to be implementable on a single VLSI chip. The scheme yields the decision symbols in blocks of length M.

For the updating part an appropriate block formulation is first derived, with a block length equal to N, and the resulting block update recursions are subsequently implemented in the frequency domain. The updating scheme is summarized in Part 3 of Table I and its block diagram is given in Fig. 2 [1] (the filtering scheme of [3] is involved in Part 2 of the same Table). Vectors $\boldsymbol{A}_{2N}(k)$ and $\boldsymbol{B}_{2N}(k)$ are the $2N$-length FFT's of the FF and FB filters respectively. F and F^{-1} represent FFT and inverse FFT operations. The computational complexity of the updating scheme is comparable to that of existing frequency domain techniques appropriate for linear equalization [2], [4]. For typical block lengths the updating scheme is implementable in today's DSPs.

The two parts are interconnected as shown in Fig. 3. At the $k-th$ block, the N decision symbols involved in relations (1.2) and (3.1) of Table I as well as the N output samples involved in vector $\boldsymbol{y}_N(k)$ of relation (3.1) are provided by the filtering part in subblocks of length M. At the same time the updating part receives also the new N input samples and then computes the new estimates of $\boldsymbol{a}_M$ and $\boldsymbol{b}_N$. These estimates are fed to the filtering part and so on. In practice the refreshment of the filtering scheme taps is not necessary to be done at every block iteration since the environment is usually stationary for several tens of block iterations.

Considerable gain in convergence speed is achieved by using the matrix step sizes $\mathcal{M}_{2N}^A(k)$ and $\mathcal{M}_{2N}^B(k)$ in the update recursions. The elements of these diagonal matrices are inversely proportional to the power levels at the corresponding frequency bins of the power spectra of $\{x\}$ and $\{d\}$ respectively. The involvement of these matrix step sizes is briefly justified as follows. Viewing DFE as a two inputs - one output system it can be shown that the convergence of the corresponding LMS is governed by the eigenvalues of the autocorrelation matrix

$$R = E\{\boldsymbol{q}\boldsymbol{q}^H\} = \begin{pmatrix} R_{xx} & R_{xd} \\ R_{xd}^H & R_{dd} \end{pmatrix} \tag{4}$$

where $\boldsymbol{q} = [\boldsymbol{x}_M^H(n+M-\tau)\ \boldsymbol{d}_N^H(n-L)]^H$ is a vector of length $M+N$. Let us now decompose matrix R as $R = R_A + R_B$, where

$$R_A = \begin{pmatrix} R_{xx} & 0 \\ 0 & R_{dd} \end{pmatrix}, \quad R_E = \begin{pmatrix} 0 & R_{xd} \\ R_{xd}^H & 0 \end{pmatrix}$$

[1] In Fig. 2: $1/N$ denotes serial to parallel and $D_1 = M - \tau + L$

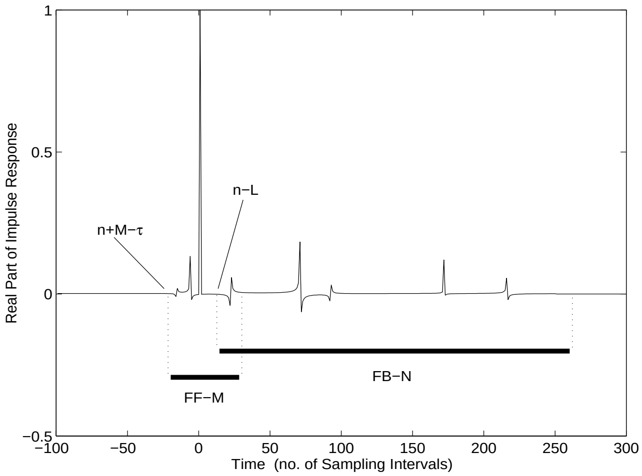

FIGURE 1

TABLE I : FREQUENCY DOMAIN DFE

Initialization

Define as $\boldsymbol{\epsilon}_{2N}$ a $2N$-length vector with a unity at the $M-\tau$ entry and zeros elsewhere, and as $\mathbf{1}_{2N}$ a $2N$-length vector with unity elements. Then

$\boldsymbol{A}_{2N}(0) = F\boldsymbol{\epsilon}_{2N}$, $\boldsymbol{B}_{2N}(0) = \mathbf{0}_{2N}$, $\boldsymbol{P}^{x}_{2N}(0) = \delta\mathbf{1}_{2N}$, $\boldsymbol{P}^{d}_{2N}(0) = \delta\mathbf{1}_{2N}$

Definition of the involved constraints

$$Q^M = \begin{bmatrix} I_{M\times M} & 0_{M\times 2N-M} \\ 0_{2N-M\times M} & 0_{2N-M\times 2N-M} \end{bmatrix} , \quad Q^N = \begin{bmatrix} I_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} \end{bmatrix}$$

$$Q^0 = [0_{N\times N} \quad I_{N\times N}]$$

For every new input block $x(kN+M-\tau), \ldots, x(kN+N+M-\tau-1)$, where k is the block index, perform the following parts:

1. Transformation of Input and Decision Samples

(1.1) $\mathcal{X}_{2M}(k) = diag\{F[x(kN-N+M-\tau), \ldots, x(kN+N+M-\tau-1)]^T\}$

(1.2) $\mathcal{D}_{2M}(k) = diag\{F[d(kN-N-L), \ldots, d(kN+N-L-1)]^T\}$

2. Filtering Part (Using the suggested fast scheme)

(2.1) Compute output of FF filter

(2.2) Compute output of FB filter

(2.3) Combine properly these outputs to compute $\boldsymbol{y}_N(k)$ and $\boldsymbol{d}_N(k)$

3. Updating Part

(3.1) $\boldsymbol{e}_N(k) = \boldsymbol{d}_N(k) - \boldsymbol{y}_N(k)$

(3.2) $\boldsymbol{E}_{2N}(k) = FQ^{0T}\boldsymbol{e}_N(k)$

(3.3) $\boldsymbol{P}^{x}_{2N}(k) = \lambda\boldsymbol{P}^{x}_{2N}(k-1) + (1-\lambda)\mathcal{X}^{H}_{2N}(k)\mathcal{X}_{2N}(k)\mathbf{1}_{2N}$

(3.4) $\boldsymbol{P}^{d}_{2N}(k) = \lambda\boldsymbol{P}^{d}_{2N}(k-1) + (1-\lambda)\mathcal{D}^{H}_{2N}(k)\mathcal{D}_{2N}(k)\mathbf{1}_{2N}$

(3.5) $\mathcal{M}^{A}_{2N}(k) = \mu^{a} diag\{[P^{-x}_{2N,1}(k), \ldots, P^{-x}_{2N,2N}(k)]^T\}$

(3.6) $\mathcal{M}^{B}_{2N}(k) = \mu^{b} diag\{[P^{-d}_{2N,1}(k), \ldots, P^{-d}_{2N,2N}(k)]^T\}$

(3.7) $\boldsymbol{A}_{2N}(k+1) = \boldsymbol{A}_{2N}(k) + 2FQ^{M}F^{-1}\mathcal{M}^{A}_{2N}(k)\mathcal{X}^{H}_{2N}(k)\boldsymbol{E}_{2N}(k)$

(3.8) $\boldsymbol{B}_{2N}(k+1) = \boldsymbol{B}_{2N}(k) + 2FQ^{N}F^{-1}\mathcal{M}^{B}_{2N}(k)\mathcal{D}^{H}_{2N}(k)\boldsymbol{E}_{2N}(k)$

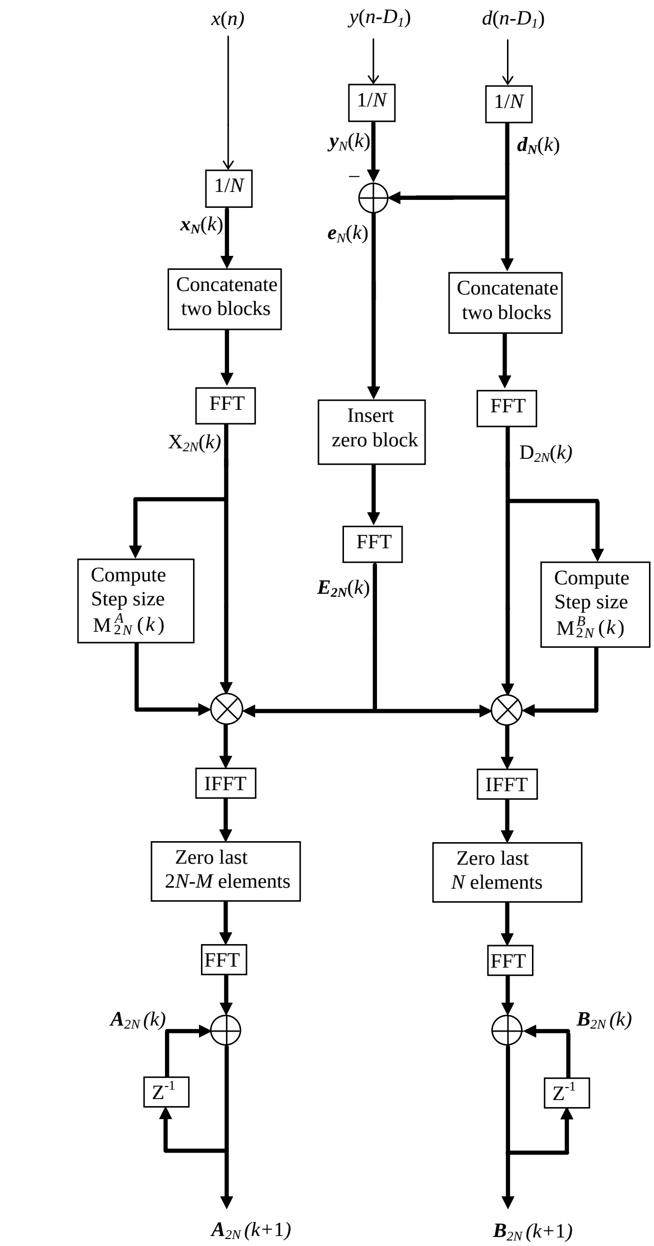

FIGURE 2

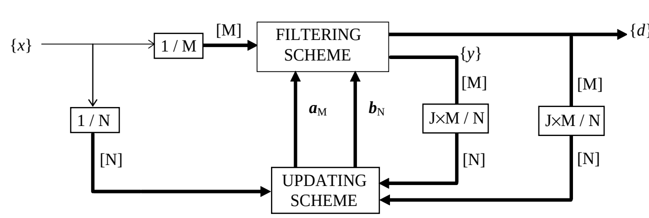

FIGURE 3

By further investigating matrix R (under the assumption that the original transmitted sequence of symbols is white) and using a well established result of linear algebra concerning the eigenvalue distribution of a sum of symmetric matrices [8] we can prove that

$$|\lambda_k(R) - \lambda_k(R_A)| \leq \sum_{i=1}^{p} |\alpha_i| \qquad (5)$$

Thus assuming the total echo energy is small with respect to that of the transmitted signal we may conclude that the eigenvalues of R may be well approximated by those of the block diagonal matrix R_A. The eigenvalue distribution of R_A may in turn be obtained through the power specta of the underlying processes, and this is exactly what is done in relations (3.3)-(3.6) of Table I.

IV EXPERIMENTAL RESULTS

The performance of the derived frequency domain block DFE was tested through a typical multipath echo cancellation experiment. The impulse response of the multipath channel consisted of 5 echoes, with amplitudes -17dB, -11dB, -14dB, -20dB, and -4dB respectively and time delays -$14T_s$, $45T_s$, $130T_s$, $210T_s$, and $230T_s$, where T_s was the symbol time interval. The echo phases were chosen randomly. Notice the presence of a strong far echo in the channel, i.e. -4dB at $210T_s$. The input to the transmitter filter was a QPSK sequence and at the output of the channel was added complex white gaussian noise.

In Fig. 4 the FD-DFE is compared with the standard Frequency Domain LMS (FLMS) algorithm as given in [2]. The additive noise power was such that $E_b/N_o = 12dB$. The orders of the FF and FB filters of the FD-DFE algorithm were taken equal to 32 and 256 respectively. The order of the FLMS algorithm was 256. The bottom (solid) curve corresponds to FD-DFE with matrix step sizes as in Table I. The top (dashed) curve corresponds to FLMS. The intermediate (dotted) curve corresponds to FD-DFE with fixed scalar step sizes for both update recursions. As it was expected, there is a striking difference in steady state MSE between FD-DFE and FLMS. Also it can be seen that using the suggested matrix step sizes a significant improvement in convergence rate of FD-DFE is achieved.

In Fig. 5 the BER curves are plotted. The bottom curve is the theoretical one for QPSK and the top curve is for the channel output without equalization. Dashed curve corresponds to FD-DFE, dotted curve corresponds to FLMS with order 256 and dashed-dotted curve corresponds to FLMS with order equal to 512. Obviously FD-DFE outperforms FLMS considerably. The latter performs very poorly even if we increase its order so that the involved adaptive filter spans a longer time interval.

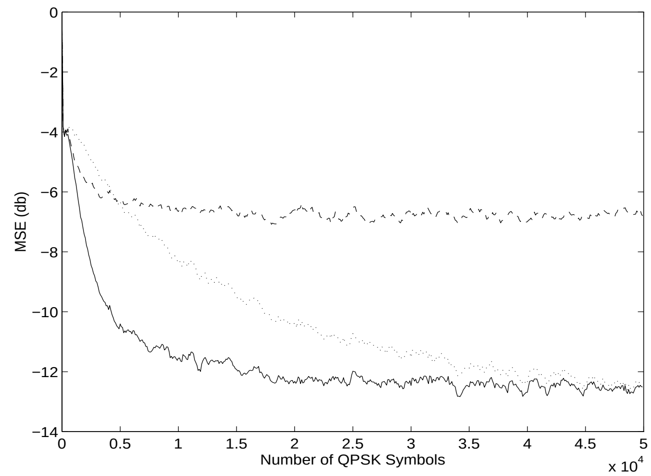

FIGURE 4

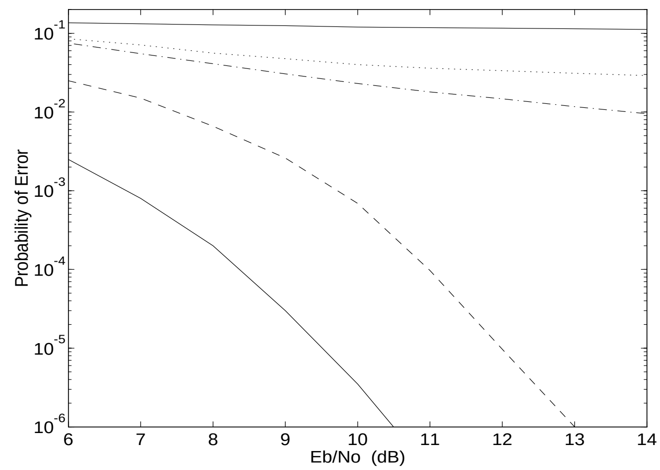

FIGURE 5

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