PERFORMANCE ANALYSIS OF A NEW ADAPTIVE ALGORITHM FOR BLIND MULTIUSER DETECTION

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ABSTRACT

In multiuser detection, the MMSE(minimum mean square error) detector is known to be effective for suppressing co-channel multiple access interference. This method needs training symbols, but recently blind adaptive methods have been proposed. In this paper we propose an adaptive algorithm for blind multiuser detection that uses a new cost function, and report theoretical and simulation results about its performance. It is seen that our new algorithm gives better performance than the existing algorithm.

1 INTRODUCTION

In a code-division multiple-access(CDMA) system, it is well known that the main source of performance degradation is the multiple-access interference(MAI). To suppress MAI, several methods have been proposed in the field of multiuser detection. If the training sequences are available, the MMSE detectors is effective for suppressing MAI. Honig et al.[1] have shown that under a certain linear constraint on the weights of the detector the solution of the MMSE criterion is equivalent to that of the MOE(minimum output error) criterion which does not require the training sequences. Then a constrained blind adaptive LMS algorithm is proposed and its performance analysis is presented by using a standard method in LMS adaptive filter theory. As another method, a linearly constrained CM(constant modulus) type algorithm for multiuser detection is proposed in [2]. However, this algorithm requires the value of the signal power of the target user and is not perfectly blind. Also, there is no performance analysis in [2].

In this paper we propose a new blind CM type algorithm with simultaneous estimation of the required signal power. Then using the general theory for performance analysis of adaptive algorithms based on the ODE(ordinary differential equation) method in [3], a theoretical expression of the signal-to-interference ratio(SIR) is derived and is compared with that of the MOE method. It is seen from the theoretical and simulation results that our new method gives better results in terms of the SIR and the convergence speed.

2 BLIND ADAPTIVE ALGORITHMS

Here, for simplicity we use a synchronous single-path time-invariant channel for K-user CDMA system. The base-band expression of the received signal $r(t)$ is

$$r(t) = \sum_{k=1}^{K} A_k b_k(t) s_k(t) + \sigma n(t) \tag{1}$$

where A_k,$b_k(t)$,$s_k(t)$ are the amplitude, the information bit, the signature waveform of user k, respectively and $n(t)$ is a white Gaussian noise with unit power independent with $b_k(t)$. Discretizing (1) in a chip rate which is n times faster than a symbol rate, we have

$$\boldsymbol{r}[i] = \sum_{k=1}^{K} A_k b_k[i] \boldsymbol{s}_k + \sigma \boldsymbol{n}[i] \tag{2}$$

where $\boldsymbol{r}[i]$,$\boldsymbol{s}_k$,$\boldsymbol{n}[i]$ are n dimentional vectors corresponding to $r(t)$,$s_k(t)$,$n(t)$, respectively. The i.i.d. information bit $b_k[i]$ takes ± 1 with equal probabilities and is independent each other ($k = 1, \cdots, K$). Also, we assume that $\|\boldsymbol{s}_k\| = 1$. The output of a linear detector is given by

$$\hat{b}_1[i] = \mathrm{sgn}(\boldsymbol{c}^T \boldsymbol{r}[i]) \tag{3}$$

where the weight vector $\boldsymbol{c}$ is adjusted to reduce the bit error rate.

The MOE(minimum output error) adaptive algorithm in [1] is obtained by minimizing the criterion

$$\mathrm{MOE}(\boldsymbol{c}) = E[(\boldsymbol{c}^T \boldsymbol{r})^2] \tag{4}$$

with the constraint

$$\boldsymbol{c}^T \boldsymbol{s}_1 = 1 \tag{5}$$

where we assume that user 1 is of interest. The algorithm is written as

$$\boldsymbol{c}[i+1] = \boldsymbol{c}[i] - \mu(\boldsymbol{c}^T[i]\boldsymbol{r}[i])(I - \boldsymbol{s}_1 \boldsymbol{s}_1^T)\boldsymbol{r}[i] \tag{6}$$

with μ a small positive step size.

Our new algorithm is based on the following criterion

$$F(P, \boldsymbol{c}) = E[\{(\boldsymbol{c}^T \boldsymbol{r})^2 - P\}^2] \tag{7}$$

where P is introduced in order to estimate the known signal power A_1^2 of user 1. The CM type algorithm in [2] assumes that A_1^2 is known and is not truely blind. Minimizing (7) with respect to P and $\boldsymbol{c}$, we obtain the following algorithm

$$P[i+1] = P[i] + \mu\{(\boldsymbol{c}^T[i]\boldsymbol{r}[i])^2 - P[i]\} \tag{8}$$

$$\boldsymbol{c}[i+1] = \boldsymbol{c}[i] - \mu \cdot 2\{(\boldsymbol{c}^T[i]\boldsymbol{r}[i])^2 - P[i]\}(\boldsymbol{c}^T[i]\boldsymbol{r}[i]) \cdot (\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{r}[i]. \tag{9}$$

Note that (9) satisfies the constraint (5).

3 ODE METHOD AND SIR

One of the most powerful methods for analyzing adaptive algorithms is the ODE(ordinary differential equation) method in [3]. This method has been successfully applied to the performance analysis of an algorithm for multiple minor components extraction in [4]. A brief summary of the method is as follows. Consider a general adaptive algorithm

$$\boldsymbol{c}[i+1] = \boldsymbol{c}[i] + \mu \boldsymbol{h}(\boldsymbol{c}[i], \boldsymbol{r}[i]) \tag{10}$$

where $\boldsymbol{c}[i]$ is a tap weight vector and $\boldsymbol{r}[i]$ is the stationary input signal vector. The function $\tilde{h}(\tilde{\boldsymbol{c}})$ is defined by

$$\tilde{\boldsymbol{h}}(\tilde{\boldsymbol{c}}) = E[\boldsymbol{h}(\tilde{\boldsymbol{c}}, \boldsymbol{r}[i])]. \tag{11}$$

Then , the ODE is given by

$$\frac{d\bar{\boldsymbol{c}}}{dt} = \tilde{\boldsymbol{h}}(\bar{\boldsymbol{c}}). \tag{12}$$

An equilibrium point of the algorithm is determined from

$$\tilde{\boldsymbol{h}}(\boldsymbol{c}_*) = \boldsymbol{0} \tag{13}$$

and we put the derivative matrix at this point as

$$\boldsymbol{H}(\boldsymbol{c}_*) = \left.\frac{\partial \tilde{\boldsymbol{h}}(\boldsymbol{c})}{\partial \boldsymbol{c}}\right|_{\boldsymbol{c}=\boldsymbol{c}_*}. \tag{14}$$

Also, we assume that this matrix is stable and the following quantity exists.

$$\boldsymbol{S}(\boldsymbol{c}) = \sum_{i=-\infty}^{\infty} E[\boldsymbol{h}(\boldsymbol{c}, \boldsymbol{r}[i])\boldsymbol{h}^T(\boldsymbol{c}, \boldsymbol{r}[0])]. \tag{15}$$

Under some regularity conditions, the covariance matrix of the estimation error $\boldsymbol{c}[i] - \boldsymbol{c}_*$ is asymptotically given by

$$E[(\boldsymbol{c}[i] - \boldsymbol{c}_*)(\boldsymbol{c}[i] - \boldsymbol{c}_*)^T] \simeq \mu \boldsymbol{Y} \tag{16}$$

where $\boldsymbol{Y}$ is a solution of the Lyapunov equation

$$\boldsymbol{H}(\boldsymbol{c}_*)\boldsymbol{Y} + \boldsymbol{Y}\boldsymbol{H}^T(\boldsymbol{c}_*) = -\boldsymbol{S}(\boldsymbol{c}_*). \tag{17}$$

As a performance measure, we use the SIR defined by

$$\mathrm{SIR} = \lim_{i\to\infty} \frac{\left(E\left[\boldsymbol{c}^T[i]\boldsymbol{r}[i]|b_1[i]\right]\right)^2}{\mathrm{Var}(\boldsymbol{c}^T[i]\boldsymbol{r}[i]|b_1[i])}. \tag{18}$$

Obviously from (4) and (5), the optimal weight vector is given by

$$\boldsymbol{c}_* = \frac{\boldsymbol{R}^{-1}\boldsymbol{s}_1}{\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1} \tag{19}$$

with the covariance matrix

$$\boldsymbol{R} = E[\boldsymbol{r}\boldsymbol{r}^T] = \sum_{k=1}^{K} A_k^2 \boldsymbol{s}_k\boldsymbol{s}_k^T + \sigma^2\boldsymbol{I}. \tag{20}$$

Assuming $\boldsymbol{c}[i] \to \boldsymbol{c}_*(i \to \infty, \mu \to 0)$, noting that $E[\boldsymbol{c}^T[i]\boldsymbol{r}[i]|b_1[i]] = A_1 b_1[i]$ and

$$\mathrm{Var}(\boldsymbol{c}_*^T\boldsymbol{r}[i]|b_1[i]) = \boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_* - A_1^2 \tag{21}$$

we have the asymptotic expression of the SIR as

$$\mathrm{SIR} = \frac{A_1^2}{\frac{1}{\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1} - A_1^2 + \mu\mathrm{tr}[\boldsymbol{R}\boldsymbol{Y}]} \tag{22}$$

where we use the fact that $\mathrm{Var}(\boldsymbol{c}^T[i]\boldsymbol{r}[i]|b_1[i]) = \mathrm{Var}(\boldsymbol{c}_*^T\boldsymbol{r}[i]|b_1[i]) + \mathrm{tr}\{\boldsymbol{R}\cdot E[(\boldsymbol{c}[i]-\boldsymbol{c}_*)(\boldsymbol{c}[i]-\boldsymbol{c}_*)^T]\}$.

4 ANALYSIS BY THE ODE METHOD

Using the standard LMS adaptive filter theory, the performance analysis of the MOE algorithm has been done in [1]. Here we show that the same result is obtained by the ODE method. For the MOE algorithm we have

$$\boldsymbol{h}(\boldsymbol{c}[i], \boldsymbol{r}[i]) = -\boldsymbol{c}^T[i]\boldsymbol{r}[i](\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{r}[i] \tag{23}$$

so that

$$\tilde{\boldsymbol{h}}(\boldsymbol{c}) = -(\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}\boldsymbol{c}. \tag{24}$$

From (24) the equilibrium point satisfies $\boldsymbol{R}\boldsymbol{c}_* = (\boldsymbol{s}_1^T\boldsymbol{R}\boldsymbol{c}_1)\boldsymbol{s}_1$ and using (5) it is given by (19). Also we have

$$\boldsymbol{H} = \frac{\partial\tilde{\boldsymbol{h}}(\boldsymbol{c})}{\partial \boldsymbol{c}} = -(\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R} \tag{25}$$

$$\begin{aligned}\boldsymbol{S}(\boldsymbol{c}) &= E[h(\boldsymbol{c}, \boldsymbol{r}[0])h^T(\boldsymbol{c}, \boldsymbol{r}[0])] \\ &= (\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)E[(\boldsymbol{c}^T\boldsymbol{r})^2\boldsymbol{r}\boldsymbol{r}^T](\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T).\end{aligned} \tag{26}$$

To calculate (26), we need to evaluate the 4th order moment of $\boldsymbol{r}$. But $\boldsymbol{r}$ is not Gaussian so that (26) will be a complicated expression.

Here we consider a rather ideal situation where the signature vector $\boldsymbol{s}_1$ is orthogonal to other signature vectors, that is,

$$\boldsymbol{s}_1^T\boldsymbol{s}_k = 0 \; (k = 2, \cdots, K). \tag{27}$$

In this case, $\boldsymbol{s}_1$ is an eigenvector of $\boldsymbol{R}$ and at the equilibrium point, the output of the detector is

$$\boldsymbol{c}_*^T\boldsymbol{r}[i] = A_1 b_1[i] + \sigma\boldsymbol{c}_*^T\boldsymbol{n}[i]. \tag{28}$$

Hence,

$$\begin{aligned}\boldsymbol{S}(\boldsymbol{c}\ ) &= (\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)(\boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_*\boldsymbol{R}+2A_1^2\sigma^2\boldsymbol{s}_1\boldsymbol{c}_*^T\\ &\quad +2A_1^2\sigma^2\boldsymbol{c}_*\boldsymbol{s}_1^T+2\sigma^4\boldsymbol{c}_*\boldsymbol{c}_*^T)(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\\ &= (\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\frac{1}{\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1}\boldsymbol{R}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\end{aligned} \tag{29}$$

and the Lyapunov equation is

$$\begin{aligned}&-(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}\boldsymbol{Y}-\boldsymbol{Y}\boldsymbol{R}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\\ &=-\frac{1}{\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T).\end{aligned} \tag{30}$$

The solution is given by

$$\boldsymbol{Y}=\frac{1}{2\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T). \tag{31}$$

Substituting (31) into (22), we have the theoretical expression of the SIR of the MOE algorithm. This expression coincides with that in [1].

Similarly, we analyze our algorithm (8) and (9) by the ODE method. Let $\boldsymbol{h}(P,\boldsymbol{c},\boldsymbol{r})=(h_1(P,\boldsymbol{r})\ \boldsymbol{h}_2^T(\boldsymbol{c},\boldsymbol{r}))^T$ with

$$h_1(P,\boldsymbol{r})=(\boldsymbol{c}^T\boldsymbol{r})^2-P \tag{32}$$

$$\boldsymbol{h}_2(\boldsymbol{c},\boldsymbol{r})=-2\{(\boldsymbol{c}^T\boldsymbol{r})^2-P\}(\boldsymbol{c}^T\boldsymbol{r})(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{r}. \tag{33}$$

Then,

$$\tilde{h}_1(P,\boldsymbol{c})=\boldsymbol{c}^T\boldsymbol{R}\boldsymbol{c}-P. \tag{34}$$

To obtain $\tilde{\boldsymbol{h}}_2(P,\boldsymbol{c})$, we evaluate the 4th order moment of $\boldsymbol{r}$. After some calculations we have

$$\begin{aligned}\tilde{\boldsymbol{h}}_2(P,\boldsymbol{c})&=(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\{-2(3\boldsymbol{c}^T\boldsymbol{R}\boldsymbol{c}-P)\boldsymbol{R}\boldsymbol{c}\\ &\quad -2\sum_{i=1}^{K}A_i^4(\boldsymbol{c}^T\boldsymbol{s}_i)^3\boldsymbol{s}_i\}.\end{aligned} \tag{35}$$

It is difficult to find an equilibrium point of (35) so that we again consider the ideal situation in (27). It is readily seen that $\boldsymbol{c}_*$ in (19) and $P_*=\boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_*$ are the equilibrium point of $\boldsymbol{c}$ and P in (34) and (35), respectively. Hence the derivative matrix is given by

$$\begin{aligned}\boldsymbol{H} &= \begin{pmatrix}\left.\frac{\partial\tilde{h}_1}{\partial P}\right|_{P=P_*} & \left.\frac{\partial\tilde{h}_1}{\partial\boldsymbol{c}}\right|_{\boldsymbol{c}=\boldsymbol{c}_*}\\ \left.\frac{\partial\tilde{\boldsymbol{h}}_2}{\partial P}\right|_{P=P_*} & \left.\frac{\partial\tilde{\boldsymbol{h}}_2}{\partial\boldsymbol{c}}\right|_{\boldsymbol{c}=\boldsymbol{c}_*}\end{pmatrix}\\ &= \begin{pmatrix}-1 & 2\boldsymbol{c}_*^T\boldsymbol{R}\\ \boldsymbol{0} & -4\boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_*(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}\end{pmatrix}.\end{aligned} \tag{36}$$

Also,

$$\boldsymbol{S} = \begin{pmatrix}E[h_1^2] & E[h_1\boldsymbol{h}_2^T]\\ E[\boldsymbol{h}_2h_1] & E[\boldsymbol{h}_2\boldsymbol{h}_2^T]\end{pmatrix} \tag{37}$$

$$\begin{aligned}\boldsymbol{S}_{22} &= E[\boldsymbol{h}_2(\boldsymbol{c},\boldsymbol{r})\boldsymbol{h}_2(\boldsymbol{c},\boldsymbol{r})^T]\\ &= 4(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)E[\{(\boldsymbol{c}^T\boldsymbol{r})^2-P\}^2\\ &\quad\cdot(\boldsymbol{c}^T\boldsymbol{r})^2\boldsymbol{r}\boldsymbol{r}^T](\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T).\end{aligned} \tag{38}$$

To calculate (38), we need to evaluate the 8th order moment of $\boldsymbol{r}$. Under the ideal situation (27) and the assumption that the SNR of user 1 A_1^2/σ^2 is sufficiently large, from (28) we finally have

$$\boldsymbol{S}_{22}\approx 16A_1^4\sigma^2\boldsymbol{c}_*^T\boldsymbol{c}_*(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T). \tag{39}$$

Let the solution $\tilde{\boldsymbol{Y}}$ of the Lyapunov equation (17) with $\boldsymbol{H}$ in (36) and $\boldsymbol{S}$ in (37) be

$$\tilde{\boldsymbol{Y}}=\begin{pmatrix}Y_{11} & \boldsymbol{Y}_{12}\\ \boldsymbol{Y}_{21} & \boldsymbol{Y}_{22}\end{pmatrix}. \tag{40}$$

Then using (36) $\boldsymbol{Y}_{22}$ is decoupled from others as

$$\boldsymbol{H}_{22}\boldsymbol{Y}_{22}+\boldsymbol{Y}_{22}\boldsymbol{H}_{22}^T=-\boldsymbol{S}_{22} \tag{41}$$

where $\boldsymbol{H}_{22}=-4\boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_*(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)\boldsymbol{R}$. Here we have

$$\boldsymbol{Y}_{22}=\frac{2A_1^4\sigma^2\boldsymbol{c}_*^T\boldsymbol{c}_*(\boldsymbol{I}-\boldsymbol{s}_1\boldsymbol{s}_1^T)}{\boldsymbol{c}_*^T\boldsymbol{R}\boldsymbol{c}_*}. \tag{42}$$

Substituting this into $\boldsymbol{Y}$ in (22), we obtain the theoretical expression of the SIR of our algorithm.

5 SIMULATION RESULTS

To see the validity of the theoretical formulas, some simulation results are presented. We set $N=31$, $K=6$, $A_1=1$, $A_2\sim A_5=\sqrt{10}$, $A_6=10$, $\sigma=0.1$ (SNR of user 1: 20dB) and $\boldsymbol{s}_1$ is taken as an M sequence and $\boldsymbol{s}_2\sim\boldsymbol{s}_6$ are generated randomly. The initial values are $\boldsymbol{c}[0]=\boldsymbol{s}_1$, $P[0]=0$. The step size parameter μ is taken as $\mu=0.0002$, 0.0001 and 0.00005. The corresponding step size for $P[i]$ is set as 32μ. Smaller values do not give good results. The upper bound of the SIR is given by

$$\mathrm{SIR}_\infty=\frac{A_1^2}{\frac{1}{\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1}-A_1^2}. \tag{43}$$

In this case, $\mathrm{SIR}_\infty=19.2791$ dB. The simulation results of the SIR are obtained by taking averages over 200 data sets.

In Table 1 the theoretical and the simulation results of the SIR at $i=20,000$ are presented for the above three values of μ. The agreements are good and our new algorithm gives much better SIR than the MOE algorithm.

Fig.1 shows the convergence characteristics of the SIR for the first 1000 iterations with the same $\mu=0.0002$. It is seen that our new algorithm converges much faster than the MOE algorithm. If we take much larger μ for the latter, it converges much faster but the resulting SIR becomes much lower. Also, our algorithm becomes unstable for $\mu\geq 0.0004$. Fig.2 shows the convergence behavior of $(\boldsymbol{c}^T[i]\boldsymbol{r}[i])^2$ and $P[i]$. The difference between these two quantities is large at the early stage of the adaptation and becomes small as $\boldsymbol{c}[i]$ converges to $\boldsymbol{c}_*$. But from (9) this difference appears in the right hand side as a time varying step size. This is one of the reasons why our algorithm converges faster.

	μ	0.0002	0.0001	0.00005
MOE	Simulation(dB)	15.9949	17.2658	18.1400
	theory(dB)	15.8779	17.2547	18.1494
NEW	Simulation(dB)	19.0581	19.2058	19.2375
	theory(dB)	19.0474	19.1617	19.2200

Table 1: Simulational and theoretical values of SIR.

6 CONCLUSION

In this paper we have proposed a new adaptive algorithm for blind multiuser detection and presented its performance analysis based on the ODE method. It has been shown that our algorithm gives better SIR and converges faster than the existing algorithm. It is a future work to give a more explicit convergence condition on the step size parameters.

References

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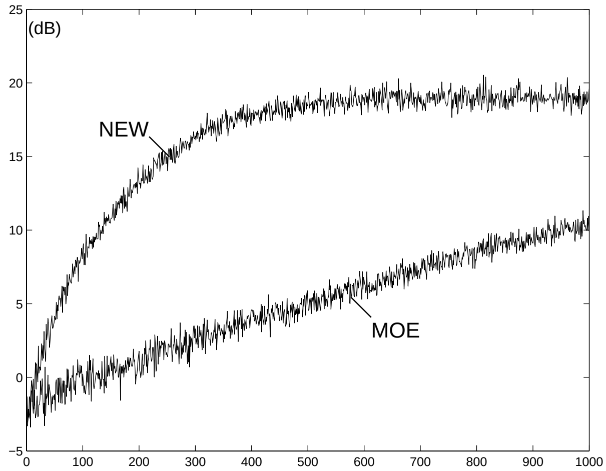

Figure 1: Convergence behavior of the SIR of MOE algorithm and our algorithm(NEW).

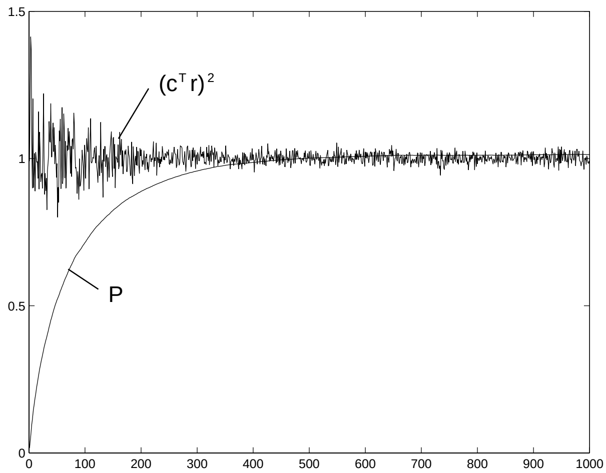

Figure 2: Convergence behavior of $(\boldsymbol{c}^T[i]\boldsymbol{r}[i])^2$ and $P[i]$.