# SIGNAL DETECTION AND CLASSIFICATION USING ATOMIC DECOMPOSITION

Gustavo López-Risueño, Jesús Grajal, Omar A. Yeste-Ojeda Dept. Señales, Sistemas y Radiocomunicaciones, ETSIT, Universidad Politécnica de Madrid, Ciudad Universitaria s/n, 28040 Madrid, Spain e-mail: risueno@gmr.ssr.upm.es

## ABSTRACT

A signal detection and classification scheme based on Atomic Decomposition (AD) is presented that allows the detection and modulation analysis of simultaneous signals with good performance at low SNR. An AD-based detector is used to detect the components of the signal acting as the feature extractor and its performance is compared to Fourier-based detectors. The classification stage consists of signal separation by clustering, reconstruction of the signal instantaneous frequency using the previous AD and modulation analysis. The system is applied to the identification of radar signals.

# **1** INTRODUCTION

Surveillance and management of the radio spectrum, radar emitter identification, or electronic warfare are applications where the goal is to extract, catalogue, and use information about signals observed in the environment. In these applications, an automatic signal classifier is often required. A critical step in automatic signal classification is the Modulation Classifier (MC), which decides the modulation type out of a given menu of possibilities based on estimated aspects of the analyzed waveform, such as instantaneous amplitude, phase and frequency [1].

In this paper, it is described a MC based on Atomic Decomposition (AD) that follows a decision theoretic approach. AD [2], also known as Matching Pursuit or Adaptive Gabor Representation [3], is an adaptive approximation technique, where the signal under analysis is expanded onto a redundant dictionary of signals called atoms. Detection is performed immediately after extraction, so that only atoms from the signal (and not from the noise) are accepted in the expansion. Thus, denoising of the signal is carried out achieving noise reduction in its instantaneous phase similarly to [4]. This allows modulation identification at lower signalto-noise ratios (SNR) than other reported approaches. AD acts as the feature extractor of the classifier.

As this MC is intended for radar applications, the modulations considered are linear frequency, phase-coded, and intrapulse frequency-hopping modulation, which comprise the main categories of modern radar transmitters [5].

Next section is devoted to AD and the associated detection scheme. The classification process is described in section 3 and simulation results are shown in section 4. Finally, the conclusions are drawn in section 5.

# 2 DETECTION BY ATOMIC DECOMPOSI-TION

AD generally provides a sparse and physically meaningful representation of a wide range of different signals [2]. Let  $D = \{h_{\gamma}(n)\}$  be the dictionary of atoms, and s(n) the signal under analysis, AD is defined as follows:

$$s(n) \approx \sum_{p} b_{p} e^{j\phi_{p}} h_{\gamma_{p}}(n) , \qquad (1)$$

where

$$\gamma_p = \arg\max_{\gamma} |\langle s_{p-1}(n), h_{\gamma}(n) \rangle|^2 \quad , \tag{2}$$

and

$$b_p \ e^{j\phi_p} = \left\langle s_{p-1}(n), h_{\gamma_p}(n) \right\rangle \ . \tag{3}$$

 $s_p(n)$  is defined as

$$s_{p}(n) = s_{p-1}(n) - b_{p} e^{j\phi_{p}} h_{\gamma_{p}}(n), \quad p > 0 , \quad (4)$$
  
$$s_{0}(n) = s(n) . \quad (5)$$

Signal detection based on AD was firstly treated in [6], where a dictionary of chirplets, i.e. chirped Gabor functions of unit energy, was used (they have good time-frequency resolution and linear frequency modulation, which is a common feature of radar signals). This detector is based on the extracted-atom-energy-to-estimated-noise ratio and is intended for extracting the components of a signal in white Gaussian noise when neither the noise power nor the characteristics and number of the components are known.

As shown in [6], the main drawback of this algorithm is the performance loss in the detection of long-duration signals. To overcome this limitation, a dictionary of chirped complex exponentials has been added in this paper. It is a two-parameter family (frequency and chirp rate) of flat-envelope signals with unit energy. The coordination of both the complex exponential and the chirplet dictionaries is shown in Fig. 1. More details about implementation, and detection improvement using complex exponentials (greater than 10 dB depending on the signal) are shown in [7].

To reduce computational cost, the long-duration signal components are firstly extracted using the complex exponential dictionary. After the extraction of a complex exponential atom, it is checked whether it comes from the noise or the signal (this noise detector is similar to that used for chirplets in



Figure 1: AD flowchart with two dictionaries and Greediness detector.

[6] with a different threshold). If the threshold is surpassed, it is checked if the atom is greedy. Greedy means that the atom does not correspond to a true signal component, i.e. it is a bad estimate of the signal. Greedy atoms are due to the greedy character of AD [2, 8] and appear when the signal consists of several components close in the time-frequency plane. In the case of the dictionary of complex exponentials, a greedy long-duration atom can be extracted when the signal is composed of several short-duration components close in time. Therefore, its greediness must be checked. This is performed by the Greediness detector described in section 2.1.

If the atom is not greedy, a descendent method on both dictionaries is used to refine its parameters. If the atom is greedy, a chirplet atom is extracted and the detection test of [6] is applied. After a chirplet extraction, another greediness test is carried out using the same Greediness detector. If the chirplet atom is greedy, a shorter-duration chirplet is extracted by a descendent technique in a smaller search space. This procedure carries on until the extracted atom is either not greedy or shorter than a given value.

Once no more complex exponential atoms can be extracted, the extraction of chirplet atoms is performed. The procedure was already explained when the detection of greedy complex exponential atoms was addressed. As for the termination condition, it has been set to 4 successive chirplet atoms coming from noise instead of only one. This is due to the probabilistic behavior of the genetic algorithm used in the extraction of chirplet atoms [6]. Another termination condition is to have extracted more than 90% of the energy.

### 2.1 Greediness detector

The Greediness detector was already presented in [7], where the reader is referred to for further information, and its foundation is similar to the similarity measure used in High Resolution Pursuit (HRP) [8]. For the *p*-th extracted atom,  $h_{\gamma_p}$ , passing the noise detector, it is defined a set of *sub-atoms*,  $\{h_{\gamma_{pi}}\}_i$ , with the same frequency law, finer scale and time support inside the time support of  $h_{\gamma_p}$ .<sup>1</sup> The Greediness detector is summarized as

$$L_{GREED}(h_{\gamma_p}) = \max_{i} \{ |X_{pi}| \} \sum_{\substack{\text{Nongreedy}}}^{Greedy} Th , \qquad (6)$$

where

$$X_{pi} = \frac{Q_{\gamma_{pi}} - \left| \langle s_{p-1}(n), h_{\gamma_p}(n) \rangle \right|}{\mu \cdot \left| \langle s_{p-1}(n), h_{\gamma_p}(n) \rangle \right| + \widehat{\sigma}_{Q_{\gamma_{pi}}}} .$$
(7)

The quotient  $Q_{\gamma_{pi}}$  comes from the similarity measure of HRP:

$$Q_{\gamma_{pi}} = \frac{\left| \langle s_{p-1}(n), h_{\gamma_{pi}(n)} \rangle \right|}{\left| \langle h_{\gamma_{p}}(n), h_{\gamma_{pi}(n)} \rangle \right|} , \qquad (8)$$

$$\widehat{\sigma}_{Q_{\gamma_{pi}}} = \frac{\sqrt{||s_p(n)||^2/N}}{\sqrt{2} \cdot \left| \langle h_{\gamma_p}(n), h_{\gamma_{pi}(n)} \rangle \right|} , \qquad (9)$$

where N is the number of samples and  $||s_p(n)||^2$  is the energy of signal  $s_p(n)$ .

In this paper, the threshold Th of eq. (6) and the constant  $\mu$  of eq. (7) have been set to 3 and  $5 \cdot 10^{-8}$  to achieve low error probability.<sup>2</sup>

# 3 CLASSIFICATION PROCEDURE

After detecting the signal components by AD, the analyzed signal is characterized by a set of atoms with a particular time of arrival, duration, frequency, chirp rate, energy and phase. The classification stage analyzes these features to identify the modulation of the different signals that can be in the received frame. The classification is split into three successive steps: 1) clustering, 2) instantaneous frequency estimation, and 3) modulation analysis. The clustering step performs the separation of the different signals allowing simultaneous-signal identification. It groups those atoms belonging to the same signal. As a clustering technique, it has been used an agglomerative hierarchical strategy consisting of grouping in a cluster those atoms whose distance is below a certain threshold. This distance or, more appropriately, clustering similarity measure has been obtained empirically and depends on the atom features. Details about the mathematical expression of them have not been included due to limitations in space.

The estimate of the instantaneous frequency (IF) is based on the finite difference of the unwrapped phase of the reconstructed signal. The reconstruction is obtained as the expansion of the atoms belonging to the same cluster. The modulation analysis is the strict-sense MC and is carried out on the IF estimate. As pointed out in section 1, three different classes are distinguished: linear frequency modulation (LFM), PSK and FSK. The LFM class comprises continuous wave and pulsed radars without or with linear frequency modulation. The PSK class refers to signals with digital phase modulation, such as PSK signals in communications or *Barker*-coded pulses in radar. FSK means those signals with digital frequency modulation, e.g. FSK signals in communications or intrapulse frequency-hopping radars.

#### 3.1 Modulation Analysis

After the separation of the simultaneous signals in the received frame and the estimation of the IF for each one of these signals, the modulation analysis is carried out in two

<sup>&</sup>lt;sup>1</sup>The details of the construction of the set of *sub-atoms* are shown in [7].

<sup>&</sup>lt;sup>2</sup>These values have been obtained through simulation and provide error probabilities below 1% for the analyzed cases.



Figure 2: Modulation Classifier.

stages. Firstly, a linear model for the IF is computed following the least-squares criterion to investigate whether the signal belongs to the LFM class. Then, the model error, i.e. the squared norm of the residuals or the sum of squared errors, is compared with a threshold,  $th_e$ . Secondly, to analyze PSK and FSK modulation, the method in [9] is used. This method consists of the median filtering of the IF, and the estimation of its variance. The median filter removes the impulsive transients due to phase changes, so that, for PSK modulation, the filtered IF becomes fairly constant and the variance very low. In the case of FSK, the median filter cannot suppress the changes of frequency, which are like steps in the IF, and the variance is higher. By means of a threshold,  $th_v$ , the discrimination between PSK and FSK is performed, as can be seen in Fig. 2 where the block diagram for the modulation classifier is shown.

## 4 SIMULATION RESULTS

The performance of the presented MC is illustrated by means of three signals whose characteristics have been taken out from real radar and communications systems. The first one is a radar chirped pulse sweeping 41 MHz in 4.8  $\mu s$ . The second one is a *Barker-13* phase-coded pulse of 5  $\mu s$  width. The third one is a 2-FSK signal with 136 ns per symbol and frequency separation equal to 3 MHz. For these signals, a frame of 1024 samples have been used in the simulations (sampling rate: 250 MHz). For the modulation analysis, it has been found that  $th_e = 10^{-4}$ ,  $th_v = 5 \cdot 10^{-4}$  and a median filter of 101 taps lead to a good system performance in terms of classification error.

Fig. 3 shows the detection performance of AD for the three signals. The thresholds of the AD detection system (Fig. 1) provide  $10^{-6}$  of false alarm probability. The AD detection scheme is compared to other classical detectors based on the *Discrete Fourier Transform* (DFT) with 1024 samples, and on the *Short-Time Fourier Transform* plus 3 non-coherent parallel integration processes in parallel<sup>3</sup> (STFT+I) [10, 11]. For these detectors, the false alarm probability in the whole frame has been set to  $10^{-6}$ . On the whole, the three detectors exhibit similar performance, although AD is better for the chirped pulse. Unlike AD, DFT and STFT+I cannot provide an estimation of the signal components.

The performance of the proposed MC to identify the modulation of the three signals is very high even for low SNR. For the chirped pulse, the probability to detect LFM modulation is 1 (conditioned to signal detection). Besides, the



Figure 3: Detection performance of AD detection scheme.

 $<sup>^3</sup>$ A 1024-sample frame is also considered. STFT of 64 bins with a 256-tap window and 32-samples decimation factor. Integrators of length 1, 6 and 24.



(a) Barker-13



(b) 2-FSK

Figure 4: Probability of detecting the right modulation and error probability in the clustering. Both conditioned to detection.

error probability in the clustering conditioned to detection, i.e. obtaining more than one cluster, is zero. For the *Barker-13* and the 2-FSK signals, the probability to detect PSK and FSK modulation, respectively, is shown in Fig. 4 along with the very low error probability in the clustering.

In the former example, signals were alone. In this case, both LFM and PSK signals are added with the same power. The PSK signal is 20 MHz above the LFM one. The results are depicted in Fig. 5, where SNR refers to the SNR of each signal. The probability of detecting LFM or PSK modulation has been defined as the probability to find a cluster with such a modulation in the frequency band of the appropriate signal. The probability of detecting PSK is slightly better than that achieved for the *Barker-13* signal alone at SNR = 0, becoming apparent that AD is an adaptive approximation technique and, therefore, depends on the signal under analysis.

The error probability in the clustering, i.e. more than 2 clusters in this case, remains low. At very low SNR, the probability to detect only one atom is very high since it is easier to detect the chirped pulse than the *Barker-13* one, and the chirped one mainly comprises one atom.



Figure 5: Probability of detecting the right modulation for the addition of the chirped and the *Barker-13* pulses. (All probabilities conditioned to detection)

## 5 CONCLUSION

It has been demonstrated the suitability of the Atomic Decomposition for signal detection and modulation classification designing a system that comprises a detection and a classification stage. The last one includes separation of simultaneous signal by clustering and modulation analysis. From the simulation results, the system features good detection performance regarding Fourier-based detectors and high probability of modulation identification for low SNR, even for simultaneous signals. The error probability in the clustering also remains very low.

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