FUZZY CELL HOUGH TRANSFORM

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ABSTRACT

In this paper a new variation of Hough Transform is proposed. It can be used to detect shapes or curves in an image, with better accuracy, especially in noisy images. It is based on a fuzzy split of the Hough Transform parameter space. The parameter space is split into fuzzy cells which are defined as fuzzy numbers. This fuzzy split of the parameter space provides the advantage to use the uncertainty of the contour points location, which is increased when noisy images have to be used. Moreover the computation time is slightly increased by this method, in comparison with classical Hough Transform.

1 INTRODUCTION

The Hough Transform (HT) [2] is a technique of fundamental importance for many applications in image processing and computer vision. It can be used to detect straight lines or circles in an image and can be generalized to detect an arbitrary shape at a given orientation and a given scale.

In this paper, we propose a novel method called Fuzzy Cell Hough Transform (FCHT), based on a fuzzy split of the Hough Transform parameter space into fuzzy cells. Each fuzzy cell is defined as a fuzzy set with a membership function $\mu(a_1, a_2, \ldots, a_p)$ of $p$ parameters. By using this fuzzy split, fuzziness is inserted to decisions through a fuzzy voting process. Each contour point in the spatial domain contributes with different voting values, in more than one fuzzy cell in the parameter space. The array that is created after the fuzzy voting process is smoother than in the classical case and the curves are estimated with better accuracy, especially when the images are corrupted by noise.

In the following we shall present the definitions of fuzzy cells in a three dimensional parameter space and will be generalized for a $p$-dimensional parameter space. Then, the corresponding fuzzy voting process for the detection of circles in an image will be described. Last, experimental results of the use of the proposed method will be presented and compared with classical Hough and Fuzzy Hough Transform (FHT) [1].

2 DEFINITION OF FUZZY CELLS

2.1 Definition of fuzzy cells for circle detection

Let us assume that a circle in a $N \times N$ image is to be detected by using the conventional HT. Since three parameters are needed to define any circle, the space that is used in HT is a three dimensional space. The parameters which are usually used are the coordinates of the circle center $a, b$ and the radius $r$. When HT is applied the parameter space has to be split in a finite number of cells. Let us assume that the three dimensional parameter space is split in $N_a \times N_b \times N_r$ cells. The crisp cell $C_{ij,k}$ can be defined as:

$$ C_{ij,k} = \{(a, b, r), a \in A_i, b \in B_j, r \in R_k\} \quad (1) $$

where $A_i$, $B_j$ and $R_k$ are classical sets.

In order to define fuzzy cells each point which belongs to the interval of confidence of the fuzzy cell corresponds to a value in $[0, 1]$ through its membership function. By using the assumption that $N_a$ is the number of partitions of parameter $a$, the fuzzy sets in coordinate $a$ can be defined by the following equation:

$$ A_i^t = \{(a, \mu_{A_i^t}(a)), a \in R\}. \quad (2) $$

where $\mu_{A_i^t}(a)$ is a membership function. Then, a fuzzy-$a$ cell $F_{ij,k}^a$ can be defined as a fuzzy number with three variables:

$$ F_{ij,k}^a = \{((a, b, r), \mu_{F_{ij,k}^a}(a, b, r)), (a, b, r) \in R^3\} \quad (3) $$

where

$$ \mu_{F_{ij,k}^a}(a, b, r) = \begin{cases} 
\mu_{A_i^t}(a) & \text{if } b \in B_j \text{ and } r \in R_k \\
0 & \text{elsewhere} 
\end{cases} \quad (4) $$

The same technique can be used to split $b$ and $r$ coordinate in fuzzy sets and similarly define the corresponding fuzzy-$b$ $F_{ij,k}^b$ and fuzzy-$r$ $F_{ij,k}^r$ cells.

If the fuzzy partitions in two coordinates, for example $a$ and $b$, are combined then a fuzzy-$ab$ cell can be defined as the following fuzzy number:

$$ F_{ij,k}^{ab} = \{((a, b, r), \mu_{F_{ij,k}^{ab}}(a, b, r)), (a, b, r) \in R^3\} \quad (5) $$
The concept of a fuzzy cell can be generalized, for the detection of straight lines.

2.2 Definition of p-dimensional fuzzy cells

The parameters used are symbolized as \( a_i \), \( i = 1, 2, \ldots, p \) and each parameter range is divided in \( N_a \), fuzzy sets symbolized as:

\[
A_{j, i} = \{ (a_i, \mu_{A_{j, i}}(a_i)), a_i \in R \}
\]

where \( i = 1, 2, \ldots, p \), \( j = 1, 2, \ldots, N_a \), and \( \mu_{A_{j, i}}(a_i) \) is a membership function. The fuzzy sets can be combined to define different kinds of fuzzy cells in the \( p \)-dimensional parameter space. In the general case, a fuzzy cell can be defined as:

\[
F_{j_1, \ldots, j_p} = \{ \{ (a_1, \ldots, a_p), \mu_{F_{j_1, \ldots, j_p}}(a_1, \ldots, a_p) \}, a_1, \ldots, a_p \in R \}
\]

where

\[
\mu_{F_{j_1, \ldots, j_p}}(a_1, \ldots, a_p) = \min(\mu_{A_{j_1, i_1}}, \ldots, \mu_{A_{j_p, i_p}})
\]

is the membership function of the \( p \)-dimensional fuzzy cell.

3 DESCRIPTION OF THE FUZZY VOTING PROCESS

Let us assume that a pixel \( x, y \) is a contour point in an image and that the parameter space is split in \( N_a \times N_b \times N_c \) fuzzy-\( r \) cells \( F_{j_1, j_2, j_3} \). The centers \( a_i \) of the crisp sets \( A_i \) and the centers \( b_j \) of the crisp sets \( B_j \) are used to compute the distances \( r_{ij} \) by solving the following equation:

\[
(x - a_i)^2 + (y - b_j)^2 = r_{ij}^2
\]

Each point \( (a_i, b_j, r_{ij}) \) belongs to one of the fuzzy cells \( F_{j_1, j_2, j_3} \). The corresponding elements of the accumulator array \( A(l, m, n) \) are increased by the values of the membership functions \( \mu_{F_{j_1, j_2, j_3}}(a_i, b_j, r_{ij}) \). Finally, the local maxima in the array \( A \) have to be detected.

When fuzzy-\( ab \) cells are used, the voting process is more complicated. The fuzzy numbers \( B_{j}^{f} \) which are symbolized by the union of their \( a \)-cuts as:

\[
B_{j}^{f} = \bigcup_{a} a \cdot [b_{j}^{(a)}, b_{j}^{(a)}], \quad a \in [0, 1]
\]

and the crisp centers \( r_{k} \) of the crisp sets \( R_k \) are used to compute the fuzzy distances \( A_{jk}^{f} \) by solving the following fuzzy equation:

\[
(x - A_{jk}^{f})^2 + (y - B_{j}^{f})^2 = r_{k}^2
\]

where all the operators are the extended fuzzy operators, through the extension principle. Then, the fuzzy distances \( A_{jk}^{f} \) which are symbolized by the union of their \( a \)-cuts as:

\[
A_{jk}^{f} = \bigcup_{a} a \cdot [a_{jk}^{(a)}, b_{jk}^{(a)}], \quad a \in [0, 1]
\]

are calculated by the equations:

\[
a_{jk}^{(a)} = \begin{cases} x - \sqrt{r_{k}^2 - \min} & \text{if } r_{k}^2 \geq \max \\ x - \frac{r_{k}^2 - \max}{\sqrt{r_{k}^2 - \max}} & \text{if } r_{k}^2 < \max \end{cases}
\]

This fuzzy process provides the ability to transfer the fuzziness of an image through the Hough Transform to the accumulator array. The array that is created after the fuzzy voting process is smoother than the crisp case. This means that local maxima which correspond to the effect of noise in an image disappear. The circles can now be detected with better accuracy. However, this method slightly increases the computation time.

Figure 1 shows an example of the two dimensions \( a, r \) of an accumulator array after a classical voting process (a) and after a fuzzy voting process (b), when classical Hough and Fuzzy Cell Hough Transform are applied in an artificial generated image with one circle in it, not corrupted by noise. The parameter space was split in 20 parts towards \( a, b \) and \( r \) coordinate in both classical and fuzzy case. Fuzzy-\( r \)-cells were used in the fuzzy case. It is obvious that the accumulator array is smoother when Fuzzy Cell Hough Transform is applied.

4 EXPERIMENTAL RESULTS

We considered that a circle was to be detected in a \( 256 \times 256 \) image. Parameters \( a, b, r \) were restricted to the sets \( a \in [-32, 32], b \in [-32, 32] \) and \( r \in [1, 21] \). The image was corrupted by uniform noise having the range \( \pm d \) pixels added to the \( p \)-dimension of a contour point \( (x, y) \). An example of such a corrupted circle is shown in Figure 2. Two criteria were used in order to compare the results. The first one was the center estimation distance error \( \sqrt{a_{ij}^2 + b_{ij}^2} - \sqrt{a_{ij}^2 + b_{ij}^2} \) of the detected circle center \( (a, b) \) to the actual one \( (a, b) \) and the second was the
radius estimation distance error \( |\hat{r} - r| \) of the detected radius \( \hat{r} \) to the actual one \( r \).

First the classical HT was used to detect the circle. The experiment was repeated for \( N = 150 \) different circles and five different values of noise range. The same circles were detected by using FHT and FCHT. In the FCHT case fuzzy-\( r \) cells were used. Fuzzy sets were chosen to be triangular and the upper and lower limits were chosen to be in equal distances from the center and equal to the distance of the centers of two neighboring fuzzy sets. In the FHT the fuzziness of contour points were supposed to have the same fuzziness as in FCHT case.

The effect of fuzziness of the chosen triangular fuzzy sets was investigated as well. The distances of the upper and lower limits of the fuzzy sets from the center were increased by a factor 1.5, 2, 2.5 and 3 in comparison with the initial case. The sum errors of the center estimation and radius estimation error are given by:

\[
E_d = \sum_{i=1}^{N_c} \left| \sqrt{a_i^2 + b_i^2} - \sqrt{\hat{a}_i^2 + \hat{b}_i^2} \right| \tag{18}
\]

\[
E_r = \sum_{i=1}^{N_c} |\hat{r}_i - \hat{r}_i| \tag{19}
\]

and are presented in Tables 1 to 3 for three different values of the noise range.

Regarding the center \( E_d \) and radius estimation \( E_r \), FCHT has always better performance in comparison to classical HT and better than FHT when the noise range is relatively small. By using FCHT method in an image without noise the center estimation error was reduced by 11\% in comparison with classical HT. The radius estimation error was also reduced by 77\%. In a noisy image with noise range \( \pm 5 \) pixels the center estimation error was reduced by 5\% and the radius estimation error was reduced by 71\%. When the noise range was \( \pm 10 \) pixels the center estimation error was reduced by 23\% and the radius estimation error was reduced by 56\%.

5 CONCLUSIONS

We introduced the Fuzzy Cell Hough Transform as a method to detect curves in an image. We proposed a fuzzy split of the Hough Transform parameter space which led us to a fuzzy voting algorithm. Each contour point voted with different values in more than one fuzzy cells in the parameter space. The array that was created after the fuzzy voting process was smoother than in classical case. Local maxima that correspond to the effect of noise or any kind of uncertainty disappeared. Curves were detected with better accuracy in comparison with classical Hough Transform and Fuzzy Hough Transform.

References


Table 1: Sum errors $E_d$ and $E_r$ in circle center and radius estimation and the corresponding computational time $t$ in seconds, by using HT, FHT and FCHT method in an image not corrupted by noise.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuzziness $(f)$</th>
<th>Errors $E_d$</th>
<th>Errors $E_r$</th>
<th>Time $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HT</td>
<td></td>
<td>485</td>
<td>256</td>
<td>23.3</td>
</tr>
<tr>
<td>FHT</td>
<td>1</td>
<td>468</td>
<td>210</td>
<td>32.4</td>
</tr>
<tr>
<td>FCHT</td>
<td>1.5</td>
<td>445</td>
<td>110</td>
<td>9.2</td>
</tr>
<tr>
<td>FHT</td>
<td>2</td>
<td>453</td>
<td>78</td>
<td>123.5</td>
</tr>
<tr>
<td>FCHT</td>
<td>2.5</td>
<td>449</td>
<td>68</td>
<td>290.2</td>
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<tr>
<td>FHT</td>
<td>3</td>
<td>444</td>
<td>62</td>
<td>97.12</td>
</tr>
<tr>
<td>FCHT</td>
<td>3</td>
<td>431</td>
<td>94</td>
<td>94.4</td>
</tr>
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Table 2: Sum errors $E_d$ and $E_r$ in circle center and radius estimation and the corresponding computational time $t$ in seconds, by using HT, FHT and FCHT method in an image corrupted by uniform noise in the range $\pm 5$ pixels.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuzziness $(f)$</th>
<th>Errors $E_d$</th>
<th>Errors $E_r$</th>
<th>Time $t$</th>
</tr>
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<tbody>
<tr>
<td>HT</td>
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<tr>
<td>FHT</td>
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<td>513</td>
<td>168</td>
<td>301</td>
</tr>
<tr>
<td>FCHT</td>
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<td>518</td>
<td>120</td>
<td>85</td>
</tr>
<tr>
<td>FHT</td>
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<td>513</td>
<td>116</td>
<td>111.7</td>
</tr>
<tr>
<td>FCHT</td>
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<td>98</td>
<td>1643</td>
</tr>
<tr>
<td>FHT</td>
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<td>493</td>
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<td>261.7</td>
</tr>
<tr>
<td>FCHT</td>
<td>3</td>
<td>514</td>
<td>60</td>
<td>87</td>
</tr>
</tbody>
</table>

Table 3: Sum errors $E_d$ and $E_r$ in circle center and radius estimation and the corresponding computational time $t$ in seconds, by using HT, FHT and FCHT method in an image corrupted by uniform noise in the range $\pm 10$ pixels.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuzziness $(f)$</th>
<th>Errors $E_d$</th>
<th>Errors $E_r$</th>
<th>Time $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HT</td>
<td></td>
<td>717</td>
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<td>637</td>
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<td>87</td>
</tr>
<tr>
<td>FCHT</td>
<td>2.5</td>
<td>552</td>
<td>116</td>
<td>180.2</td>
</tr>
<tr>
<td>FHT</td>
<td>3</td>
<td>588</td>
<td>100</td>
<td>264.5</td>
</tr>
<tr>
<td>FCHT</td>
<td>3</td>
<td>551</td>
<td>120</td>
<td>88</td>
</tr>
</tbody>
</table>

Figure 1: An accumulator array after a classical Hough Transform voting process (a) and a Fuzzy Cell Hough Transform voting process (b).

Figure 2: An artificially generated circle in a $256 \times 256$ image corrupted by uniform noise with range $\pm 5$ pixels added to the $\rho$-dimension of a contour point.