EFFECTS OF CHANNELS INTRODUCING TIME-VARIANT DELAYS ON GENERALIZED ALMOST-CYCLOSTATIONARY SIGNALS

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ABSTRACT

In this paper, the effects of linear time-variant channels introducing a time-variant delay are studied with reference to the class of the generalized almost-cyclostationary input signals. Specifically, the Doppler channels existing between a transmitter and a receiver in relative motion is considered in both the cases of constant relative radial speed and constant relative radial acceleration. It is shown that, when excited by an almost cyclostationary signal, in the first case the channel generates an almost-cyclostationary output signal whereas in the second case it generates a generalized almost-cyclostationary output signal.

1 INTRODUCTION

Linear time variant (LTV) channels introducing a time-variant delay on the input signal are of great interest in communication problems since they model the case in which transmitter and receiver are in relative motion [1].

In this paper, the effects of LTV channels introducing time-variant delays are studied with reference to the class of the generalized almost-cyclostationary (GACS) input signals recently introduced in [3]. The GACS signals exhibit multivariate statistical functions that are almost-periodic functions of time whose Fourier series expansions present coefficients and frequencies that can depend on the lag shifts of the signals. GACS signals have been characterized in terms of generalized cyclic statistics in [3]. They include as a special case (when the frequencies do not depend on lag shifts) the almost-cyclostationary (ACS) signals.

The LTV channels are characterized in terms of the system temporal moment function which is the operator that transforms the almost-periodic component contained in the input signal lag product into the almost-periodic component of the output signal lag product [4]. Thus, such a characterization turns out to be particularly useful for GACS signals since their Nth-order lag product can be decomposed into an almost-periodic component, which is called the Nth-order temporal moment function, and a residual term not containing any additive sinewave component.

Two LTV channels introducing a time-variant delay on the input signal are considered here. The former is the Doppler channel existing between a transmitter and a receiver with constant relative radial speed. It is shown that for an ACS input signal the output signal is still ACS but with cyclic statistics exhibiting cycle frequencies and lags scaled with respect to those of the input cyclic statistics. The latter is the Doppler channel existing between a transmitter and a receiver with constant relative radial acceleration. For such a channel, ACS input signals are transformed into GACS output signals, that is, the output cycle frequencies depend on the lag shifts of the signals.

2 BACKGROUND

A continuous-time finite-power possibly complex-valued time-series $x(t)$ is said Nth-order generalized almost-cyclostationary in the wide sense (for the considered conjugation configuration) if its Nth-order lag product

$$L_{\boldsymbol{x}}(\mathbf{1}t+\boldsymbol{\tau}) \triangleq \prod_{n=1}^{N} x^{(*)_n}(t+\tau_n)\,, \qquad (1)$$

where $(*)_n$ denotes the nth optional complex conjugation, $\boldsymbol{\tau} \triangleq [\tau_1, \cdots, \tau_N]^{\mathsf{T}}$ and $\boldsymbol{x} \triangleq [x^{(*)_1}(t), \ldots, x^{(*)_N}(t)]^{\mathsf{T}}$, can be decomposed into an almost-periodic component plus a residual term not containing any additive sinewave. The almost-periodic component, which is referred to as the Nth-order temporal moment function (TMF), can be expressed as

$$\mathcal{R}_{\boldsymbol{x}}(\mathbf{1}t+\boldsymbol{\tau}) = \sum_{\zeta\in W_{\boldsymbol{x}}} \mathcal{R}_{\boldsymbol{x},\zeta}(\boldsymbol{\tau}) e^{j2\pi\alpha_\zeta(\boldsymbol{\tau})t}\,, \qquad (2)$$

where $W_{\boldsymbol{x}}$ is a countable set, the functions $\alpha_\zeta(\boldsymbol{\tau})$ are referred to as (moment) lag-dependent cycle frequencies, and the functions $\mathcal{R}_{\boldsymbol{x},\zeta}(\boldsymbol{\tau})$, which are called the generalized cyclic temporal moment functions (GCTMFs), are given by [3]

$$\mathcal{R}_{\boldsymbol{x},\zeta}(\boldsymbol{\tau}) \triangleq \begin{cases} \left\langle L_{\boldsymbol{x}}(\mathbf{1}t+\boldsymbol{\tau})e^{-j2\pi\alpha_\zeta(\boldsymbol{\tau})t}\right\rangle_t \\ \forall\boldsymbol{\tau} \text{ such that } \alpha_\zeta(\boldsymbol{\tau}) \text{ is defined} \\ 0 \quad \text{elsewhere,} \end{cases} \tag{3}$$

where $\langle\cdot\rangle_t$ denotes infinite averaging with respect to t whose convergence is assumed in the temporal mean-square sense. In the special case where all the lag-dependent cycle frequencies $\alpha_\zeta(\boldsymbol{\tau})$ are constant with respect to $\boldsymbol{\tau}$, the signal $x(t)$ is Nth-order wide-sense ACS and the GCTMFs are coincident with the cyclic temporal moment functions (CTMF) [2].

In [4], it is shown that the output TMF of a LTV system can be expressed as

$$\mathcal{R}_{\boldsymbol{y}}(\mathbf{1}t+\boldsymbol{\tau}) = \int_{\mathbb{R}^N} \mathcal{R}_{\boldsymbol{h}}(\mathbf{1}t+\boldsymbol{\tau},\mathbf{1}t+\boldsymbol{v})\mathcal{R}_{\boldsymbol{x}}(\mathbf{1}t+\boldsymbol{v})\,\mathrm{d}\boldsymbol{v}, \tag{4}$$

where the function

$$\mathcal{R}_{\boldsymbol{h}}(\mathbf{1}t+\boldsymbol{\tau},\boldsymbol{u}) = \sum_{\theta\in\Theta}\int_{\mathbb{R}^N} G_\theta(\boldsymbol{\lambda},\boldsymbol{\tau})e^{j2\pi\varphi_\theta(\boldsymbol{\lambda},\boldsymbol{\tau})t}e^{-j2\pi\boldsymbol{\lambda}^{\mathsf{T}}\boldsymbol{u}}\,\mathrm{d}\boldsymbol{\lambda} \tag{5}$$

is referred to as the system temporal moment function and is the kernel of the linear (with respect to the lag product) operator that transforms the almost-periodic component of the input lag product into the almost-periodic component of the output lag product. In (4) and (5), $\boldsymbol{u} \triangleq [u_1,\ldots,u_N]^{\mathsf{T}}$, $\boldsymbol{v} \triangleq [v_1,\ldots,v_N]^{\mathsf{T}}$, $\boldsymbol{\lambda} \triangleq [\lambda_1,\ldots,\lambda_N]^{\mathsf{T}}$, Θ is a countable set and $G_\theta(\boldsymbol{\lambda},\boldsymbol{\tau})$ and $\varphi_\theta(\boldsymbol{\lambda},\boldsymbol{\tau})$ are the coefficients and the frequencies, respectively, of the additive sinewaves contained in the output lag product when the input lag product is $e^{j2\pi\boldsymbol{\lambda}^{\mathsf{T}}(\mathbf{1}t+\boldsymbol{s})}$.

The above statistical characterization of signals and systems is carried out in the fraction-of-time (FOT) probability framework, where statistical functions are defined through infinite-time averages of a single time-series or products of time- and/or frequency-shifted versions of this time-series, instead of ensemble averages of a stochastic process, as in the classical stochastic process framework [2]. Note that, all the results of this paper can be interpreted in both the stochastic and FOT frameworks, provided that the FOT operator that extracts the almost-periodic component of its argument is interpreted as the stochastic expectation (ensemble average) and the involved stochastic processes are ergodic for the considered generalized cyclic statistical functions. However, the FOT approach turns out to be the only useful when a system is modelled as stochastic and transforms ergodic input processes into nonergodic output processes.

3 TIME-VARIANT DELAY CHANNELS

A channel introducing a time-variant delay is a LTV system with impulse-response function

$$h(t,u) = \delta(u-t+D(t)) , \tag{6}$$

where $\delta(\cdot)$ denotes Dirac's delta function, and $D(t)$ is the time-variant delay. Therefore, the input/output relationship is

$$\begin{aligned} y(t) &\triangleq \int_{\mathbb{R}} h(t,u)\,x(u)\,\mathrm{d}u \\ &= x(t-D(t)) . \end{aligned} \tag{7}$$

3.1 Constant radial speed

Let us consider the Doppler channel existing between a transmitter and a receiver with constant relative speed, that is, a LTV system characterized by the impulse-response function (6) with

$$D(t) = d_0 + d_1 t . \tag{8}$$

Such a system performs a time-scale changing and introduces a delay in the output signal:

$$y(t) = x((1-d_1)t-d_0) . \tag{9}$$

By assuming, for the sake of simplicity, $d_0 = 0$ and by substituting (9) into (2) and (3), it can be shown that the output TMF and CTMFs are given by

$$\mathcal{R}_{\boldsymbol{y}}(\mathbf{1}t+\boldsymbol{\tau}) = \mathcal{R}_{\boldsymbol{x}}(s(\mathbf{1}t+\boldsymbol{\tau})) \tag{10}$$

$$\mathcal{R}^{\alpha}_{\boldsymbol{y}}(\boldsymbol{\tau}) = \mathcal{R}^{\alpha/s}_{\boldsymbol{x}}(s\boldsymbol{\tau}) , \tag{11}$$

where $s \triangleq 1-d_1$. Therefore, the considered time-variant delay modifies the input CTMFs introducing a scale factor into the output lags and cycle frequencies. Moreover, from (11) it follows that if the input signal is ACS, then the output is in turn ACS.

A simulation experiment has been conducted to illustrate the previous result. In this experiment (as well as in that presented in the following section), time has been discretized with sampling increment $T_s = T/M$, where T is the data-record length and M is the number of samples. The parameters of the Doppler channel have been assumed to be $d_0 = 0T_s$ and $d_1 = 0.20$. Moreover, it has been selected, as input signal to the Doppler channel, the double side-band (DSB) signal

$$x(t) = a(t)\cos(2\pi f_0 t) , \tag{12}$$

where $f_0 = 0.1/T_s$ and $a(t)$ is a colored fourth-order wide-sense stationary signal which has been obtained by passing white noise through the filter with transfer function $H_0(f) = (1+jf/B_0)^{-4}$, with $B_0 = 0.01/T_s$. It is well known that $x(t)$ is wide-sense cyclostationary with fourth-order cyclic temporal moment functions nonzero only in correspondence of the cycle frequencies $\alpha = \pm 2f_0$, $\alpha = \pm 4f_0$ (and $\alpha = 0$). Thus, the support in the (α,τ_1) plane of a slice with τ_2 and τ_3 fixed of the reduced-dimension (i.e., $\tau_4 = 0$) CTMF of $x(t)$ is confined in the five lines with equations $\alpha = 0$, $\alpha = \pm 2f_0$, and $\alpha = \pm 4f_0$. The output signal $y(t)$ is in turn DSB with cycle frequencies $\pm 2f_0 s$ and $\pm 4f_0 s$.

Figure 1 shows the support in the (α, τ_1) plane [Fig. 1(a)] and the magnitude [Fig. 1(b)] of the slice for $\tau_1 = \tau_2 = \tau_3$ of the reduced-dimension fourth-order CTMF of the input DSB signal estimated by $M = 2^{14}$ samples. The estimated support and magnitude of the slice of the reduced-dimension CTMF of the output DSB signal $y(t)$ are reported in Fig.s 2(a) and 2(b), respectively. The simulation results are in accordance with the theoretical results predicted by (10) and (11).

3.2 Constant radial acceleration

Let us consider the Doppler channel existing between a transmitter and a receiver with constant relative radial acceleration, that is, characterized by the impulse-response function (6) with

$$D(t) \triangleq d_0 + d_1 t + d_2 t^2 \,, \quad d_2 \neq 0 \,. \tag{13}$$

The almost-periodic component of the output lag product when the input lag product is $e^{j2\pi\boldsymbol{\lambda}^\mathsf{T}(\mathbf{1}t+\boldsymbol{s})}$ is given by

$$\begin{aligned} &\mathrm{E}^{\{\alpha\}}\left\{L_{\boldsymbol{y};\boldsymbol{\lambda}}(\mathbf{1}t+\boldsymbol{\tau})\right\} \\ &= e^{j2\pi\boldsymbol{\lambda}^{(-)\mathsf{T}}(\mathbf{1}t+\boldsymbol{\tau})}\,\mathrm{E}^{\{\alpha\}}\left\{e^{-j2\pi\boldsymbol{\lambda}^\mathsf{T}\boldsymbol{D}^{(-)}(\mathbf{1}t+\boldsymbol{\tau})}\right\} \,, \end{aligned} \tag{14}$$

where $\boldsymbol{D}^{(-)}(\mathbf{1}t+\boldsymbol{\tau}) \triangleq [(-)_1 D(t+\tau_1), \ldots, (-)_N D(t+\tau_N)]^\mathsf{T}$. Moreover, when $D(t)$ is given by (13), it results that

$$\begin{aligned} &\mathrm{E}^{\{\alpha\}}\left\{e^{-j2\pi\boldsymbol{\lambda}^\mathsf{T}\boldsymbol{D}^{(-)}(\mathbf{1}t+\boldsymbol{\tau})}\right\} \\ &= e^{-j2\pi\boldsymbol{\lambda}^\mathsf{T}[d_0\mathbf{1}^{(-)}+d_1\boldsymbol{\tau}^{(-)}+d_2\boldsymbol{\tau}^{(-)(2)}]} \\ &\quad e^{-j2\pi\boldsymbol{\lambda}^\mathsf{T}[d_1\mathbf{1}^{(-)}+2d_2\boldsymbol{\tau}^{(-)}]t}\,\delta_{\boldsymbol{\lambda}^\mathsf{T}\mathbf{1}^{(-)}d_2} \,, \end{aligned} \tag{15}$$

where $\mathbf{1}^{(-)} \triangleq [(-)_1 1, \ldots, (-)_N 1]^\mathsf{T}$, $\boldsymbol{\tau}^{(-)} \triangleq [(-)_1\tau_1, \ldots, (-)_N\tau_N]^\mathsf{T}$, and $\boldsymbol{\tau}^{(-)(2)} \triangleq [(-)_1\tau_1^2, \ldots, (-)_N\tau_N^2]^\mathsf{T}$.

From (14) and (15), it follows that for the LTV system with impulse-response function (6), (13), the set Θ contains only one element and

$$G_\theta(\boldsymbol{\lambda}, \boldsymbol{\tau}) = e^{j2\pi\boldsymbol{\lambda}^{(-)\mathsf{T}}[(1-d_1)\boldsymbol{\tau}-d_2\boldsymbol{\tau}^{(2)}]}\,\delta_{\boldsymbol{\lambda}^{(-)\mathsf{T}}\mathbf{1}} \tag{16}$$

$$\varphi_\theta(\boldsymbol{\lambda}, \boldsymbol{\tau}) = -2d_2\boldsymbol{\lambda}^{(-)\mathsf{T}}\boldsymbol{\tau} \,, \quad \text{for } \boldsymbol{\lambda}^{(-)\mathsf{T}}\mathbf{1} = 0 \,. \tag{17}$$

Thus, accounting for (5), it results that

$$\mathcal{R}_{\boldsymbol{h}}(\mathbf{1}t+\boldsymbol{\tau}, \boldsymbol{u}) = \int_{\mathbb{R}^N} e^{-j2\pi\boldsymbol{\lambda}^\mathsf{T}[\boldsymbol{u}-\boldsymbol{\Phi}^{(-)}(t,\boldsymbol{\tau})]}\,\delta_{\boldsymbol{\lambda}^{(-)\mathsf{T}}\mathbf{1}}\,\mathrm{d}\boldsymbol{\lambda} \,, \tag{18}$$

where

$$\begin{aligned} \boldsymbol{\Phi}^{(-)}(t,\boldsymbol{\tau}) &\triangleq [(-)_1\Phi(t,\tau_1), \ldots, (-)_N\Phi(t,\tau_N)]^\mathsf{T} \\ &= (1-d_1)\boldsymbol{\tau}^{(-)} - d_2\boldsymbol{\tau}^{(-)(2)} - 2d_2\boldsymbol{\tau}^{(-)}t \,. \end{aligned} \tag{19}$$

By substituting the expression of the system temporal moment function (18) into the input/output relationship (4), one obtains that

$$\mathcal{R}_{\boldsymbol{y}}(\mathbf{1}t+\boldsymbol{\tau}) = \int_{\mathbb{R}^N} e^{j2\pi\boldsymbol{\lambda}^\mathsf{T}\boldsymbol{\Phi}^{(-)}(t,\boldsymbol{\tau})}\delta_{\boldsymbol{\lambda}^{(-)\mathsf{T}}\mathbf{1}}\mathcal{S}_{\boldsymbol{x}}(\boldsymbol{\lambda})\,\mathrm{d}\boldsymbol{\lambda}, \tag{20}$$

where $\mathcal{S}_{\boldsymbol{x}}(\boldsymbol{\lambda})$, referred to as the spectral moment function [3], is the Fourier transform, in the sense of the distributions (generalized functions), of the TMF $\mathcal{R}_{\boldsymbol{x}}(\boldsymbol{\tau})$. The presence of the Kronecker delta in the integrand function implies that the output TMF can contain additive sinewave components only if the input spectral moment function $\mathcal{S}_{\boldsymbol{x}}(\boldsymbol{\lambda})$ contains impulsive terms, that is, if the input signal contains an ACS component. In particular, if the input signal is ACS, it results that [2]

$$\mathcal{S}_{\boldsymbol{x}}(\boldsymbol{\lambda}) = \sum_{\alpha \in A_{\boldsymbol{x}}} S_{\boldsymbol{x}}^{\alpha}(\boldsymbol{\lambda}')\,\delta(\alpha - \boldsymbol{\lambda}^\mathsf{T}\mathbf{1}) \,, \tag{21}$$

where prime denotes the operator that transforms a vector $\boldsymbol{u} = [u_1, \ldots, u_K]^\mathsf{T}$ into $\boldsymbol{u}' = [u_1, \ldots, u_{K-1}]^\mathsf{T}$ and $S_{\boldsymbol{x}}^{\alpha}(\boldsymbol{\lambda}')$ is the reduced-dimension cyclic spectral moment function, which, in general, is impulsive [2].

If all the optional conjugations are absent, from (21) it follows that

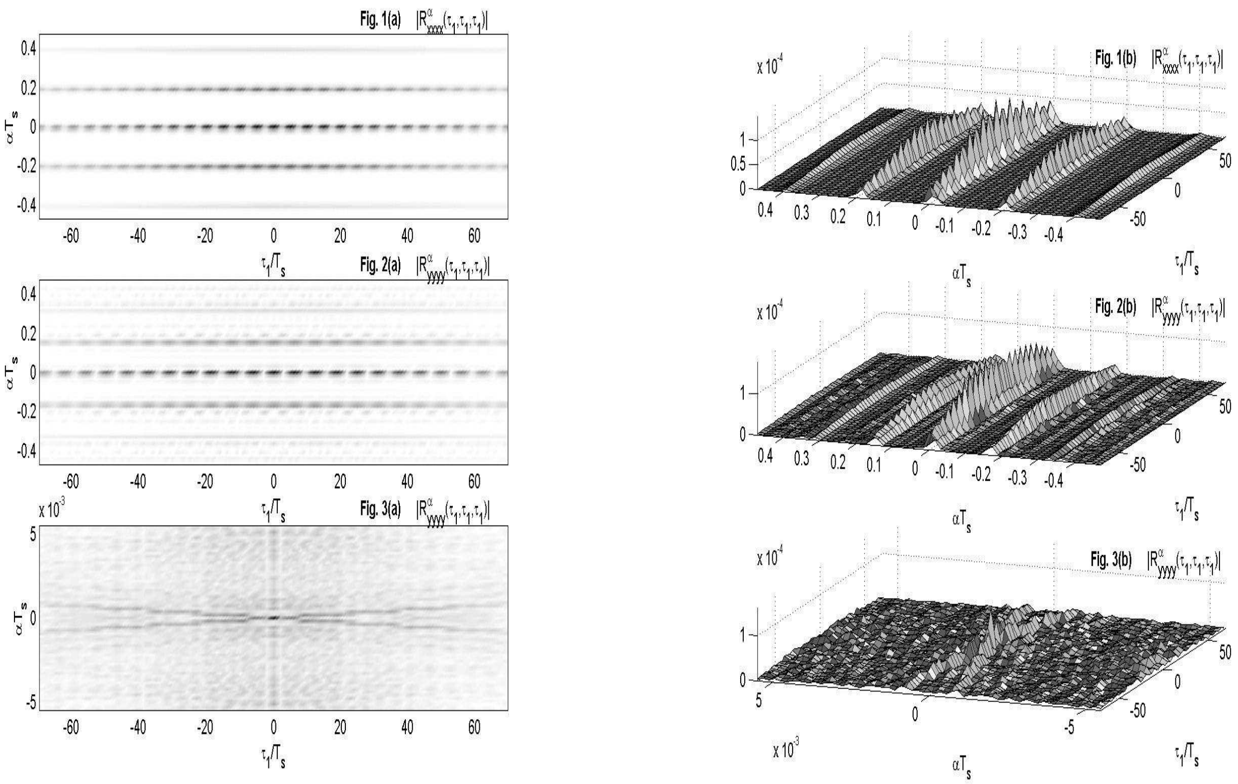
$$\mathcal{S}_{\boldsymbol{x}}(\boldsymbol{\lambda})\,\delta_{\boldsymbol{\lambda}^\mathsf{T}\mathbf{1}} = S_{\boldsymbol{x}}^{0}(\boldsymbol{\lambda}')\delta(\boldsymbol{\lambda}^\mathsf{T}\mathbf{1}) \,. \tag{22}$$

Thus, by substituting (22) into (20) and accounting for the relationship between CTMFs and cyclic temporal cumulant functions [2], one obtains that

$$\begin{aligned} &\mathcal{R}_{\boldsymbol{y}}(\mathbf{1}t+\boldsymbol{\tau}) \\ &= C_{\boldsymbol{x}}^{0}(\boldsymbol{\Phi}'(t,\boldsymbol{\tau}') - \mathbf{1}\Phi(t,\tau_N)) + \sum_{\substack{\mathsf{P} \\ p\neq 1}} \\ &\quad \left[\sum_{\boldsymbol{\beta}^\mathsf{T}\mathbf{1}=0} C_{\boldsymbol{x}_{\mu_p}}^{\beta_{\mu_p}}(\boldsymbol{\Phi}'_{\mu_p}(t,\boldsymbol{\tau}'_{\mu_p}) - \mathbf{1}\Phi(t,\tau_N)) \right. \\ &\quad \prod_{i=1}^{p-1} e^{j2\pi\beta_{\mu_i}[\Phi(t,\tau_{r_i})-\Phi(t,\tau_N)]} \\ &\quad \left. C_{\boldsymbol{x}_{\mu_i}}^{\beta_{\mu_i}}(\boldsymbol{\Phi}'_{\mu_i}(t,\boldsymbol{\tau}'_{\mu_i}) - \mathbf{1}\Phi(t,\tau_{r_i}))\right] \,. \end{aligned} \tag{23}$$

In (23), $C_{\boldsymbol{x}}^{\beta}(\boldsymbol{\tau}')$ is the reduced-dimension cyclic temporal cumulant function [2], P is the set of distinct partitions of $\{1, \ldots, N\}$, each constituted by the subsets $\{\mu_i \,:\, i = 1, \ldots, p\}$, $|\mu_i|$ is the number of elements in μ_i, and $\boldsymbol{x}_{\mu_i}$ is the $|\mu_i|$-dimensional vector whose components are those of $\boldsymbol{x}$ having indices in μ_i. Furthermore, it is assumed, without loss of generality, that each partition is ordered such that μ_p always contains N as its last element and r_i indicates the last element of each subset μ_i $(i = 1, \ldots, N-1)$.

Finally, by substituting the expression of $\Phi(t,\tau)$ given by (19) into (23) with $\tau_N = 0$ and taking the coefficient of the additive sinewave component at frequency α, one obtains the expression for the Nth-order reduced-dimension CTMF $R_{\boldsymbol{y}}^{\alpha}(\boldsymbol{\tau}')$. For example, in the case of a zero mean signal $x(t)$, that is, a signal not containing any additive sinewave component and, moreover, such that $x(t)$ and $x(t+\tau)$ are asymptotically $(|\tau| \to \infty)$



independent, for $N = 4$, one has

$$
\begin{aligned}
&R^{\alpha}_{yyyy}(\tau_1, \tau_2, \tau_3) \\
&\quad = C^{0}_{xxxx}(0,0,0)\, \delta_{\tau_1}\, \delta_{\tau_2}\, \delta_{\tau_3}\, \delta_{\alpha} \\
&\qquad + \sum_{\boldsymbol{\beta}^{\mathsf{T}}\mathbf{1}=0} R^{\beta_2}_{xx}(0) R^{\beta_1}_{xx}(0) \\
&\qquad \Big[\delta_{\tau_3}\, \delta_{\tau_1-\tau_2}\, e^{j2\pi\beta_1[(1-d_1)\tau_2 - d_2\tau_2^2]}\, \delta_{\alpha+\beta_1 2 d_2 \tau_2} \\
&\qquad + \delta_{\tau_2}\, \delta_{\tau_1-\tau_3}\, e^{j2\pi\beta_1[(1-d_1)\tau_3 - d_2\tau_3^2]}\, \delta_{\alpha+\beta_1 2 d_2 \tau_3} \\
&\qquad + \delta_{\tau_1}\, \delta_{\tau_2-\tau_3}\, e^{j2\pi\beta_1[(1-d_1)\tau_3 - d_2\tau_3^2]}\, \delta_{\alpha+\beta_1 2 d_2 \tau_3} \Big] . \quad (24)
\end{aligned}
$$

In the derivation of (24), the fact that the equality in (23) is in the temporal mean-square sense and the fact that, for an asymptotically independent time-series, $C^{\beta}_{\boldsymbol{x}}(\boldsymbol{\tau}')$ is infinitesimal as $\|\boldsymbol{\tau}'\| \to \infty$, have been accounted for. Note that, in such a case, the GCTMFs are discontinuous functions of τ_1, τ_2, and τ_3.

In order to corroborate the effectiveness of the previously presented theoretical results, a simulation experiment has been realized. In this experiment, the parameters of the Doppler channel have been assumed to be $d_0 = 1772T_s$, $d_1 = 0.28$, and $d_2 = 6.104 \cdot 10^{-5}/T_s$. Moreover, it has been selected, as input signal $x(t)$ to the Doppler channel, the same DSB signal used in the previous experiment.

Figure 3 shows a zoom within the neighborhoods of $\alpha = 0$ of the support in the (α, τ_1) plane [Fig. 3(a)] and the magnitude [Fig. 3(b)] of the slice for $\tau_1 = \tau_2 = \tau_3$ of the fourth-order reduced-dimension CTMF of the output signal $y(t)$ estimated by $M = 2^{14}$ samples. The generalized cyclostationary nature of $y(t)$ is evident.

Finally, it is worthwhile to note the different behavior of the two Doppler channels with $D(t)$ linear (see (8)) or quadratic (see (13)) in the presence of an ACS input signal. Specifically, the former generates an ACS signal, whereas the latter generates a GACS signal.

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