SPLINE PULSE SHAPING WITH ISI-FREE MATCHED FILTER RECEIVER

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ABSTRACT

Although the raised cosine pulse shaping filter is a well-established standard in digital communications, in many practical systems simpler approaches are useful. In this paper a new family of pulse shaping filters with zero intersymbol interference after matched filtering is proposed. The method, based on the spline interpolation of the symbol data, exploits two properties of splines: that the B-spline coefficients can be efficiently obtained via digital filtering, and that the B-spline kernels of order n can be constructed from the convolution of $n + 1$ rectangular pulses. These two facts suggest how to decompose the filtering operations needed to obtain any spline interpolation of odd-order into two matched filters. The proposed architecture allows an efficient implementation, produces optimal performance and shows better spectral characteristics than other suboptimal approaches.

1 INTRODUCTION

In digital communications it is important to select signal shapes that produce zero intersymbol interference (ISI) after the matched filtering stage when the channel is bandlimited. In his seminal paper, Nyquist set down the necessary conditions to achieve ISI-free transmission [1]. Among them, the first Nyquist criterion, which states that the equivalent impulse response of the transmitting and receiving filters should have zero crossings at multiples of the symbol period T, is the most used in practice. Also due to Nyquist, the vestigial symmetry theorem allows the design of realizable filters that satisfy the first Nyquist criterion, including the raised cosine filter, that is used in many practical systems when ISI is a relevant problem. Despite the success of the raised cosine pulse shape, still some research is conducted in the design of filters that satisfy the Nyquist-I constraint, searching for different objectives such as maximizing the energy in a certain time interval [2], or having ISI-free properties with or without matched filtering [3, 4]. In some cases, however, due to the computational cost associated to this optimal raised cosine filtering, practical applications rely on much simpler shaping filters with worse spectral characteristics, mainly rectangular waves or cosine functions.

Although not commonly stated, the first Nyquist criterion is equivalent to interpolating the communication symbols using a basis function with the interpolatory property [5], i.e., with zero crossings at the sampling points. Following this idea, in this paper we propose to interpolate the discrete sequence of symbols using splines. In this way, we construct an ISI-free signal when sampled at the symbol rate. Although any reconstruction technique from the communications symbols that exactly goes through the symbols is a zero ISI communications signal, splines have a number of advantages that make them an interesting choice when spectral content as well as complexity are of concern. The main advantage is that any spline interpolation model of odd-order may be computed by filtering the sequence of symbols with two matched filters, so it is quite simple to design an optimal receiver. Besides this advantage, spline interpolation achieves better spectral characteristics than rectangular pulses with a moderate increase in complexity.

The rest of the paper is organized as follows. In Section 2 we review some basic notions of spline interpolation. In Section 3 we show how to use splines as ISI-free shaping filters. Section 4 is concerned with implementation aspects of the proposed spline shaping technique. Section 5 shows some simulation results to illustrate the advantages of spline conformation. The paper ends with the conclusions presented in Section 6.

2 SPLINE INTERPOLATION

In this section we review the fundamental properties of spline interpolation; a more detailed review is carried out in [6, 7, 8]. A B-spline of order n, $\beta^n(t)$, is a symmetrical, bell-shaped function that can be constructed from the convolution of $n + 1$ identical rectangular pulses

$$\beta^0(t) = \begin{cases} 1, & |t| < 1/2; \\ 0, & |t| \geq 1/2. \end{cases}$$

Spline interpolation of a given input data signal $s[k]$ consists in determining a set of coefficients $c[k]$ so that the continuous time signal

$$s(t) = \sum_{k \in \mathbb{Z}} c[k]\beta^n\left(\frac{t}{T} - k\right) \tag{1}$$

goes exactly through the data points; i.e.,

$$s[k] = s(kT), \qquad k \in \mathbb{Z}. \tag{2}$$

One of the advantages of the B-spline expansion (1) is that the coefficients $c[k]$ can be easily obtained by digital filtering techniques. To clarify this idea, it is convenient to introduce the discrete B-spline kernel $b^n[k]$, which is obtained by sampling the B-spline of degree n at integer instants

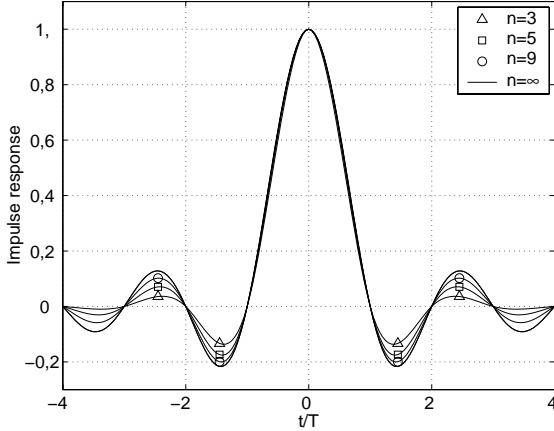$$b^n[k] = \beta^n(k) = \beta^n(t)|_{t=k \in \mathbb{Z}}.$$

Figure 1: Impulse response of the pulse shaping filter $\varphi^n(t)$ for $n = 3$, 5, 9 and ∞.

Using the discrete B-spline kernel, (2) may be expressed as the convolution

$$s[k] = \sum_{m \in \mathbb{Z}} c[m] b^n[k-m] = c[k] * b^n[k].$$

According to this equation, $s[k]$ is obtained by filtering $c[k]$ with $b^n[k]$. Hence, to obtain $c[k]$ from $s[k]$ it is enough to apply the filter $d^n[k]$ that verifies $b^n[k] * d^n[k] = \delta[k]$. The filter $d^n[k]$, known as the direct filter, is all-pole and has symmetry properties that permit to implement it as a cascade of a causal, $d^{n+}[k]$, and an anticausal, $d^{n-}[k]$, recursive filters. As we show in the next section, this property is crucial to obtain a ISI-free matched filter using splines.

The overall interpolation process may be expressed as a convolution using the discrete sequence $s[k]$ —the communication symbols— as coefficients

$$s(t) = \sum_{k \in \mathbb{Z}} s[k] \eta^n \left(\frac{t}{T} - k \right),$$

where $\eta^n(t)$ is the cardinal spline of degree n, which is obtained as the convolution of the direct filter and the B-spline kernel

$$\eta^n(t) = \sum_{k \in \mathbb{Z}} d^n[k] \beta^n(t-k).$$

If, for notational convenience, we define our zero ISI pulse-shaping filter as

$$\varphi_T^n(t) = \eta^n \left(\frac{t}{T} \right), \tag{3}$$

the communications signal can be rewritten as

$$s(t) = \sum_{k \in \mathbb{Z}} s[k] \varphi_T^n(t - kT).$$

Fig. 1 shows the impulse response of the pulse shaping filters $\varphi_T^n(t)$ of degree 3, 5, 9 and ∞. In particular, for $n = \infty$, $\varphi_T^n(t)$ is the ideal Nyquist filter. It is clear from the zero crossings at the multiples of T that the proposed spline interpolation technique produces ISI-free signals.

3 CUBIC SPLINE FILTER

In this section we discuss how to split the zero ISI cardinal spline (3) between the transmitter and the receiver in order to match the transmitting and receiving filters and achieve a practical implementation. Even if we restrict our exposition to cubic spline interpolation, all the results may be extended to any odd-order spline.

The direct B-spline filter for the cubic spline is

$$d^3[k] = \frac{1 - z_1}{1 + z_1} z_1^{|k|},$$

where $z_1 = -2 + \sqrt{3}$ is the filter pole [7]. It is easy to show that the direct cubic B-spline filter can be factored into two filters, one causal ($d^{3+}[k]$) and one anticausal ($d^{3-}[k]$), given by

$$d^{3\pm}[k] = (1 - z_1) z_1^{\pm k} u[\pm k]. \tag{4}$$

Since $d^{3+}[k] = d^{3-}[-k]$, their Fourier transforms are conjugated and hence both filters are matched. As a pair of matched filters we propose a transmission filter $h_T(t)$, composed by the cascade connection of $d^{3+}[k]$ and $\beta^1(t/T)$,

$$h_T(t) = \sum_{k \in \mathbb{Z}} d^{3+}[k] \beta^1 \left(\frac{t}{T} - k \right),$$

and a receiving filter $h_R(t)$, composed by the cascade of $\beta^1(t/T)$ and $d^{3-}[k]$

$$h_R(t) = h_T(-t) = \sum_{k \in \mathbb{Z}} d^{3-}[k] \beta^1 \left(-\frac{t}{T} + k \right).$$

We will denote $h_R(t)$ and $h_T(t)$ as square root cubic spline (SQRCS) filters.

Since the frequency response of the SQRCS transmitting filter is

$$H_T(\omega) = (1 - z_1) \frac{\mathrm{sinc}^2(\omega T/2\pi)}{1 - z_1 e^{-j\omega T}},$$

if we assume that the symbol sequence $s[k]$ is white with variance σ_S^2, then, the power spectral density (PSD) of the transmitted signal $s_T(t)$ will be

$$S_T(\omega) = \frac{\sigma_S^2}{T} |H_T(\omega)|^2 = \frac{3\sigma_S^2}{T} \frac{\mathrm{sinc}^4(\omega T/2\pi)}{2 + \cos(\omega T)}.$$

4 IMPLEMENTATION ASPECTS

In this section we briefly discuss some implementation aspects of the proposed spline pulse shaping technique. We focus on the cubic spline conformation, but the results are easily extended to any odd-order spline.

In principle, the major drawback of the proposed approach is the implementation of the IIR (infinite impulse response) digital filters $d^{3\pm}[k]$. The causal filter $d^{3+}[k]$ may be implemented using just one multiplication and one addition for each input symbol

$$s_{d+}[k] = s[k] + z_1 s_{d+}[k-1].$$

The anticausal filter, however, must be truncated. Moreover, to keep a matched transmission system, the causal filter should also be truncated. In this way, both filters $d^{3+}[k]$ and $d^{3-}[k]$ are implemented as finite impulse response (FIR) filters. The truncation procedure introduces some degradation; however, $d^{3+}[k]$ and $d^{3-}[k]$ are both fast falling filters, and we have found by simulation that with just three coefficients almost optimal performance is obtained (see Fig. 2). Note
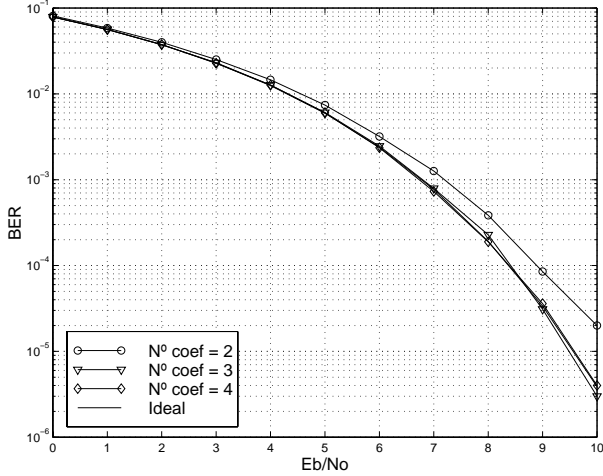
Figure 2: BER degradation due to the truncation of the transmitting and receiving digital filters.

that, when the input symbol sequence is binary, no multiplications are necessary, only additions and subtractions. Additionally, the discrete-time FIR transmission filtering may be implemented in the form of a look-up table.

In the proposed system the transmitter and receiver filters are the cascade of a triangular shaped filter with $d^{3-}[k]$ and $d^{3+}[k]$, respectively. In a practical setting the transmitter can be implemented using a differentiator and an integrator based on operational amplifiers, as Fig. 3 shows. In this case, the transmitter must perform digitally the differentiation using a two-tap filter, which can be combined with $d^{3-}[k]$ and implemented through a look-up table. Note, finally, that in this implementation the D/A converter works at the symbol rate and it does not require an analog reconstruction filter, as opposed to traditional schemes.
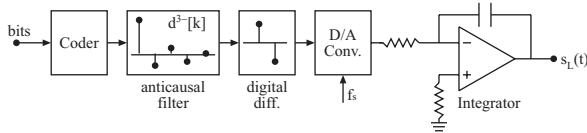

Figure 3: Transmitter implementation.

In a digital implementation, the receiver will sample the incoming signal at a multiple of the symbol rate. The matched filter in this case will be a sampled triangular filter. After this filter the signal may be decimated and then the anticausal filter may be applied at the symbol rate. This implementation has a computational cost lower than other approaches where infinite length matched filters are truncated and implemented through FIR filters. Besides, this receiver needs a less restrictive antialiasing filter, and has lower ISI than, for example, the NRZ, because its sidelobes are lower.

5   NUMERICAL RESULTS

From a practical point of view, the main contribution of this paper is to show how to implement an ISI-free triangle-shaped communication system. This offers a new alterna-
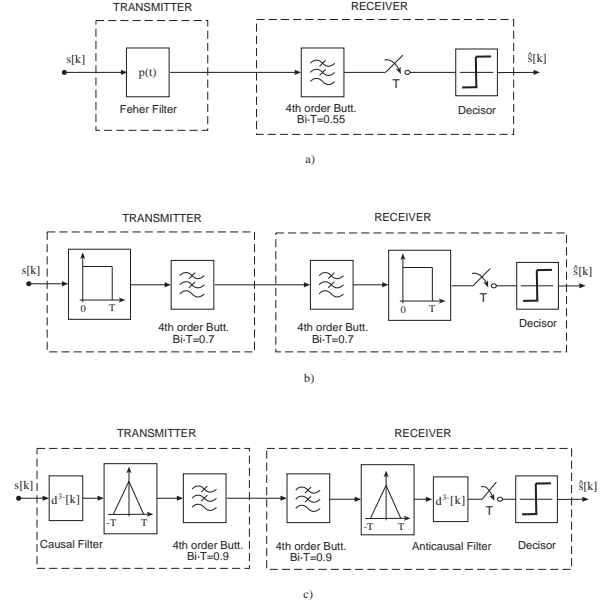

Figure 4: The three simulated systems: a) F-QPSK, b) NRZ with 4th-order Butterworth, c) SQRCS.

tive to implement communication systems with better performance than conventional NRZ based systems and with a lower computational cost than square-root raised cosine filters. As a numerical proof of this improved performance, we have simulated the three communication links shown in Fig. 4.

The three systems are designed to comply with a similar spectral mask. Specifically, the first system is based on the following shaping filter

$$p(t) = \begin{cases} 0.5[1 + \cos(\pi t/T)], & \text{if } |t| \leq T, \\ 0, & \text{otherwise.} \end{cases}$$

that is used in the F-PQSK modulation proposed in [9]. The lowpass reception filter is a 4th-order Butterworth filter with a 3dB bandwidth of $0.55Rb$, with Rb denoting the binary data rate. As discussed in [9], this filter, although not matched, provides a good BER performance in a Gaussian noise channel. The second system is a NRZ pulse-shaping filter followed by a 4th-order Butterworth lowpass filter with a 3dB bandwidth of $0.7Rb$. The same lowpass filter followed by the NRZ pulse shaping is used in reception to match both the transmitter and receiver. Finally, the third system is our proposed cubic spline pulse shaping, using also two 4th-order Butterworth lowpass filters in transmission and reception, in this case of bandwidth $0.9Rb$. The three power spectral densities in the channel are shown in Fig. 5, where it can be seen that the three structures attenuate more than 30 dB the first sidelobe, and that our proposed cubic spline shaping has the fastest decay in the lateral lobes.

The bit error rates for the three systems in an ideal channel with additive white Gaussian noise (AWGN) are shown in Fig. 6. It can be seen that no degradation from optimal performance is observed for the proposed spline shaping, while the first system (denoted as Feher filter) has a degradation
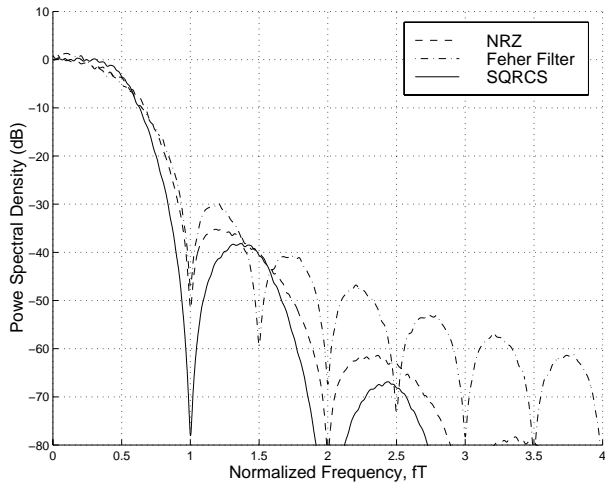
Figure 5: Transmitted power spectral density for the three systems: NRZ, Feher filter, and the proposed SQRCS shaping.
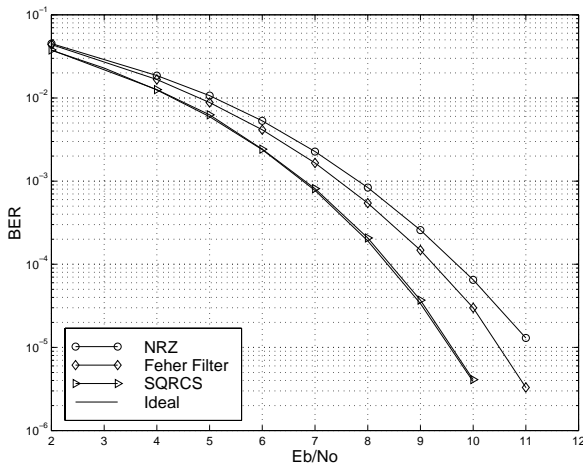

Figure 6: BER for the three communication systems: NRZ, Feher filter, and the proposed SQRCS shaping.

of approximately 1 dB for a bit error rate of 10^{-5}, and for the NRZ shaping the degradation is about 1.5 dB.

6   CONCLUSIONS

In this paper we have proposed a new family of pulse shaping filters, which produce zero ISI after matched filtering. The proposed pulse-shaping technique produces at the receiver a spline interpolation of odd-order of the symbol sequence. The fact that the spline interpolation may be implemented as a digital filtering operation, which can be expressed as the convolution of two matched filters, means that optimal performance in an AWGN channel can be achieved. In the particular case of cubic spline interpolation, the communications system is composed of two digital FIR filters operating at the symbol rate, and two triangle-shaped filters. The resulting system may be considered an improved alternative to the NRZ conformation with a small increase in computational cost.

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