DIRECTIONAL ANISOTROPIC DIFFUSION

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ABSTRACT

In this paper we present a novel diffusion filter, designed for strongly oriented patterns. Unlike most previous diffusion methods, our filter employs global orientation information, the diffusion process being steered according to this information. Using a scalar diffusivity, as used in the classical Perona Malik filter, we are able to eliminate the inherent corner rounding, commonly observed when tensor or mean curvature driven diffusions are performed. We will demonstrate the performance of our filter on both synthetic and real images, showing applications in noise removal for digital images of ancient engravings.

1 INTRODUCTION

The non-linear classical diffusion processes have been widely used in the last decade in edge preserving denoising. Perona and Malik proposed in [8] the following PDE for the non-linear diffusion filter:

$$\frac{\partial U}{\partial t} = div(c(x,y,t)\nabla U) . \quad (1)$$

The diffusivity *c(x,y,t)* in a pixel of coordinates *x,y* is a non-increasing function of the image gradient ∇U ,such as:

$$c(x,y,t) = g(|\nabla U|) = e^{-(\frac{|\nabla U|}{K})^2} , \quad (2)$$

where K is a gradient threshold. Even though several drawbacks of the classical process were mentioned in the literature [12], [13], practical implementations of the Perona Malik (P-M) filter give impressive results: the noise is eliminated and the edges remain stable for a long period of time. The only observable instability is a staircasing effect for slow varying edges [13]. The P-M filter can also enhance edges, when the local contrast is greater than the threshold *K*.

In [3], Catte et al. have proved the ill-posedness of the P-M process, showing that the noise can also be enhanced introducing strong oscillations. They proposed a spatial regularization, using a smoothed version of the gradient:

$$c_\sigma(x,y,t) = e^{-(\frac{|\nabla(U * G_\sigma)|}{K})^2} , \quad (3)$$

G_σ being a Gaussian of standard deviation σ .

The authors established the existence, uniqueness and regularity of the solution of their improved filter. Noise and structures with size smaller than the size of the gaussian kernel are removed.

Another undesired effect of the P-M filter is the pinhole effect [7]. Typically such an effect occurs when a group of pixels with intermediate gray level is placed near a high contrast zone.

In [1] Alvarez et al. proposed a mean curvature motion approach for the non-linear diffusion filter:

$$\frac{\partial U}{\partial t} = g(|G_\sigma * \nabla U|)[(1-h(|\nabla U|))\Delta U + h(|\nabla U|)|\nabla U| div(\frac{\nabla U}{|\nabla U|})] \quad (4)$$

g(.) is a decreasing function of the smoothed gradient controlling the diffusion, h(.) is a smooth nondecreasing function. For gradient values $|\nabla U|$ inferiors to some threshold value *e* (4) acts like an isotropic diffusion; for values of the gradient $|\nabla U| \geq 2e$ the diffusion is anisotropic, depending on the directional derivative:

$$U_{\xi\xi} = |\nabla U| div(\frac{\nabla U}{|\nabla U|}) . \quad (5)$$

In (5) ξ represents the direction orthogonal to the gradient

A detailed analysis of the classical diffusion methods can be found in [4], [5] and [6]. The authors show that all the classical diffusion filters can be put under the general form:

$$\frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} + c_\eta U_{\eta\eta} . \quad (6)$$

In (6) η represents the gradients direction.

Under this formalism the authors explain the behaviour of the classical diffusion schemes and, in [6], through a SNR criterion, the models are compared in the presence of various types of noise.

Recently, Kornprobst et al. proposed in [5] a combined diffusion-reaction-coupling model the filter uses:

- a diffusion term according to (4),
- a reaction term based on the theory of shock filters developed by Osher and Rudin [10] and Alvarez et Mazorra [2],
- a coupling term that keeps the solution close to the original image.

As the authors noticed both in [1] and [5], when running diffusion for a long time corner smoothing is produced.

A global, coherence based method, was proposed by Weickert in [12]. Starting from a smoothed version of the gradient at a local scale σ, by convolving the components of the gradient at a global scale ρ, a structure tensor is constructed as in [11]. The author builds up a tensor driven diffusion, imposing the way to diffuse along the smallest contrast direction and the orthogonal one, weighting the diffusion according to a coherence measure:

$$\frac{\partial U}{\partial t} = div[D(J_\rho(\nabla U_\sigma))\nabla U] . \quad (7)$$

Such a filter is capable of closing interrupted line-like structures but, at junctions, corner rounding is also observed and the filter can develop false, anisotropic structures.

In this paper we develop a method for filtering oriented patterns like engravings. For this type of images, the patterns are dark line-like structures; the noise consists in dark or clear stains. We want to design a filter capable of removing noise and closing small gaps without corner smoothing. It is easy to show that using only local information we cannot expect such a result. We propose a modified Catte et al. model that includes global orientation information in order to obtain accurate results including a reduction of the pinhole effect. The diffusion process is steered, depending on the global orientation. In Section 2 we present our novel filter, Section 3 gives some experimental results. In the final section we present some conclusions and potential future research directions.

2 DIRECTIONAL DIFFUSION

An example of the images we are dealing with is presented in Fig.2. Such types of images usually contain a large amount of details that have to be preserved. Inspecting Fig.2, one can see that junctions are also present, mean curvature or tensor driven models are not suitable; as mentioned, such methods tend to smooth transitions between patterns.

We present the construction of our filter progressively. The global orientation is independent of the time evolution of the diffusion filter, and it will be presented in subsection 2.1. In subsection 2.2 we give the mathematical model that includes the orientation previously estimated.

2.1 Orientation estimation

We use classical, gradient based, local orientation estimation. The orientation is each pixel is defined to be orthogonal to the gradient. Let U_x and U_y denote the components of the gradient vector. The associated local orientation is defined modulo π as is computed using the inverse tangent [11]:

$$\theta = \arctan\left(\frac{U_y}{U_x}\right) + \frac{\pi}{2} . \quad (8)$$

In order to average the local orientation over larger neighborhoods we use a Principal Component Analysis. Let $V = \{V_i\}_{1 \le i \le n}$ a field of n gradient vectors, the sign of V_i being ignored. The average orientation of the field is obtained as the principal axis of the moment tensor [11]:

$$M = \frac{1}{n}\sum_{i=1}^{n} V_i V_i^T \quad (9)$$

The solution can be viewed in terms of a Principal Component Analysis of the covariance matrix, computed considering the vectors V_i together with the symmetrical vectors $-V_i$-.

The orientation of the eigenvector corresponding to the smallest eigenvalue is the desired mean orientation. The support of the Principal Component Analysis is a rectangular window with an odd number of elements.

2.2 Directional filter

The average orientation information is included in a Catte et al. filter in order to weight the contribution of the horizontal and vertical components of the gradient vector. Let $\theta \in [0, \pi]$ denote the global orientation as computed in 2.1.

We propose the following PDE:

$$\frac{\partial U}{\partial t} = div(c(x, y, t)\nabla U^\theta) . \quad (10)$$

In (12), ∇U^θ represents a modified version of the gradient vector, it`s components simply represent the projections of the components of the original gradient vector onto the global orientation axis:

$$\nabla U^\theta : (U_x^{(\xi)}, U_y^{(\xi)}) . \quad (11)$$

As the global orientation turns, the diffusion has a strong directional tendency. For horizontal profiles, for instance, we can only have horizontal diffusion. As θ approaches $\frac{\pi}{2}$, vertical diffusion becomes more important, and for $\theta = \frac{\pi}{2}$ only vertical diffusion is allowed.

The choice of the diffusivity strongly influences the results. We choose a scalar diffusivity, as in the Catte et al. process:

$$c_\sigma(x, y, t) = e^{-\left(\frac{|\nabla(U * G_\sigma)|}{K}\right)^2} . \quad (12)$$

Unlike in Weickert’s method, the value of the local smoothed gradient weights the diffusion. Such a process will not develop false anisotropic structures and, as in the classical diffusion processes, corner rounding is avoided.

The geometrical interpretation of the proposed diffusion model is presented in Fig. 1.

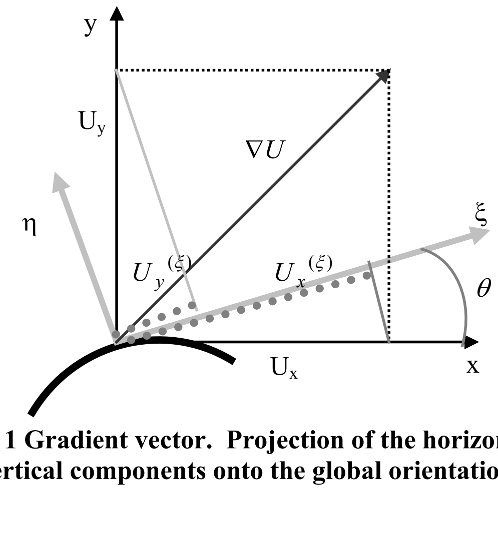

Fig. 1 Gradient vector. Projection of the horizontal and vertical components onto the global orientation axis

Let θ^* be:

$$\theta^* = \begin{cases} \pi - \theta, if \quad \theta > \dfrac{\pi}{2} \\ \theta \quad if \quad \theta <= \dfrac{\pi}{2} \end{cases} . \qquad (13)$$

We can rewrite (13):

$$\nabla U^{\theta} : (U_x \cos\theta^*, U_y \sin\theta^*) . \qquad (14)$$

The discrete filter that corresponds to (11) is based on a 4 neighbour explicit numerical scheme of Perona and Malik:

$$U^{t+1}(x,y) = U^t(x,y) + dt(c_E \nabla_E \cos\theta^* + c_N \nabla_N \sin\theta^* + c_S \nabla_S sin\,\theta^* + c_W \nabla_W cos\,\theta^*) \qquad (15)$$

with:

$$\begin{aligned} \nabla_E U &= U_{i+1,j} - U_{i,j} \\ \nabla_w U &= U_{i-1,j} - U_{i,j} \\ \nabla_N U &= U_{i,j-1} - U_{i,j} \\ \nabla_N I &= U_{i,j+1} - U_{i,j} \end{aligned} \qquad (16)$$

and:

$$\begin{aligned} c_E &= g[\nabla_E (U * G_\sigma)] \\ c_W &= g[\nabla_W (U * G_\sigma)] \\ c_N &= g[\nabla_N (U * G_\sigma)] \\ c_S &= g[\nabla_S (U * G_\sigma)] \end{aligned} \qquad (17)$$

A time step:

$$dt \leq \frac{1}{4} \qquad (18)$$

assures the stability of the discrete scheme.

3 EXPERIMENTAL RESULTS

We first present some comparative results on synthetic images with junctions. All the results were obtained using the discrete schemes originally proposed by the authors.

The original image in Fig.2 is corrupted by white gaussian noise Fig.2 (b).

The results given in Fig.2 are compared after a fixed number of 125 iterations. The time step is dt=0.2.

Given the original image I_1 and the filtered image I_2, we compute the signal to noise ratio (SNR):

$$SNR(I_1 / I_2) = 10 \log_{10} \frac{\sigma^2(I_2)}{\sigma^2(I_1 - I_2)} \qquad (19)$$

The computed SNR are summarized in Table 1. For this type of images in terms of SNR our method outperforms the classical methods.

The curvature-based method of Alvarez et al. and the global Weickert method remove noise but also are producing corner rounding.

Using the proposed model corner smoothing is avoided and compared to a classical Catte et al filter; the staircasing effect is also reduced.

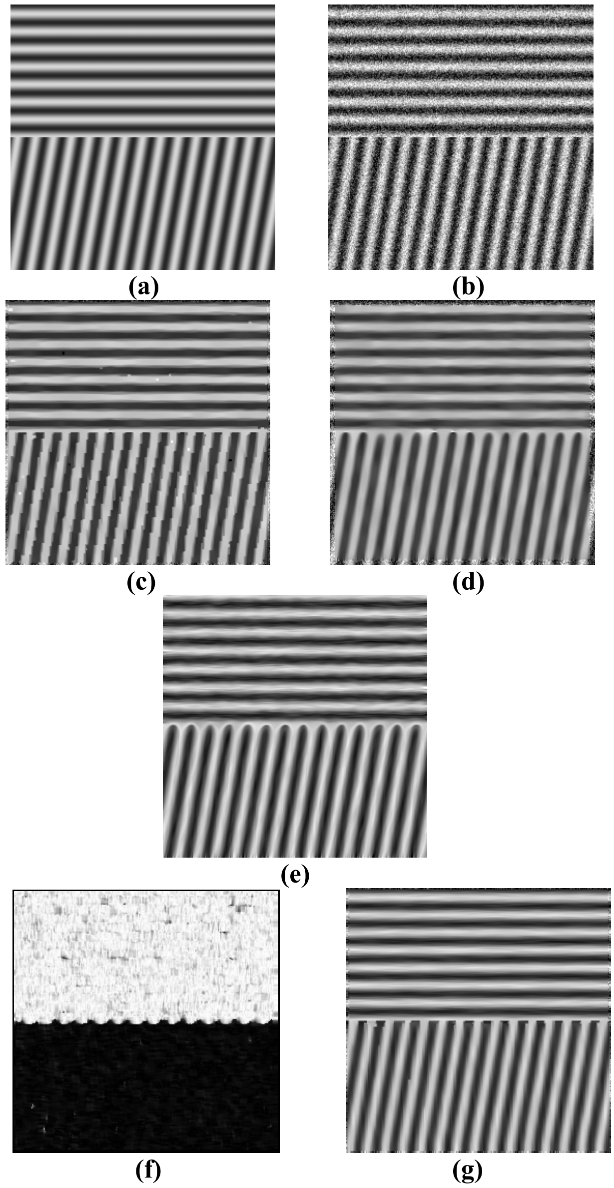

Fig. 2 (a) Noise free image (b) Noisy image- gaussian noise standard deviation 20 SNR=4.97 dB (c) Catte et al. (d) Alvarez et al. (e) Weickert coherence filter (f) Global orientation using a [7x7] Principal Component Analysis, represented using a circular palette of colors (g) Results using the proposed method

Diffusion filter	Parameters	SNR
Catte et al.	σ=0.75, K=5	SNR= 13.59 dB
Alvarez et al.	σ=1.0, e=40	SNR=13.06 dB
Weickert	σ=0.5, ρ=2	SNR=15.01 dB
Proposed method	σ=0.75, K=10	SNR=17.04 dB

Table 1 Computed SNR's for the image in Fig. 3

We present also some comparative results for real ancient engraving pictures in Fig.3 and 4.

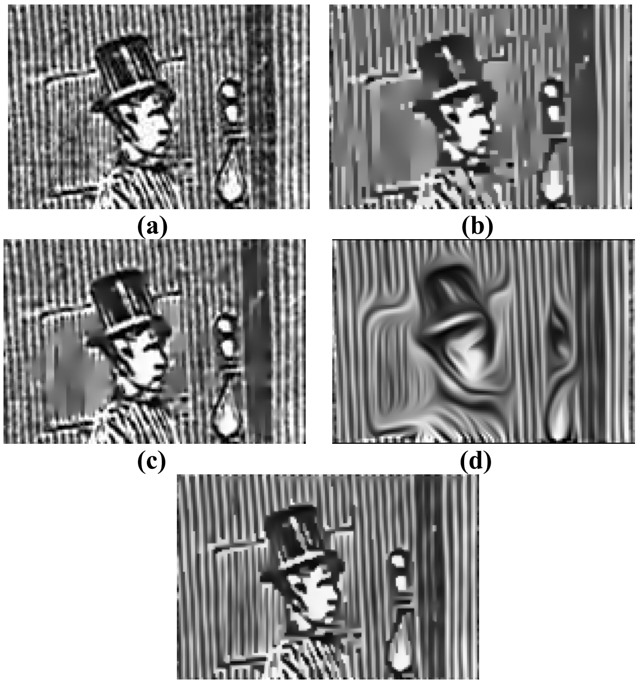

Fig.3 (a) Original image (b) Catte et al. result (c) Alvarez et al. result (d) Weickert result (e) Proposed method result

In Fig.3 and 4 we represented the best results for each type of filter discussed in Section 1. Analysing Fig.3 (b) and Fig. 4 (b) we can observe a typical pinhole effect for classical implementations of the P-M process (or, like here the regularized Catte et al version). Using the global orientation information, the results in Fig.3 (e) and Fig. 4 (f) show that this effect is reduced significantly.

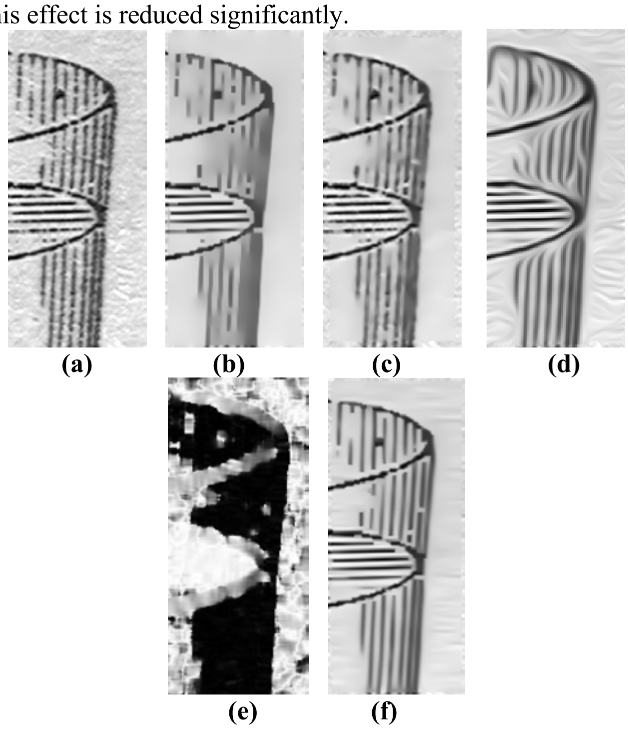

Fig.4 (a) Original image (b) Catte et al method (c) Alvarez et al. method (d) Weickert method (e) Global orientation 7x7 support window (f) Proposed method

4 CONCLUSIONS

The nonlinear diffusion process proposed in this paper provides an efficient tool for oriented textures denoising. The orientation estimation step is separated from the diffusion process. Then, the diffusion process steers according to this information. The only observable instability is a staircasing effect, as presented in the literature for the classical P-M filter. We addressed two undesirable effects of corner smoothing and pinhole effect and we showed through application examples that using the proposed method these effects are greatly reduced.

Future work will be devoted in searching for an adaptive gradient threshold value in each pixel and in proving the existence and the uniqueness of the solution of the associated PDE.

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