BIORTHOGONAL B-SPLINE FILTER BANKS FOR LOW BIT RATE VIDEO CODING

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ABSTRACT

In this paper we investigate the performance of B-Spline filter banks for low bit rate image coding. The influence of certain characteristics of the analysis and synthesis of *FIR* filters are studied. These include the B-Spline polynomial order, the effects of coefficient truncation, coding quantisation and the distortion introduced by the filters themselves. Due to the high concentration of energy in the low frequency band, these biorthogonal filter banks have better capabilities to reconstruct signals from the lower frequency band than their counterparts. As a result a very low bit rate video codec can be designed by coarse quantisation of the higher bands.

1 INTRODUCTION

Subband coding [1] techniques are becoming more popular for compression of speech and video signals. In this technique, an input signal is decomposed into a set of frequency bands by their proper analysis filter banks. In the reconstruction process, these bands are decoded, interpolated, filtered (*synthesis*) and are added together to represent a replica of the original signal. Several design procedures for the perfect and aliase free reconstruction of the filter banks are known [2, 3]. However, for very low bit rate coding applications higher frequency bands are coarsely quantised, then the signal is mainly reconstructed with the lower band signal. Hence the coding efficiency is improved by using filters that concentrate higher energy in the lower frequency band. Such filters can be of B-Spline type. We have designed a two-channel analysis and synthesis filter banks which are biorthogonal and use B-Spline *FIR* filters. In this paper we analyse the error characteristics of these filters and compare their performance for image coding applications against the commonly used filters [4, 5].

2 B-SPLINES FUNCTIONS

Polynomials are widely used in approximation theory and numerical analysis to approximate smooth functions. However, as the number of interpolation points increases, the necessarily the flexibility of the interpolating polynomial is achieved by increasing both the degree and the risk of severe oscillations of the fitted curves. In order to prevent this problem, an alternative approach is to keep the degree of the polynomial low and provide flexibility by joining sufficient number of low order polynomial segments. The resulting interpolating function is called a *piecewise polynomial function*. B-Spline basis functions, first introduced by Schoenberg [6], have such property and are commonly used to represent curves. Letting $X(u)$ be the parametric position vector along a curve, as a function of the parameter u, a definition for B-Spline curve is given by

$$X(u) = \sum_{i=1}^{n+1} Y_i B_{i,k}(u),\ u_{min} \le u < u_{max},\ 2 \le k \le n+1 \tag{1}$$

where Y_i are the position vectors of the *n+1* polygon vertices and the $B_{i,k}$ are the normalised B-Spline basis functions of order k. These polygon vertices are a set of data values which represent the original curve by approximation. The *piecewise polynomial* curve is formed by a group of consecutive polynomial segments. The joints between these segments form a set of values designed by *knot vectors*. The most important properties of the B-Spline basis functions are:

- The continuity (C) of the curve in each knot is two less (C^{k-2}) than the order (k).
- The B-Spline order of the curve is restricted to the number of control vertices.
- Each control vertex influences $\pm\frac{k}{2}$ segments of the polygon in u.
- B-Spline basis functions assume always non-negative values.
- The curve is invariant with respect to an affine transformation, which means that any transformation can be applied to the curve by applying it to the defining control vertices.

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The 2^{nd} and 3^{rd} properties, define the local functional characteristic of the B-Spline. This gives an indication of how an error in a control vertex can be spread into the neighbourhood of the reconstructed data point. Finally, one of the most interesting property is the ability to perform translations, rotations and scalings over the control vertice's domain, with the same accuracy as applying these transformations in the data points domain. Evaluation of the control vertices of a B-Spline curve or two-dimensional signals (surfaces) for interpolation, however, requires the solution of large system of equations [7]. For this reason, we approximate the B-Spline functions by filters and study their behaviour in a perfect reconstruction subband coding system.

2.1 Discrete B-Splines

This functions can be derived using a signal processing interpolation method [8], where a function is interpolated by analytic functions which are a piecewise polynomial of a fixed degree. Based on the normalised k order B-Spline for $n+2$ uniformly spaced knots [9], Unser [10] obtained the discrete B-Splines representation in the z-transform form,

$$B_M^k(z) = \frac{1}{M^k} B_1^k(z) (B_M^0(z))^{k+1}, \qquad (2)$$

where B_1^k is the discrete signal at the B-Spline knots, $B_M^0(z)$ is the rectangular pulse of width M (interpolation order), and k is the B-Spline order. The evaluation of the original digital signal, F_M, using the polygon vertices (or coefficients), Y, is as follows,

$$X_M(z) = B_M^k(z) Y(z^M), \qquad (3)$$

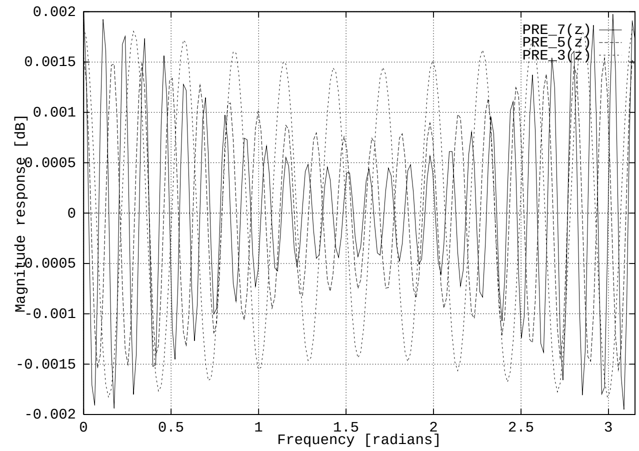
and the coefficients can be obtained by convolving the data points with $(B_M^k(z))^{-1}$.

3 BIORTHOGONAL B-SPLINE FILTER BANKS

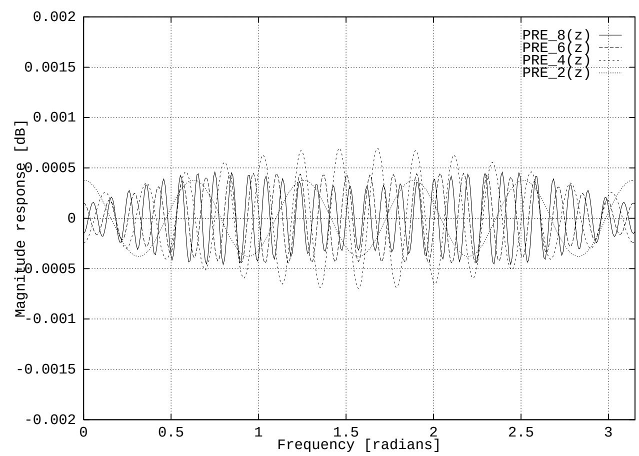
The biorthogonal filter banks of a two-channel (M=2) subband coding system are characterised by *FIR* low pass synthesis filter, $G_0(z)$, and analysis filter, $H_0(z)$. The z-transform of the synthesis filter is given by Eqn. 2. $H_0(z)$ is obtained by approximation, using the inverse B-Spline basis functions [7]. This leads to an infinite-duration impulse response filter. In order to achieve an *FIR* analysis filter, less significant coefficients of the B-Spline filters can be discarded. Since these filters are both symmetric and linear phase, then perfect reconstruction (*PR*) and a zero overall delay can be achieved [11] if the high frequency band filters are $H_1(z) = -z^{2d-1} G_0(-z)$ and $G_1(z) = -z^{2d-1} H_0(-z)$, where $2d-1$ is the resultant delay.

3.1 Error analysis

Due the fast rate of decay of B-Spline filter coefficients the $H_0(z)$ filter can be truncated without introducing an apparent error (PRE_k) in the reconstructed signal. Let's assume that the coefficients of these filters with magnitudes less than 10^{-4} have small influence in the signal reconstruction. The magnitude response error of these filters, with B-Spline orders between 2 and 8, is shown in Figs. 1a and 1b for odd and even orders respectively. Filters with odd B-Spline orders,



(a) Odd order



(b) Even order

Figure 1: Reconstruction error of odd and even order B-Splines.

which correspond to even degree polynomials have larger reconstruction error in the lower and higher frequencies than in the midrange frequencies. On the other hand, even order B-Spline filters show a larger reconstruction error at the midrange frequencies than the lower or higher frequencies. Despite the similar error at midrange frequencies, the odd order B-Splines have a 10 times larger error at lower and higher frequencies than the even order B-Splines. This implies that even order B-Splines concentrate image energy at the lower band, leading to a better image compression. Thus, in our simulations B-Splines of even order are used, namely cubic B-Spline ($k = 4$). The concentration of the energy in the lower band rises by the B-Spline order, but for orders larger than 6, the improvement in energy compaction is not significant.

3.2 Low frequency subband image reconstruction

The general form for the reconstructed signal of a subband coding system is given by,

$$\begin{aligned}\hat{X}(z) &= \frac{1}{2}[H_0(z)G_0(z)X(z) + H_0(-z)G_0(z)X(-z)] \\ &+ H_1(z)G_1(z)X(z) + H_1(-z)G_1(z)X(-z)]. \quad (4)\end{aligned}$$

where $X(z)$ is the input signal. The first and second terms correspond to the low and high frequency branches, 0 and 1 respectively. If only the lower frequency subband image is used for reconstruction of the signal ($G_1(z) = 0$), then in the first term of Eqn. 4 the first term corresponds to the error in the signal and the second term is the aliasing effect. Using the *PR* condition

$$H_0(z)G_0(z) = 2 - H_0(-z)G_0(-z) \quad (5)$$

and substituting in Eqn. 4, we obtain the error for the low frequency band,

$$E\_k(z) = \frac{H_0(-z)}{2}[G_0(-z)X(z) - G_0(z)X(-z)]. \quad (6)$$

Considering $X(z)$ a first order Gauss-Markov process with an autocorrelation coefficient of 0.95, the magnitude response of this error ($E\_k(z)$) is mainly observed at the high frequencies, except for order 2 that also appears at the low frequencies. On the other hand, as

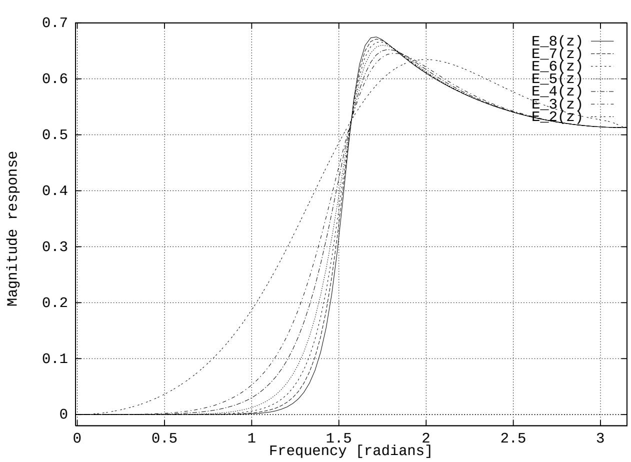


Figure 2: Output error at the low frequency channel.

the B-Spline order increases, the error in the low frequencies is reduced, as can be seen in Fig. 2.

4 EXPERIMENTAL RESULTS

In order to compare the energy compaction of B-Spline with other kernels, *Lena* picture (*512×512* pixels) was reconstructed using only the lowest frequency band under all the filters. The experiments were carried by comparing B-Spline biorthogonal filters with some filters described in the literature as: the *QMF 16TAP(A)*, *near PR*; the biorthogonal *LeGall*, *PR* [5] and the linear phase biorthogonal wavelet *Edu*, *PR* [12]. In order to obtain *PR*, this image is symmetrically extended at the borders and the overall subband system have zero delay.

This comparison is performed in the absence of coding distortion of the low frequency subband image (Si_0). The performance of B-Spline filters banks were assessed, by computing *PSNR* to measure the reconstruction quality of the *Lena* image in different situations like:

- the effect of coefficient's truncation, when using cubic B-spline filters in a biorthogonal filter bank,
- using different orders of B-Splines to decompose an image in a subband coding system,
- reconstruction of images without quantisation of the subimages, showing the magnitude distortion due to the analysis and synthesis process,
- reconstruction of the image without quantisation using only information from the low frequency band. This characterises the capability of the filter in concentrating the image energy in the low frequency subimage.

It is observed that for truncation of filter coefficients with an accuracy of 10^{-3} (No of taps of $H_0(z)$=41) or larger the introduced error is insignificant. Thus, for this accuracy in the coefficient truncation, the reconstruction quality for B-Spline orders between 3 and 8 are measured. This experiment has shown that odd order B-Spline filters have less reconstruction capabilities compared with their even counterparts and, despite of increase of the quality with the increase of the B-Spline order, for $k > 6$ the quality improvement in terms of *PSNR(dB)* is not significant.

In order to give an idea of the B-Spline filter performance in the context of subband image decomposition, these type of filters are compared with the filters described above. This comparison is related to the filter capability to concentrate the energy in the low frequency subimage. Such property is important as this subband coding system might be used for very low bit rate coding, where the system might be forced to use only information from the subimage Si_0 to reconstruct the image.

As can be seen in Fig. 3, using Eqn. 6 and the first order Gauss-Markov process input signal $X(z)$ to represent an image, it is shown that the cubic B-Spline filter introduces less error than the other filters used in the

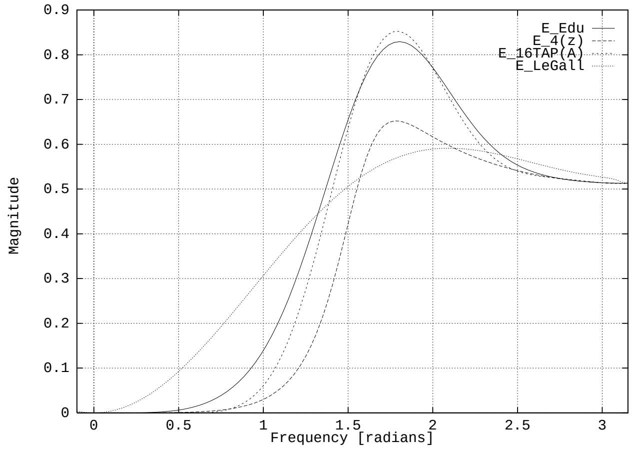


Figure 3: Single band reconstruction error.

comparison. The short tap filter introduces large error at the low frequencies, being penalised in more that 1 dB. As shown in Tab. 1, the remaining filters have an error shape similar to the cubic B-Spline, but magnified in the midrange frequencies, which introduces larger error in the image reconstruction.

Filter type	*No of filter taps*		Si_0
	$H_0(z)$	$G_0(z)$	
Cubic B-Spline	41	9	35.77428
16TAP(A)	16	16	35.52514
LeGall	5	3	34.47334
Edu	11	5	35.45363

Table 1: Comparison between various filters.

5 CONCLUSIONS

B-Spline *FIR* filters were designed for the subband coding applications that achieve near perfect reconstruction. The introduced error from different orders of B-Spline filters were analysed and a relationship between the size of the truncated analysis filter of different B-Spline orders and the introduced error in the reconstruction was established. Furthermore, the possibility of encoding only the lowest frequency subimage is considered, as well as, the capability of the B-Spline filters to concentrate the energy of the picture in this subimage. The results show that B-Spline filter banks are capable of reconstructing images using only information from the lowest frequency subimage, with better quality than other known filters reported in the literature. Hence, these type of filters can be used for very low bit rate video coding, as the B-Spline coefficients are invariant to geometric affine transformations.

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