DATA-DEPENDENT FILTERING BASED ON IF-THEN RULES AND ELSE RULE

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ABSTRACT

We have proposed fuzzy filters based on local characteristics, in order to remove additive noise while preserving signal edges. Fuzzy filters were constructed by only IF-THEN rules. This paper shows a novel fuzzy filter which is constructed by not only IF- THEN rules but also ELSE rule. A lot of IF-THEN rules which have the same consequent, can be integrated into one ELSE rule. As a results, introducing the ELSE rule can realize increasing the local characteristics for the fuzzy filter without increasing the number of IF-THEN rules.

1 INTRODUCTION

Recently, fuzzy techniques have been successfully applied to image processing field. Russo and Ramponi introduced FIRE (Fuzzy Inference Ruled by Else-action) operators[1-3]. Many applications by using FIRE operators have been proposed addressing image enhancement, filtering and edge extraction. We have proposed data-dependent fuzzy (DDF) filters[4,5] which are constructed by fuzzy rules, in order to remove additive noise while preserving signal edges. Since the antecedents of fuzzy rules can be composed of several local characteristics, it is possible for the DDF filter to adjust its weights to adapt to local data. However, the increase the local characteristics for the DDF filter causes the increase the number of rules rapidly. It is difficult to optimize DDF filters constructed by a large number fuzzy rules. This paper shows a new data-dependent fuzzy filter which is constructed by not only IF-THEN rules but also ELSE rule. A lot of IF-THEN rules which have the same consequent, can be integrated into one ELSE rule. In fact many fuzzy rules of the DDF filter proposed in [5] (27 IF-THEN rules) have the same consequent. 18 IF-THEN rules will be integrated in one ELSE rule in this paper. As a results, introducing the ELSE rule can realize increasing the local characteristics for the DDF filter without increasing the number of IF-THEN rules. Thus, tuning of the DDF filter is performed by the experimental method.

2 DDF FILTERS CONSTRUCTED BY IF-THEN RULES AND ELSE RULE

2.1 Reducing IF-THEN Rules by Using ELSE Rules

Fuzzy inference is expressed by the following form:

IF (x_1 is A_{11}) AND $\cdots$ AND (x_M is A_{1M}) THEN (y is ω_1)
....
IF (x_1 is A_{n1}) AND $\cdots$ AND (x_M is A_{nM}) THEN (y is ω_n)
IF (x_1 is A_{n+11}) AND $\cdots$ AND (x_M is A_{n+1M}) THEN (y is ω_{n+1})
....
IF (x_1 is A_{N1}) AND $\cdots$ AND (x_M is A_{NM}) THEN (y is ω_N)

If many rules having the same consequent, for example $\omega_{n+1} = \cdots = \omega_N$, these IF-THEN rules can be integrated into one ELSE rule as follows:

IF (x_1 is A_{11}) AND $\cdots$ AND (x_M is A_{1M}) THEN (y is ω_1)
....
IF (x_1 is A_{n1}) AND $\cdots$ AND (x_M is A_{nM}) THEN (y is ω_n)
ELSE (y is ω_{n+1})

The degree of activation of the IF-THEN rules is given by

$$\mu_i = \min\{\mu_{A_{i1}}(x_1),\cdots,\mu_{A_{iM}}(x_M)\} \quad (i = 1,\ldots,n) \quad (1)$$

where $\mu_{A_{ij}}(x_j)$ is membership function on the fuzzy set A_{ij}. Then, the degree of activation of the ELSE rule is calculated by

$$\mu_{n+1} = \min\{(1-\mu_1),\cdots,(1-\mu_n)\} \quad (2)$$

The output y can be evaluated by using the following relationship

$$y = \{\sum_{i=1}^{n+1}\mu_i\cdot\omega_i\} \Big/ \{\sum_{i=1}^{n+1}\mu_i\} \quad (3)$$

2.2 DDF Filters

Let a noisy signal $x(i,j)$ be given by

$$x(i,j) = s(i,j) + n(i,j) \quad (4)$$

where $s(i,j)$ is an original signal and $n(i,j)$ is only an zero mean Gaussian noise with variance σ_n^2.

The proposed fuzzy filter which is called the data-dependent fuzzy (DDF) filter with input $x(i,j)$ produces output $y(i,j)$ according to the relation:

$$y(i,j) = \frac{\sum_{k=-N}^{N}\sum_{l=-N}^{N} w(k,l)x(i+k,j+l)}{\sum_{k=-N}^{N}\sum_{l=-N}^{N} w(k,l)} \quad (5)$$

The weights $w(k,l)$ are determined by fuzzy techniques at each point (i,j) based on local characteristics of image data (i.e., data-dependent filtering). $w(k,l)$ is also considered as a probability that $x(i,j)$ and $x(i+k,j+l)$ are belonging to the same flat region. Thus, $w(0,0)=1$ and if $x(i,j)$ and $x(i+k,j+l)$ are belonging to different flat regions, $w(k,l)$ should be set 0.

The probability is estimated by fuzzy inference. In [4], we show 9 IF-THEN rules' DDF filters with two local characteristics (i.e., $e(k,l)$ and $d(k,l)$) reviewed later. In [5], we further introduce the local characteristic $K(i,j)$ [6] into DDF filters. The number of IF-THEN rules has increased from 9 to 27, by introducing $K(i,j)$. It is impossible to optimize 27 IF-THEN rules' DDF filters by the experimental method.

In this paper, we investigate a novel DDF filter which is constructed by IF-THEN rules and ELSE rule. The DDF filter with three local characteristics (i.e., $K(i,j)$, $e(k,l)$ and $d(k,l)$) can be realized by only 9 IF-THEN rules pulse 1 ELSE rule.

2.3 Local Characteristics

We review typical local characteristics which are used in DDF filters as follows:

Local characteristics 1 : K(i,j) [5,6]

$$K(i,j) = \begin{cases} \dfrac{\mathrm{var}(i,j)-\sigma_n^2}{\mathrm{var}(i,j)} & \mathrm{var}(i,j) \ge \sigma_n^2 \\ 0 & otherwise \end{cases} \quad (6)$$

where $\mathrm{var}(i,j)$ is the sample variance of the data inside the filter window. In (6), roughly speaking, if there exist edges within the filter window, then $\mathrm{var}(i) \rangle\rangle \sigma_n^2$ and $K(i,j) \approx 1$. On the other hand, in gradually varying partitions of the input, $\mathrm{var}(i) \langle\langle \sigma_n^2$ and $K(i,j) \approx 0$

Local characteristics 2 : e(k,l) [4,5]

$$e(k,l) = |x(i+k,j+l) - x(i,j)| / \sigma_n \quad (7)$$

The smaller the value $e(k,l)$ indicates the more probability of $x(i+k,j+l)$ is involved in the same flat part involving $x(i,j)$.

Local characteristics 3 : d(k,l) [4,5]

$$d(k,l) = \sqrt{k^2+l^2} \,/\, N\sqrt{2} \quad (8)$$

The autocorrelation function of image signals is generally non-increasing symmetric. Therefore, the nearer distance signals from processed position is more important for filtering.

2.4 Structure of a Novel DDF Filter

The DDF filter with 27 IF-THEN rules was proposed in [5], that is expressed by:

IF ($K(i,j)$ is A_{p1}) AND ($e(i,j)$ is A_{p2}) AND ($d(i,j)$ is A_{p3})
THEN ($w(k,l)$ is ω_p) $\quad (p = 1,\ldots,27)$

The consequent action of 18 IF-THEN rules can be set equally one another. Thus, we can integrate these IF-THEN rules into one ELSE rule.

We express a novel DDF filter with 9 IF-THEN rules and one ELSE rule, in the following form:

IF ($K(i,j)$ is $Large$) AND ($e(i,j)$ is A_{q2}) AND ($d(i,j)$ is A_{q3})
THEN ($w(k,l)$ is ω_q) $\quad (q=1,\cdots,9)$
ELSE ($w(k,l)$ is ω_{10})

where A_q and B_q are fuzzy sets for *e(k,l)* and *d(k,l)* , respectively. IF-THEN rules are shown in Table 1. The relation between the set of local characteristics and the corresponding output (i.e., the coefficients of DDF filters) can be calculated by using from (1) to (3), as follows:

$$\mu_q(k,l) = \min\{\mu_L(K(i,j)),\ \mu_{A_{q1}}(e(k,l)),\ \mu_{A_{q2}}(d(k,l))\} \quad q=1,......,9 \qquad (9)$$

$$\mu_{10}(k,l) = \min\{1-\mu_1(k,l),\ \cdots,1-\mu_9(k,l)\} \qquad (10)$$

$$w(k,l) = (\sum_{q=1}^{10}\mu_q(k,l)\cdot\omega_q)/(\sum_{q=1}^{10}\mu_q(k,l)) \qquad (11)$$

3 EXPERIMENTAL RESULTS

3.1 Tuning of the Membership functions

We will show practical effects on image processing of the proposed DDF filter with 5x5 window. The fuzzy sets of *K(i,j)* , *e(k,l)* and *d(k,l)* are presented Fig.1. Furthermore, the output singletons are shown in Table 1.

Table 1 IF-THEN rules

	Small	Medium	Large	*d(k,l)*
Small	ω_1: 1.0	ω_2: 0.75	ω_3: 0.5	
Medium	ω_4: 0.75	ω_5: 0.5	ω_6: 0.25	
Large	ω_7: 0.5	ω_8: 0.25	ω_9: 0.0	
e(k,l)				

(ELSE rule: ω_{10} =1.0)

We tune up the membership functions experimentally by using upper left image "Lenna" corrupted by zero mean Gaussinan noise with standard deviation σ_n=20. We tune up the membership functions by using 6 parameters shown in Fig.1. We evaluate the filter performance by "increment of S/N" which is given by

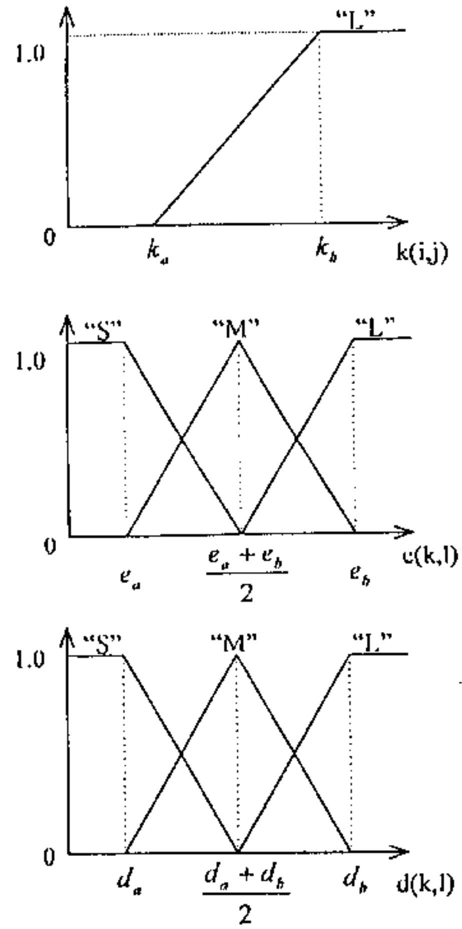

Fig.1 Fuzzy sets of each local characteristics

$$(\textit{Increment of S/N}) = (\textit{S/N of the output signal}) - (\textit{S/N of the input signal}) \qquad (12)$$

Figure 2 shows the filtering results for various cases. We obtain the tuning results as $(K_a,K_b,e_a,e_b,d_a,d_b)$=(0~0.1, 0.5~0.7, 0~2, 3~4, 0.2~0.5, 0.3~0.7). From these results, each optimal range is fairy wide. Therefore, even if these values are roughly set, the performance is not so degrade for them.

3.2 Experimental Results

Computer simulations have been carried out to compare the performance of the proposed filter with the optimal L-filter, the Wiener filter and conventional DDF filters [4,5]. All DDF filters with 5x5 window, are tuned by using upper left image "Lenna" corrupted by zero Gaussian noise with standard deviation σ_n=20. The image "Lenna" is corrupted by zero mean Gaussian noise with standard deviation $\sigma_n = 10,20,30$, are used for processing. From Table 2, the performance of the proposed filter almost equal to 27 IF-THEN rules' DDF[5]. We can decrease IF-THEN rules by using ELSE rule, without falling the performance of DDF filters..

Table 2(a) MSE results of DDF filters

	Proposed	DDF:9 rules	DDF:27 rules
$\sigma_n = 10$	43.2	50.1	40.9
$\sigma_n = 20$	90.6	96.8	90.3
$\sigma_n = 30$	145.9	151.1	143.6

(b) MSE results of filters

	Optimal- L	Wiener	DW-MTM[7]
$\sigma_n = 10$	146.2	54.1	42.9
$\sigma_n = 20$	176.6	120.5	105.3
$\sigma_n = 30$	217.8	175.5	171.1

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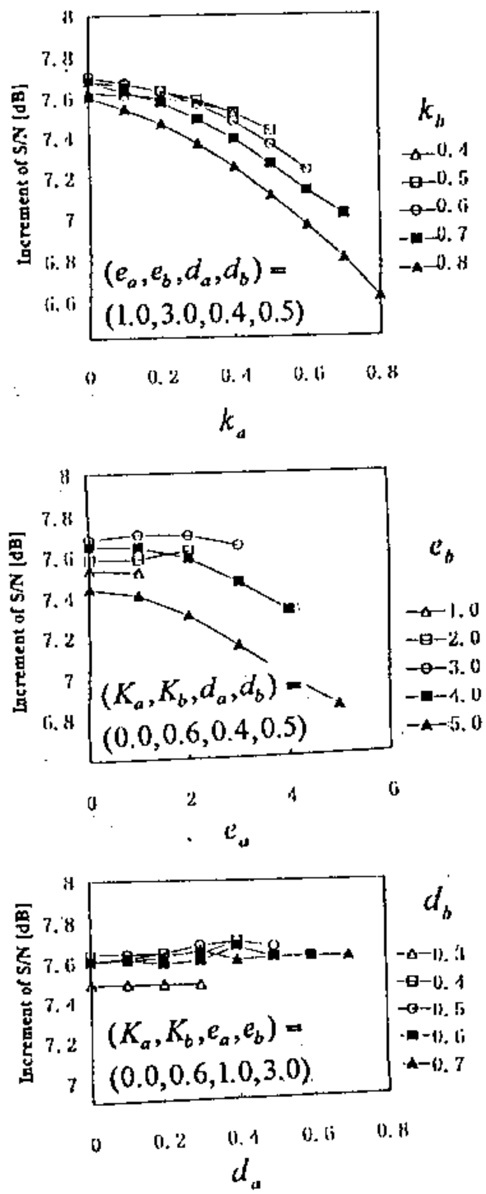

Fig.2 Tuning results of each membership function