FUZZY CELL HOUGH TRANSFORM

Vassilios Chatzis *Ioannis Pitas*
Department of Informatics
University of Thessaloniki, 54006 Thessaloniki, GREECE
Tel, fax: +30-31-996304
e-mail: pitas@zeus.csd.auth.gr

ABSTRACT

In this paper a new variation of Hough Transform is proposed. It can be used to detect shapes or curves in an image, with better accuracy, especially in noisy images. It is based on a fuzzy split of the Hough Transform parameter space. The parameter space is split into fuzzy cells which are defined as fuzzy numbers. This fuzzy split of the parameter space provides the advantage to use the uncertainty of the contour points location, which is increased when noisy images have to be used. Moreover the computation time is slightly increased by this method, in comparison with classical Hough Transform.

1 INTRODUCTION

The Hough Transform (HT) [2] is a technique of fundamental importance for many applications in image processing and computer vision. It can be used to detect straight lines or circles in an image and can be generalized to detect an arbitrary shape at a given orientation and a given scale.

In this paper, we propose a novel method called Fuzzy Cell Hough Transform (FCHT), based on a fuzzy split of the Hough Transform parameter space into fuzzy cells. Each fuzzy cell is defined as a fuzzy set with a membership function $\mu(a_1, a_2, \ldots, a_p)$ of p parameters. By using this fuzzy split, fuzziness is inserted to decisions through a fuzzy voting process. Each contour point in the spatial domain contributes with different voting values, in more than one fuzzy cell in the parameter space. The array that is created after the fuzzy voting process is smoother than in the classical case and the curves are estimated with better accuracy, especially when the images are corrupted by noise.

In the following we shall present the definitions of fuzzy cells in a three dimensional parameter space and will be generalized for a p-dimensional parameter space. Then, the corresponding fuzzy voting process for the detection of circles in an image will be described. Last, experimental results of the use of the proposed method will be presented and compared with classical Hough and Fuzzy Hough Transform (FHT) [1].

2 DEFINITION OF FUZZY CELLS

2.1 Definition of fuzzy cells for circle detection

Let us assume that a circle in a $N \times N$ image is to be detected by using the conventional HT. Since three parameters are needed to define any circle, the space that is used in HT is a three dimensional space. The parameters which are usually used are the coordinates of the circle center a, b and the radius r. When HT is applied the parameter space has to be split in a finite number of cells. Let us assume that the three dimensional parameter space is split in $N_a \times N_b \times N_r$ cells. The crisp cell C_{ijk} can be defined as:

$$C_{ijk} = \{(a, b, r), a \in A_i, b \in B_j, r \in R_k\} \tag{1}$$

where A_i, B_j and R_k are classical sets.

In order to define fuzzy cells each point which belongs to the interval of confidence of the fuzzy cell corresponds to a value in $[0, 1]$ through its membership function. By using the assumption that N_a is the number of partitions of parameter a, the fuzzy sets in coordinate a can be defined by the following equation:

$$A_i^f = \{(a, \mu_{A_i^f}(a)), a \in R\}. \tag{2}$$

where $\mu_{A_i^f}(a)$ is a membership function. Then, a fuzzy-a cell F_{ijk}^a can be defined as a fuzzy number with three variables:

$$F_{ijk}^a = \{((a, b, r), \mu_{F_{ijk}^a}(a, b, r)), (a, b, r) \in R^3\} \tag{3}$$

where

$$\mu_{F_{ijk}^a}(a, b, r) = \begin{cases} \mu_{A_i^f}(a) & \text{if } b \in B_j \text{ and } r \in R_k \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

The same technique can be used to split b and r coordinate in fuzzy sets and similarly define the corresponding fuzzy-b F_{ijk}^b and fuzzy-r F_{ijk}^r cells.

If the fuzzy partitions in two coordinates, for example a and b, are combined then a fuzzy-ab cell can be defined as the following fuzzy number:

$$F_{ijk}^{ab} = \{((a, b, r), \mu_{F_{ijk}^{ab}}(a, b, r)), (a, b, r) \in R^3\} \tag{5}$$

where

$$\mu_{F^{ab}_{ijk}}(a,b,r) = \begin{cases} \min(\mu_{A^f_i}(a), \mu_{B^f_j}(b)) & \text{if } r \in R_k \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

Similarly fuzzy-ar, fuzzy-br cells and fuzzy-abr cells can be defined. Fuzzy cells can also easily be defined in a restricted two dimensional parameter space for the detection of straight lines.

2.2 Definition of p-dimensional fuzzy cells

The concept of a fuzzy cell can be generalized, for the detection of any curve that can be described with p parameters, in a p-dimensional parameter space as follows. The parameters used are symbolized as a_i, $i = 1, 2, \ldots p$ and each parameter range is divided in N_{a_i} fuzzy sets symbolized as:

$$A^f_{j_i} = \{(a_i, \mu_{A^f_{j_i}}(a_i)), a_i \in R\} \tag{7}$$

where $i = 1, 2, \ldots, p$, $j_i = 1, 2, \ldots, N_{a_i}$ and $\mu_{A^f_{j_i}}(a_i)$ is a membership function. The fuzzy sets can be combined to define different kinds of fuzzy cells in the p-dimensional parameter space. In the general case, a fuzzy cell can be defined as:

$$F^p_{j_1,\ldots j_p} = \{((a_1, \ldots a_p), \mu^p_{j_1,\ldots j_p}(a_1, \ldots a_p)), (a_1, \ldots a_p) \in R^p\} \tag{8}$$

where

$$\mu^p_{j_1,\ldots j_p}(a_1, \ldots a_p) = \min(\mu_{A^f_{j_1}}, \ldots \mu_{A^f_{j_p}}) \tag{9}$$

is the membership function of the p dimensional fuzzy cell.

3 DESCRIPTION OF THE FUZZY VOTING PROCESS

Let us assume that a pixel x, y is a contour point in an image and that the parameter space is split in $N_a \times N_b \times N_r$ fuzzy-r cells F^r_{lmn}. The centers a_i of the crisp sets A_i and the centers b_j of the crisp sets B_j are used to compute the distances r_{ij} by solving the following equation:

$$(x - a_i)^2 + (y - b_j)^2 = r^2_{ij} \tag{10}$$

Each point (a_i, b_j, r_{ij}) belongs to more than one fuzzy cells F^r_{lmn}. The corresponding elements of the accumulator array $A(l, m, n)$ are increased by the values of the membership functions $\mu_{F^r_{lmn}}(a_i, b_j, r_{ij})$. Finally, the local maxima in the array A have to be detected.

When fuzzy-ab cells are used, the voting process is more complicated. The fuzzy numbers B^f_j which are symbolized by the union of their α-cuts as:

$$B^f_j = \bigcup_\alpha \alpha \cdot [b^{(\alpha)}_{j_l}, b^{(\alpha)}_{j_r}], \qquad \alpha \in [0,1] \tag{11}$$

and the crisp centers r_k of the crisp sets R_k are used to compute the fuzzy distances A^f_{jk} by solving the following fuzzy equation:

$$(x - A^f_{jk})^2 + (y - B^f_j)^2 = r^2_k \tag{12}$$

where all the operators are the extended fuzzy operators, through the extension principle. Then, the fuzzy distances A^f_{jk} which are symbolized by the union of their α-cuts as:

$$A^f_{jk} = \bigcup_\alpha \alpha \cdot [a^{(\alpha)}_{jk_l}, a^{(\alpha)}_{jk_r}], \qquad \alpha \in [0,1] \tag{13}$$

are calculated by the equations:

$$a^{(\alpha)}_{jk_l} = x - \sqrt{r^2_k - \min} \tag{14}$$

$$a^{(\alpha)}_{jk_r} = \begin{cases} x - \sqrt{r^2_k - \max} & \text{if } r^2_k \geq \max \\ x - \frac{r^2_k - \max}{\sqrt{r^2_k - \min}} & \text{if } r^2_k < \max \end{cases} \tag{15}$$

where

$$\max = \max((y - b^{(\alpha)}_{j_r})^2, (y - b^{(\alpha)}_{j_l})^2) \tag{16}$$

$$\min = \min((y - b^{(\alpha)}_{j_r})^2, (y - b^{(\alpha)}_{j_r})(y - b^{(\alpha)}_{j_l}), (y - b^{(\alpha)}_{j_l})^2). \tag{17}$$

This fuzzy process provides the ability to transfer the fuzziness of an image through the Hough Transform to the accumulator array. The array that is created after the fuzzy voting process is smoother than in the crisp case. This means that local maxima which correspond to the effect of noise in an image disappear. The circles can now be detected with better accuracy. However, this method slightly increases the computation time.

Figure 1 shows an example of the two dimensions a, r of an accumulator array after a classical voting process (a) and after a fuzzy voting process (b), when classical Hough and Fuzzy Cell Hough Transform are applied in an artificial generated image with one circle in it, not corrupted by noise. The parameter space was split in 20 parts towards a, b and r coordinate in both classical and fuzzy case. Fuzzy r-cells were used in the fuzzy case. It is obvious that the accumulator array is smoother when Fuzzy Cell Hough Transform is applied.

4 EXPERIMENTAL RESULTS

We considered that a circle was to be detected in a 256×256 image. Parameters a, b, r were restricted to the sets $a \in [-32, 32]$, $b \in [-32, 32]$ and $r \in [1, 21]$. The image was corrupted by uniform noise having the range $\pm d$ pixels added to the $\rho-$dimension of a contour point (x, y). An example of such a corrupted circle is shown in Figure 2. Two criteria were used in order to compare the results. The first one was the center estimation distance error $|\sqrt{a^2_i + b^2_i} - \sqrt{\hat{a}^2_i + \hat{b}^2_i}|$ of the detected circle center $(\hat{a}, \hat{b})$ to the actual one (a, b) and the second was the radius estimation distance error $|r - \hat{r}|$ of the detected radius $\hat{r}$ to the actual one r.

First the classical HT was used to detect the circle. The experiment was repeated for $N_c = 150$ different circles and five different values of noise range. The same circles were detected by using FHT and FCHT. In the FCHT case fuzzy-r cells were used. Fuzzy sets were chosen to be triangular and the upper and lower limits were chosen to be in equal distances from the center and equal to the distance of the centers of two neighbouring fuzzy sets. In the FHT the fuzziness of contour points were supposed to have the same fuzziness as in FCHT case.

The effect of fuzziness of the chosen triangular fuzzy sets was investigated as well. The distances of the upper and lower limits of the fuzzy sets from the center were increased by a factor 1.5, 2, 2.5 and 3 in comparison with the initial case. The sum errors of the center estimation and radius estimation error are given by:

$$E_d = \sum_{i=1}^{N_c} |\sqrt{a_i^2 + b_i^2} - \sqrt{\hat{a}_i^2 + \hat{b}_i^2}| \tag{18}$$

$$E_r = \sum_{i=1}^{N_c} |r_i - \hat{r}_i| \tag{19}$$

and are presented in Tables 1 to 3 for three different values of the noise range.

Regarding the center E_d and radius estimation E_r, FCHT has always better performance in comparison to classical HT and better than FHT when the noise range is relatively small. By using FCHT method in an image without noise the center estimation error was reduced by 11% in comparison with classical HT. The radius estimation error was also reduced by 77%. In a noisy image with noise range ± 5 pixels the center estimation error was reduced by 5% and the radius estimation error was reduced by 71%. When the noise range was ± 10 pixels the center estimation error was reduced by 23% and the radius estimation error was reduced by 56%.

5 CONCLUSIONS

We introduced the Fuzzy Cell Hough Transform as a method to detect curves in an image. We proposed a fuzzy split of the Hough Transform parameter space which led us to a fuzzy voting algorithm. Each contour point voted with different values in more than one fuzzy cells in the parameter space. The array that was created after the fuzzy voting process was smoother than in classical case. Local maxima that correspond to the effect of noise or any kind of uncertainty disappeared. Curves were detected with better accuracy in comparison with classical Hough Transform and Fuzzy Hough Transform.

References

[1] J. H. Han, L. Koczy, T. Poston. *"Fuzzy Hough Transform"*, Pattern Recognition Letters 15, pp. 649-658, July 1994.

[2] P. V. C. Hough. *"Method and Means for Recognizing Complex pattern"*, U.S.Patent 3069654, Dec. 18, 1962.

[3] V. F. Leavers. *"Shape Detection in Computer Vision Using the Hough Transform"*, Springer-Verlag, London, 1992.

[4] J Illingworth, J. Kittler. *"The Adaptive Hough Transform"*, IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 9, no 5, pp. 690-698, Sep. 1987.

[5] N. Guil, J. Villalba, E. L. Zapata. *A Fast Hough Transform for Segment Detection"*, IEEE Trans. on Image Processing, vol. 4, no 11, pp. 1541-1548,Nov. 1995.

[6] R. O. Duda, P. E. Hart. *"Use of the Hough Transform to detect lines and curves in pictures"*, Commun. Assoc. Computing. Math., vol. 15, no 1, pp. 204-208, 1972.

[7] S. Tsuji, F. Matsumoto. *"Detection of ellipses by a modified Hough transformation"*, IEEE Trans. Comput., vol. 27, no. 8, pp. 777-781, Aug. 1978.

[8] M. Atiquzzaman. *"Multiresolution Hough Transform - An efficient method of detecting patterns in images"*, IEEE Trans. Pattern Anal. Mach. Intel., vol. 14, no. 11, pp. 1090-1095, 1992.

[9] D. Chang, S. Hashimoto. *"An inverse voting algorithm for Hough Transform"*, Proceedings ICIP-94, vol.1, pp. 223-227, 1994.

[10] L. Zadeh. *"Fuzzy Sets"*, IEEE Inform. and Control, vol 8, no 3, pp. 338-353, 1965.

[11] S. G. Tzafestas, A. N. Venetsanopoulos. *"Fuzzy Reasoning in Information, Decision and Control Systems"*, Kluwer Academic Publishers, Dordrecht, 1994.

[12] L. H. Tsoukalas, R. E. Uhrig. *"Fuzzy and Neural Approaches in Engineering"*, 1993.

[13] J. C. Bezdek, S. K. Pal. *"Fuzzy Models for Pattern Recognition"*, IEEE Press, New York, 1992.

[14] A. Kaufmann, M. M. Gupta. *"Introduction to Fuzzy Arithmetic: Theory and Applications"*, Van Nostrand Reinhold, New York, 1985.

[15] D. Dubois, H. Prade. *"Fuzzy Sets and Systems, Theory and Applications"*, Academic Press, Boston, 1980.

Table 1: Sum errors E_d and E_r in circle center and radius estimation and the corresponding computational time t in seconds , by using HT, FHT and FCHT method in an image not corrupted by noise.

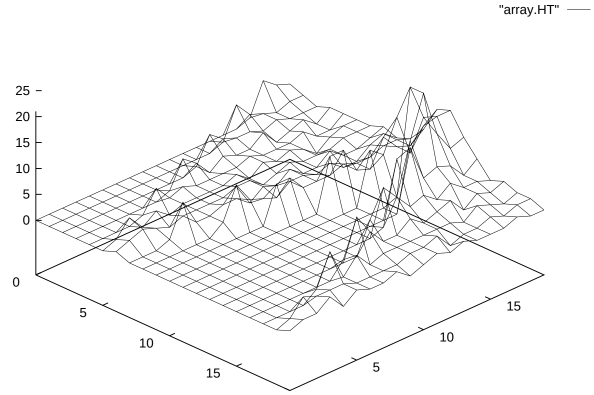
Method	Fuzziness (f)	Errors E_d	Errors E_r	Time t
HT		483	256	23 √
FHT	1	460	210	324
FCHT	1	460	228	93
FHT	1.5	435	130	721
FCHT	1.5	444	110	92
FHT	2	453	78	1225
FCHT	2	449	62	94
FHT	2.5	449	68	2002
FCHT	2.5	435	60√	94
FHT	3	444	62	2712
FCHT	3	431√	94	94

Table 2: Sum errors E_d and E_r in circle center and radius estimation and the corresponding computational time t in seconds , by using HT, FHT and FCHT method in an image corrupted by uniform noise in the range ± 5 pixels.

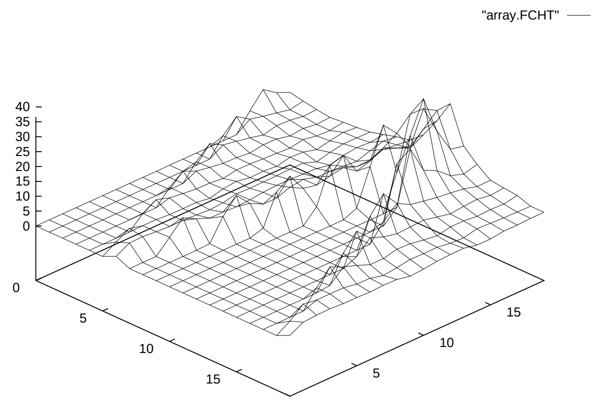
Method	Fuzziness (f)	Errors E_d	Errors E_r	Time t
HT		514	180	21 √
FHT	1	515	168	301
FCHT	1	523	180	85
FHT	1.5	523	134	662
FCHT	1.5	518	120	85
FHT	2	513	116	1117
FCHT	2	496	58	86
FHT	2.5	499	96	1843
FCHT	2.5	486√	52 √	86
FHT	3	493	94	2617
FCHT	3	514	60	87

Table 3: Sum errors E_d and E_r in circle center and radius estimation and the corresponding computational time t in seconds , by using HT, FHT and FCHT method in an image corrupted by uniform noise in the range ± 10 pixels.

Method	Fuzziness (f)	Errors E_d	Errors E_r	Time t
HT		717	262	22√
FHT	1	678	222	301
FCHT	1	698	232	86
FHT	1.5	682	146	675
FCHT	1.5	710	202	86
FHT	2	674	194	1129
FCHT	2	637	160	87
FHT	2.5	610	116	1892
FCHT	2.5	552	116√	87
FHT	3	589	166	2645
FCHT	3	551√	120	88



(a)



(b)

Figure 1: An accumulator array after a classical Hough Transform voting process (a) and a Fuzzy Cell Hough Transform voting process (b)

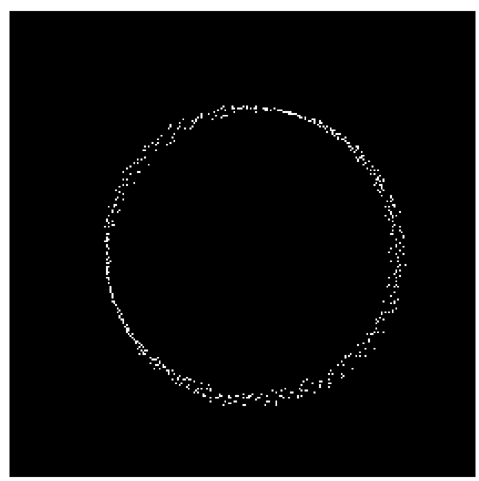


Figure 2: An artificially generated circle in a 256×256 image corrupted by uniform noise with range ± 5 pixels added to the ρ-dimension of a contour point.