AN LCMV-BASED APPROACH FOR DOWNLINK BEAMFORMING IN FDD SYSTEMS IN PRESENCE OF ANGULAR SPREAD*

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ABSTRACT

An approach based on constrained filtering for downlink beamforming for FDD systems is presented. The impact of the angular spread in the calculation of the beamforming weights is evaluated for such approach, as well as for that based on the maximization of the signal to interference ratio. It is shown that the LCMV-based method leads to an efficient solution with the introduction of eigenvector constraints in the optimization procedure. The simulation results illustrate the good performance of the proposed technique, which presents an acceptable computational complexity and is suitable for adaptive implementation.

1 INTRODUCTION

In mobile communications the downlink beamforming plays an important role in the increasing of link quality and/or system capacity. Beamforming may be performed with or without cancellation of interferers. Further, the use of joint beamforming and cancellation provides better results [1].

In this context, the main drawback concerns the downlink channel parameters (reverse link) which are not available prior to transmission and must be estimated in order to provide a correct beamforming. Such parameters can be the spatial covariance matrix or the DOAs (directions of arrival) together with the angular spread (AS) and the powers of each multipath. For TDD systems both parameters (spatial covariance matrix and DOA) are the same in up- and downlink, provided that time duplex distance is small enough, i.e., smaller than the coherence time of fading. However, for FDD systems where frequency duplex distance is greater than coherence bandwidth, an useful approach is to use uplink parameters, obtained from received signal, to estimate the downlink ones [1, 2].

After the up- to downlink mapping, the beamforming weights may be obtained from the estimated covariance matrix by means of the Summed Inverse Carrier to interference Ratio (SICR) criterion, as posed in [2, 3]. An alternative strategy is the Linearly Constrained Minimum Variance (LCMV) criterion. Such method consists on the minimization of the transmitted power with point constraints in the array response, so that the directions of the desired signals be enhanced [4, 5]. In the original method of LCMV the angular spread (AS) was not considered in order to find the weights for beamforming. In this paper we investigate the inclusion of the AS on the LCMV-based solution by means of some additional considerations over the constraints to obtain satisfactory interference cancellation in presence of AS. The most suitable solution is the use of the so-called eigenvector constraints instead of the point one. AS can also be included in the previous method, based on the SCIR criterion. This allows us to evaluate the performance for both methods, providing comparative results. Such results indicate some performance equivalency between both LCMV-based and SICR methods in presence of angular spread (AS).

The paper is organized as follows. Section 2 presents the LCMV solution with the introduction of eigenvector constraints. Section 3 recalls the SICR criterion, so that the simulations and performance comparison for both methods are presented in Section 4. Finally, our conclusions are stated in Section 5.

2 LCMV-BASED SOLUTION

2.1 No Angular Spread

The LCMV is a criterion that minimizes both pollution[1] and interference. In addition, the array response is constrained to some values at defined directions in order to force the array to transmit for the desired user. This approach requires the knowledge of the DOA and the

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[1]By pollution we mean the transmitted power in all directions.

power of each user multipath, in order to compute the corresponding constraints.

For each user, the beamforming weights are obtained by

$$\mathbf{w}_k = \arg\min_{\mathbf{w}_k} \left\{ \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \right\} \mid \mathbf{C}_k^T \mathbf{w}_k = \mathbf{f}_k \tag{1}$$

where $\mathbf{R}$ is the total (for all users) downlink covariance matrix, $\mathbf{C}_k$ is the constraint matrix for the k-th user and $\mathbf{f}_k$ is the response vector for the k-th user.

Such method just takes into account the corresponding DOAs for each user, by introducing point constraints for each direction. That is, the signal (angular) spread around its DOA is not considered. The matrix $\mathbf{C}_k$ is formed as follows:

$$\mathbf{C}_k = \left[\begin{array}{cccc} \mathbf{d}(\theta_1) & \mathbf{d}(\theta_2) & \cdots & \mathbf{d}(\theta_L) \end{array} \right] \tag{2}$$

where $\mathbf{d}(\theta_l)$ is the steering vector corresponding to the l-th considered DOA and L is the number of constraints. The response vector is given by:

$$\mathbf{f}_k = \left[\begin{array}{cccc} 1 & \sqrt{\gamma_2} & \cdots & \sqrt{\gamma_L} \end{array} \right]^T \tag{3}$$

where γ_l is the relative power of the l-th path with respect to the first one.

2.2 Angular Spread - Point Constraints

To deal with AS, a number of point constraints must be applied over the angle bandwidth. However, the number of antenna in the array limits the number of constraints, since each constraint requires two antennas and a certain degree of freedom is necessary to minimize the transmitted power in other directions. So, there are two possibilities: the use of the mean DOA or considering a number of discrete DOAs in the angle bandwidth. Figure 1 depicts an example of 3 point constraints for a given AS.

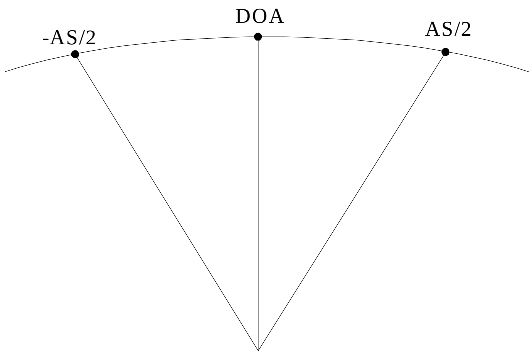

Figure 1: Point constraints for angular spread consideration.

A different strategy consists in avoiding point constraints, as presented in the sequel.

2.3 AS - Eigenvector Constraints

As mentioned in the previous section, the use of point constraints is limited to the number of degree of freedom. In order to increase the degree of freedom, the so-called eigenvector (EV) constraints have been introduced by [6, 5].

EV constraints proceed from a lower rank orthonormal representation of the signal space, based on the Karhunen-Loève discrete expansion [5]. Such representation is obtained using the set of eigenvectors corresponding to the most significant eigenvalues of the desired user spatial covariance matrix. This is the most efficient representation of the signal space, in the statistical second order sense.

Furthermore, it can be also shown that EV constraints are based in a minimum square approximation of the desired array response over the desired user angle bandwidth. The EV constraints are optima in the sense that the mean square error between the obtained array response and the desired one is minimized for a given number of constraints [7]. Hence the EV constraints provide a more direct control over the array response than the point constraints.

In practice, the singular value decomposition (SVD) is directly applied to the point constraints matrix and a more simple representation of $\mathbf{C}_k$ is given as follows:

$$\mathbf{C}_k^H = \mathbf{U}_k \left[\begin{array}{cc} \mathbf{\Sigma}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right] \mathbf{V}_k^H \tag{4}$$

and

$$\begin{aligned} \mathbf{U} &= \left[\begin{array}{cccc} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_L \end{array} \right] \\ \mathbf{V} &= \left[\begin{array}{cccc} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_M \end{array} \right] \\ \mathbf{\Sigma} &= \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_P) \end{aligned} \tag{5}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, containing the left and right singular vectors, respectively; and $\mathbf{\Sigma}$ is the diagonal matrix composed by the singular values of $\mathbf{C}_k$ sorted as $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_P > 0$.

For a given number $p \leq P$ of constraints, the matrices $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{\Sigma}$ are reduced to:

$$\begin{aligned} \widetilde{\mathbf{U}} &= \left[\begin{array}{cccc} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_p \end{array} \right] \\ \widetilde{\mathbf{V}} &= \left[\begin{array}{cccc} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_p \end{array} \right] \\ \widetilde{\mathbf{\Sigma}} &= \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_p) \end{aligned} \tag{6}$$

Thus the EV constraints matrix $\widetilde{\mathbf{C}}_k$ and EV response vector $\widetilde{\mathbf{f}}_k$ are given by:

$$\begin{aligned} \widetilde{\mathbf{C}}_k^H &= \widetilde{\mathbf{V}}^H \\ \widetilde{\mathbf{f}}_k &= \widetilde{\mathbf{\Sigma}}^{-1} \widetilde{\mathbf{U}}^H \mathbf{f}_k \end{aligned} \tag{7}$$

The $\mathrm{LCMV_{EV}}$ is then described by equations (6) and (7) applied to (1).

3 SICR SOLUTION

In [2, 3], a criterion for maximization of the carrier-to-interference ratio (CIR) for all co-channel users is proposed. The aim of this method is to maximize the transmitted power for the desired user and to cancel the others by placing spatial nulls in their direction. This criterion can be described as

$$\gamma_k = (\mathrm{CIR})_k = \max_{\mathbf{w}_1 \ldots \mathbf{w}_K} \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\sum\limits_{\substack{j=1 \\ j \neq k}}^{K} \mathbf{w}_j^H \mathbf{R}_k \mathbf{w}_j} \tag{8}$$

A sub-optimum solution for this problem can be found by means of a new criterion, defined as

$$\arg\max_{\mathbf{w}_1 \ldots \mathbf{w}_K} \left[\sum_{k=1}^{K} \gamma_k^{-1} \right] \tag{9}$$

With this new criterion, the sub-optimum beamforming weights can be independently performed as follows:

$$\mathbf{w}_k = \arg\max_{\mathbf{w}_k} \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{R}_{k,\mathrm{int}} \mathbf{w}_k} \tag{10}$$

where $\mathbf{R}_k$ and $\mathbf{R}_{k,\mathrm{int}} = \mathbf{R} - \mathbf{R}_k = \sum\limits_{i, i \neq k} \mathbf{R}_i + \sigma_n^2 \mathbf{I}$ are respectively the downlink covariance matrix of the k-th user and k-th's interferers plus noise, while $\mathbf{w}_k$ is the beamformer weight vector for the k-th user.

This minimization procedure can be solved by using Lagrange multipliers. The solution is the unit norm generalized eigenvector of $[\mathbf{R}_k, \mathbf{R}_{k,\mathrm{int}}]$ corresponding to the largest eigenvalue. Such criterion also corresponds to

$$\mathbf{w}_k = \arg\min_{\mathbf{w}_k} \left\{ \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \right\} \bigm| \mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k = c \tag{11}$$

where c is an arbitrary constant.

One can easily note that equations (1) and (11) are quite similar. Both criteria perform a minimization of $\mathbf{w}_k^H \mathbf{R} \mathbf{w}_k$, however LCMV imposes gain constraints while SICR imposes a flexible power constraint. The set of constraints in (1) supposes the knowledge of the k-th user DOAs, while its covariance matrix $\mathbf{R}_k$ is required in (11). These features make interesting a performance comparison between both criteria.

4 SIMULATION RESULTS

In order to evaluate the performance of the LCMV-based solution and compare it with the SICR solution, we have used the following wireless communication scenario, as depicted in Figure 2: one user and one interferer in a 120° sector, with two considered paths for each one and random DOAs for each path. The angular separation was set to 30°. A linear array with $M = 8$ antennas, spaced by half downlink carrier wavelength is considered. The other parameters are AS = 10°, CIR = 0dB, SNR = 20dB. We have simulated over 10^4 trials and computed the cumulative function distribution (CDF) of the beamforming quality (BQ) which is defined as:

$$\mathrm{BQ} = \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{R}_{k,\mathrm{int}} \mathbf{w}_k} \tag{12}$$

It worths to mention that BQ is a sort of signal to pollution ratio.

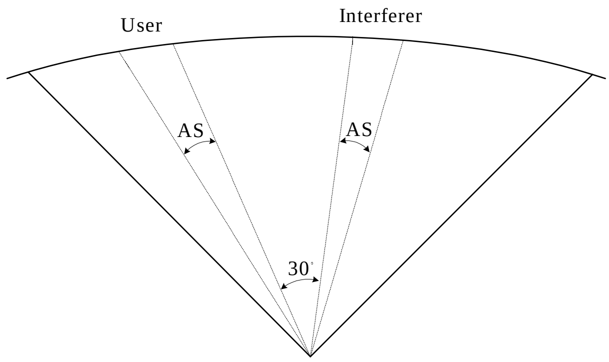

Figure 2: Simulation scenario.

The required parameters for finding the optimum beamforming weight vectors for both methods, i.e. DOA, $\mathbf{R}$ and $\mathbf{R}_k$ on downlink, are assumed to be perfectly known.

Figure 3 shows the comparison between the LCMV-based solutions for different AS values. As constraints we have considered: the mean DOAs (LMCV); 3 point constraints as depicted in Figure 1 (LCMV_3); and one EV constraint ($\mathrm{LCMV}_{\mathrm{EV1}}$), obtained from a constraints matrix formed by a 1° sampling over the angle bandwidth. It can be noted that the increasing of AS causes LCMV and LCMV_3 performance to decrease, while $\mathrm{LCMV}_{\mathrm{EV1}}$ performance remains practically unchanged.

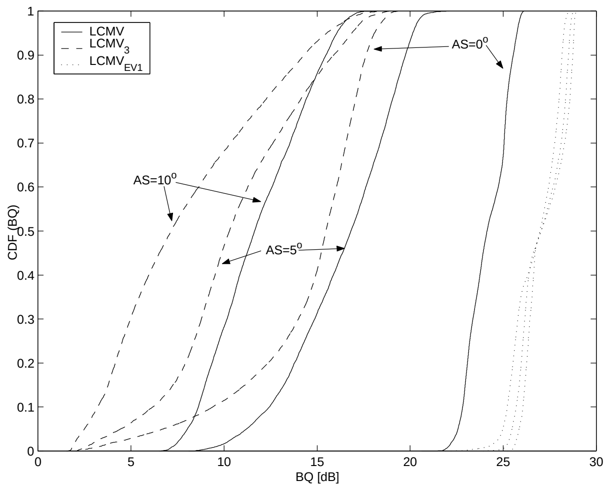

Figure 3: LCMV-based solutions performance comparison for AS $= 1°, 5°$and$10°$.

The behavior of the $\mathrm{LCMV}_{\mathrm{EV}}$ faced with different number of EV constraints compared with SICR is shown in Figure 4. One can easily see that the best performance is achieved with only one EV constraint, leading to a performance quite similar to the SICR. in fact, the performance of LCMV-based methods is related to the number of degrees of freedom, which depends on the number of constraints.

Finally, the SICR outperforms the $LCMV_{EV1}$ for all AS with a performance gap that increases with the AS, as shown in Figure 5.

5 CONCLUSIONS

An LCMV-based method for downlink beamforming was presented and compared with the SICR method. The method allows to consider the angular spread in the beamforming calculation, which is efficiently achieved by introducing eigenvector constraints in the optimization procedure. Moreover, a study into the number of constraints for the LCMV-based methods was provided. In fact, the performance of the point constrained beamforming has shown to be strongly dependent on the number of constraints, while the eigenvectorial method is much more robust. Besides, the best solution is achieved by the LCMV with only one eigenvector constraint.

When compared with the SICR, the $LCMV_{EV1}$ has shown a lower but satisfactory performance. Both methods have approximately the same computational complexity due to the eigenanalysis for the SICR and the singular value decomposition for the $LCMV_{EV1}$. However, the $LCMV_{EV1}$ solution is suitable for an adaptive version while the SICR does not have one yet.

Then, a natural extension of this work is the investigation of the adaptive solution for $LCMV_{EV1}$ in order to reduce the computational burden, specially when the interferers position change.

Finally, an interesting aspect to proceed with the investigation concerns the mathematical relations between both LCMC and SICR criteria. Clearly there is not a mathematical equivalence, but a number of situations where they present similar performance was verified. An interesting task seems to be the analytical derivation of the conditions for which such similarities hold.

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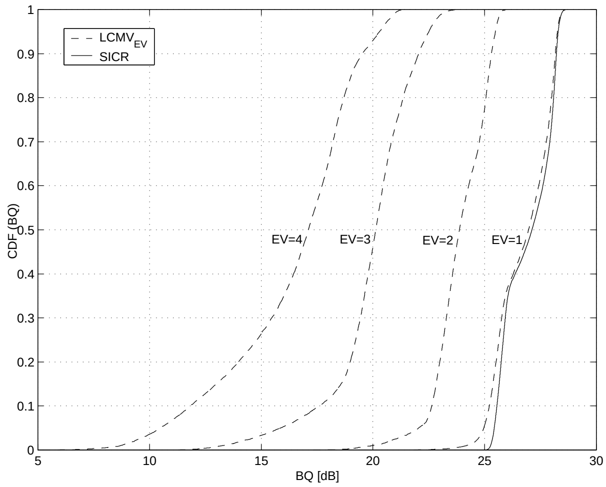

Figure 4: $LCMV_{EV}$ performance for $AS = 10°$ and different number of EV constraints.

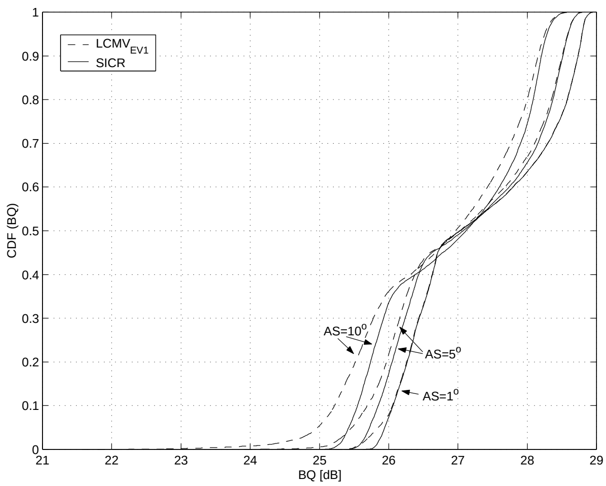

Figure 5: Comparison between the SICR and $LCMV_{EV1}$ for $AS = 1°, 5°$ and $10°$.